

# 6.2 PROPORTIONS

## Objectives

- A1** Write Sentences as Proportions.
- B1** Determine Whether Proportions Are True.
- C1** Find an Unknown Number in a Proportion.

### Objective **A1** Writing Proportions

A **proportion** is a statement that two ratios or rates are equal.

#### Proportion

A proportion states that two ratios are equal. If  $\frac{a}{b}$  and  $\frac{c}{d}$  are two ratios, then

$$\frac{a}{b} = \frac{c}{d}$$

is a proportion.

For example,

$$\frac{5}{6} = \frac{10}{12}$$

is a proportion. We can read this as, “5 is to 6 as 10 is to 12.”

**EXAMPLE 1** Write each sentence as a proportion.

- a. 12 diamonds is to 15 rubies as 4 diamonds is to 5 rubies.
- b. 5 hits is to 9 at bats as 20 hits is to 36 at bats.

#### Solution:

- a. diamonds  $\rightarrow$   $\frac{12}{15} = \frac{4}{5}$   $\leftarrow$  diamonds  
     rubies  $\rightarrow$   $\frac{15}{5}$   $\leftarrow$  rubies
- b. hits  $\rightarrow$   $\frac{5}{9} = \frac{20}{36}$   $\leftarrow$  hits  
     at bats  $\rightarrow$   $\frac{9}{36}$   $\leftarrow$  at bats

### **C1** Work Practice Problem 1

#### Helpful Hint

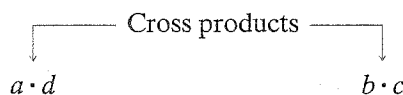
Notice in the above examples of proportions that the numerators contain the same units and the denominators contain the same units. In this text, proportions will be written so that this is the case.

### Objective **B1** Determining Whether Proportions Are True

Like other mathematical statements, a proportion may be either true or false. A proportion is true if its ratios are equal. Since ratios are fractions, one way to determine whether a proportion is true is to write both fractions in simplest form and compare them.

Another way is to compare cross products as we did in Section 4.2.

#### Using Cross Products to Determine Whether Proportions Are True or False



$$\frac{a}{b} = \frac{c}{d}$$

#### Answers

1. a.  $\frac{24}{6} = \frac{4}{1}$     b.  $\frac{560}{25} = \frac{112}{5}$

If cross products are *equal*, the proportion is *true*. If cross products are *not equal*, the proportion is *false*.

**EXAMPLE 2** Is  $\frac{2}{3} = \frac{4}{6}$  a true proportion?

**Solution:**

2 · 6	Cross products	3 · 4
↓		↓
$\frac{2}{3} = \frac{4}{6}$		

$2 \cdot 6 \stackrel{?}{=} 3 \cdot 4$  Are cross products equal?  
 $12 = 12$  Equal, so proportion is true

Since the cross products are equal, the proportion is true.

### Work Practice Problem 2

**EXAMPLE 3** Is  $\frac{4.1}{7} = \frac{2.9}{5}$  a true proportion?

**Solution:**

4.1 · 5	Cross products	7 · 2.9
↓		↓
$\frac{4.1}{7} = \frac{2.9}{5}$		

$4.1 \cdot 5 \stackrel{?}{=} 7 \cdot 2.9$  Are cross products equal?  
 $20.5 \neq 20.3$  Not equal, so proportion is false

Since the cross products are not equal,  $\frac{4.1}{7} \neq \frac{2.9}{5}$ . The proportion is false.

### Work Practice Problem 3

**EXAMPLE 4** Is  $\frac{1\frac{1}{6}}{10\frac{1}{2}} = \frac{1}{4\frac{1}{2}}$  a true proportion?

**Solution:**

$$\frac{1\frac{1}{6}}{10\frac{1}{2}} = \frac{1}{4\frac{1}{2}}$$

$1\frac{1}{6} \cdot 4\frac{1}{2} \stackrel{?}{=} 10\frac{1}{2} \cdot \frac{1}{2}$  Are cross products equal?

$\frac{7}{6} \cdot \frac{9}{2} \stackrel{?}{=} \frac{21}{2} \cdot \frac{1}{2}$  Write mixed numbers as improper fractions.

$\frac{21}{4} = \frac{21}{4}$  Equal, so proportion is true

Since the cross products are equal, the proportion is true.

### Work Practice Problem 4

**✓ Concept Check** Using the numbers in the proportion  $\frac{21}{27} = \frac{7}{9}$ , write two other true proportions.

### Objective Finding Unknown Numbers in Proportions

When one number of a proportion is unknown, we can use cross products to find the unknown number. For example, to find the unknown number  $x$  in the proportion

$\frac{2}{3} = \frac{x}{30}$ , we use cross products.

### PRACTICE PROBLEM 2

Is  $\frac{4}{8} = \frac{10}{20}$  a true proportion?

### PRACTICE PROBLEM 3

Is  $\frac{4.2}{6} = \frac{4.8}{8}$  a true proportion?

### PRACTICE PROBLEM 4

Is  $\frac{3\frac{3}{10}}{1\frac{5}{6}} = \frac{4\frac{1}{5}}{2\frac{1}{3}}$  a true proportion?

#### Answers

2. yes 3. no 4. yes

#### ✓ Concept Check Answer

possible answers:  $\frac{27}{21} = \frac{9}{7}$  and  $\frac{9}{27} = \frac{7}{21}$

## PRACTICE PROBLEM 5

Solve  $\frac{2}{5} = \frac{x}{25}$  for  $x$ .**EXAMPLE 5** Solve  $\frac{2}{3} = \frac{x}{30}$  for  $x$ .**Solution:** If the cross products are equal, then the proportion is true. We begin, then, by setting cross products equal to each other.

$$\frac{2}{3} = \frac{x}{30}$$

$$2 \cdot 30 = 3 \cdot x \quad \text{Set cross products equal.}$$

$$60 = 3x \quad \text{Multiply.}$$

Recall that to find  $x$ , we divide both sides of the equation by 3.

$$\frac{60}{3} = \frac{3x}{3} \quad \text{Divide both sides by 3.}$$

$$20 = x \quad \text{Simplify.}$$

**Check:** To check, we replace  $x$  with 20 in the original proportion to see if the result is a true statement.

$$\frac{2}{3} = \frac{x}{30} \quad \text{Original proportion}$$

$$\frac{2}{3} \stackrel{?}{=} \frac{20}{30} \quad \text{Replace } x \text{ with 20.}$$

$$\frac{2}{3} = \frac{2}{3} \quad \text{True}$$

Since  $\frac{2}{3} = \frac{2}{3}$  is a true statement, 20 is the solution.**Work Practice Problem 5**

## PRACTICE PROBLEM 6

Solve  $\frac{15}{2} = \frac{60}{x}$  for  $x$ .**EXAMPLE 6** Solve  $\frac{51}{34} = \frac{-3}{x}$  for  $x$ .**Solution:**

$$\frac{51}{34} = \frac{-3}{x}$$

$$51 \cdot x = 34 \cdot -3 \quad \text{Set cross products equal.}$$

$$51x = -102 \quad \text{Multiply.}$$

$$\frac{51x}{51} = \frac{-102}{51} \quad \text{Divide both sides by 51.}$$

$$x = -2 \quad \text{Simplify.}$$

$$\text{Check: } \frac{51}{34} = \frac{-3}{x} \quad \text{Original proportion}$$

$$\frac{51}{34} \stackrel{?}{=} \frac{-3}{-2} \quad \text{Replace } x \text{ with } -2.$$

$$\frac{51}{34} \stackrel{?}{=} \frac{-3 \cdot -17}{-2 \cdot -17}$$

$$\frac{51}{34} = \frac{51}{34} \quad \text{True}$$

**Work Practice Problem 6**

## Answers

5. 10   6. 8

**EXAMPLE 7**

Solve for  $y$ :  $\frac{1}{2} = \frac{3}{4y}$

**Solution:**

$$\frac{1}{2} = \frac{3}{4y}$$

$$\frac{1}{2} \cdot y = \frac{4}{5} \cdot \frac{3}{4}$$
 Set cross products equal.

$$\frac{1}{2}y = \frac{3}{5}$$
 Multiply.

$$2 \cdot \frac{1}{2}y = 2 \cdot \frac{3}{5}$$
 Multiply both sides by 2.

$$y = \frac{6}{5}$$
 Simplify.

Verify that  $\frac{6}{5}$  is the solution.**Check:** We will check by cross products.

$$\frac{1}{2} = \frac{3}{4y}$$

$$\frac{1}{2} \stackrel{?}{=} \frac{3}{4 \cdot \frac{6}{5}}$$

Replace  $y$  with  $\frac{6}{5}$ .

$$\frac{1}{2} \cdot \frac{6}{5} \stackrel{?}{=} \frac{4}{5} \cdot \frac{3}{4}$$
 Set cross products equal.

$$\frac{3}{5} = \frac{3}{5}$$
 True

True, so the solution is  $\frac{6}{5}$ .**Work Practice Problem 7****EXAMPLE 8**

Solve for  $x$ :  $\frac{x}{3} = \frac{0.8}{1.5}$

**Solution:**

$$\frac{x}{3} = \frac{0.8}{1.5}$$

$$x \cdot 1.5 = 3(0.8)$$
 Set cross products equal.

$$1.5x = 2.4$$
 Multiply.

$$\frac{1.5x}{1.5} = \frac{2.4}{1.5}$$
 Divide both sides by 1.5.

$$x = 1.6$$
 Simplify.

**Check:**

$$\frac{x}{3} = \frac{0.8}{1.5}$$

$$\frac{1.6}{3} \stackrel{?}{=} \frac{0.8}{1.5}$$
 Replace  $x$  with 1.6.

$$(1.6)(1.5) \stackrel{?}{=} 3(0.8)$$
 Set cross products equal.

$$2.4 = 2.4$$
 True

True, so the solution is 1.6.

**Work Practice Problem 8****PRACTICE PROBLEM 7**Solve for  $z$ :

$$\frac{7}{8} = \frac{2}{\frac{3}{4z}}$$

**PRACTICE PROBLEM 8**

Solve for  $y$ :  $\frac{y}{9} = \frac{0.6}{1.2}$

**Answers**

7.  $\frac{3}{4}$  8. 4.5

**PRACTICE PROBLEM 9**Solve for  $z$ :  $\frac{17}{z} = \frac{8}{10}$ **EXAMPLE 9** Solve for  $y$ :  $\frac{14}{y} = \frac{12}{16}$ **Solution:**

$$\frac{14}{y} = \frac{12}{16}$$

$$14 \cdot 16 = y \cdot 12 \quad \text{Set cross products equal.}$$

$$224 = 12y \quad \text{Multiply.}$$

$$\frac{224}{12} = \frac{12y}{12} \quad \text{Divide both sides by 12.}$$

$$\frac{56}{3} = y \quad \text{Simplify.}$$

Check to see that the solution is  $\frac{56}{3}$ .**Work Practice Problem 9****Helpful Hint**In Example 9, the fraction  $\frac{12}{16}$  may be simplified to  $\frac{3}{4}$  before solving the equation. The solution will remain the same.**✓ Concept Check** True or false: the first step in solving the proportion  $\frac{4}{z} = \frac{12}{15}$  yields the equation  $4z = 180$ . If false, give the correct cross product equation.**PRACTICE PROBLEM 10**Solve for  $y$ :  $\frac{4.5}{1.8} = \frac{y}{3}$ . Round the solution to the nearest tenth.**EXAMPLE 10** Solve for  $x$ :  $\frac{1.6}{1.1} = \frac{x}{0.3}$ . Round the solution to the nearest hundredth.**Solution:**

$$\frac{1.6}{1.1} = \frac{x}{0.3}$$

$$(1.6)(0.3) = 1.1 \cdot x \quad \text{Set cross products equal.}$$

$$0.48 = 1.1x \quad \text{Multiply.}$$

$$\frac{0.48}{1.1} = \frac{1.1x}{1.1} \quad \text{Divide both sides by 1.1.}$$

$$0.44 \approx x \quad \text{Round to the nearest hundredth.}$$

**Work Practice Problem 10****Answers**9.  $\frac{85}{4}$  or 21.25    10. 7.5**✓ Concept Check Answer**false;  $60 = 12z$ .

## Vocabulary and Readiness Check

Use the words and phrases below to fill in each blank.

ratio      cross products      true  
false      proportion

- $\frac{4.2}{8.4} = \frac{1}{2}$  is called a \_\_\_\_\_ while  $\frac{7}{8}$  is called a \_\_\_\_\_.
- In  $\frac{a}{b} = \frac{c}{d}$ ,  $a \cdot d$  and  $b \cdot c$  are called \_\_\_\_\_.
- In a proportion, if cross products are equal, the proportion is \_\_\_\_\_.
- In a proportion, if cross products are not equal, the proportion is \_\_\_\_\_.

## 6.2 EXERCISE SET

FOR EXTRA HELP



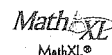
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Objective **A** **Translating** Write each sentence as a proportion. See Example 1.

- 10 diamonds is to 6 opals as 5 diamonds is to 3 opals.      2. 1 raisin is to 5 cornflakes as 8 raisins is to 40 cornflakes.
3. 20 students is to 5 microscopes as 4 students is to 1 microscope.      4. 4 hit songs is to 16 releases as 1 hit song is to 4 releases.
5. 6 eagles is to 58 sparrows as 3 eagles is to 29 sparrows.      6. 12 errors is to 8 pages as 1.5 errors is to 1 page.
7.  $2\frac{1}{4}$  cups of flour is to 24 cookies as  $6\frac{3}{4}$  cups of flour is to 72 cookies.      8.  $1\frac{1}{2}$  cups milk is to 10 bagels as  $\frac{3}{4}$  cup milk is to 5 bagels.
9. 22 vanilla wafers is to 1 cup of cookie crumbs as 55 vanilla wafers is to 2.5 cups of cookie crumbs. (Source: Based on data from *Family Circle* magazine)      10. 1 cup of instant rice is to 1.5 cups cooked rice as 1.5 cups of instant rice is to 2.25 cups of cooked rice. (Source: Based on data from *Family Circle* magazine)

Objective **B** Determine whether each proportion is true or false. See Examples 2 through 4.

11.  $\frac{15}{9} = \frac{5}{3}$
12.  $\frac{8}{6} = \frac{20}{15}$
13.  $\frac{5}{8} = \frac{4}{7}$
14.  $\frac{7}{3} = \frac{9}{5}$
15.  $\frac{9}{36} = \frac{2}{8}$
16.  $\frac{8}{24} = \frac{3}{9}$
17.  $\frac{5}{8} = \frac{625}{1000}$
18.  $\frac{30}{50} = \frac{600}{1000}$

19.  $\frac{0.8}{0.3} = \frac{0.2}{0.6}$

20.  $\frac{0.7}{0.4} = \frac{0.3}{0.1}$

21.  $\frac{4.2}{8.4} = \frac{5}{10}$

22.  $\frac{8}{10} = \frac{5.6}{0.7}$

23.  $\frac{\frac{3}{4}}{\frac{4}{3}} = \frac{\frac{1}{2}}{\frac{8}{9}}$

24.  $\frac{\frac{2}{3}}{\frac{8}{5}} = \frac{\frac{1}{7}}{\frac{3}{10}}$

25.  $\frac{\frac{2\frac{2}{5}}{2}}{\frac{5}{3}} = \frac{\frac{10}{9}}{\frac{1}{4}}$

26.  $\frac{\frac{5\frac{5}{8}}{5}}{\frac{3}{3}} = \frac{4\frac{1}{2}}{1\frac{1}{5}}$

27.  $\frac{\frac{4}{5}}{\frac{5}{6}} = \frac{\frac{6}{5}}{\frac{5}{9}}$

28.  $\frac{\frac{6}{7}}{\frac{7}{3}} = \frac{\frac{10}{7}}{\frac{5}{5}}$

Objectives **A** **B** **Mixed Practice—Translating** Write each sentence as a proportion. Then determine whether the proportion is a true proportion. See Examples 1 through 4.

29. ten is to fifteen as four is to six

30. six is to eight as nine is to twelve

31. eleven is to four as five is to two

32. five is to three as seven is to five

33. fifteen hundredths is to three as thirty-five hundredths is to seven

34. one and eight tenths is to two as four and five tenths is to five

35. two thirds is to one fifth as two fifths is to one ninth

36. ten elevenths is to three fourths as one fourth is to one half

Objective **C** Solve each proportion for the given variable. Round the solution where indicated. See Examples 5 through 10.

37.  $\frac{x}{5} = \frac{6}{10}$

38.  $\frac{x}{3} = \frac{12}{9}$

39.  $\frac{-18}{54} = \frac{3}{n}$

40.  $\frac{25}{100} = \frac{-7}{n}$

41.  $\frac{30}{10} = \frac{15}{y}$

42.  $\frac{16}{20} = \frac{z}{35}$

43.  $\frac{8}{15} = \frac{z}{6}$

44.  $\frac{12}{10} = \frac{z}{16}$

45.  $\frac{24}{x} = \frac{60}{96}$

46.  $\frac{26}{x} = \frac{28}{49}$

47.  $\frac{-3.5}{12.5} = \frac{-7}{n}$

48.  $\frac{-0.2}{0.7} = \frac{-8}{n}$

49.  $\frac{n}{0.6} = \frac{0.05}{12}$

50.  $\frac{7.8}{13} = \frac{n}{2.6}$

51.  $\frac{8}{\frac{1}{3}} = \frac{24}{n}$

52.  $\frac{12}{\frac{3}{4}} = \frac{48}{n}$

53.  $\frac{\frac{1}{3}}{\frac{3}{8}} = \frac{\frac{2}{5}}{n}$

54.  $\frac{\frac{7}{9}}{\frac{8}{27}} = \frac{\frac{1}{4}}{n}$

55.  $\frac{12}{n} = \frac{\frac{2}{3}}{\frac{6}{9}}$

56.  $\frac{24}{n} = \frac{\frac{8}{15}}{\frac{5}{9}}$

57.  $\frac{n}{1\frac{1}{5}} = \frac{4\frac{1}{6}}{6\frac{2}{3}}$

58.  $\frac{n}{3\frac{1}{8}} = \frac{7\frac{3}{5}}{2\frac{3}{8}}$

59.  $\frac{25}{n} = \frac{3}{\frac{7}{30}}$

60.  $\frac{9}{n} = \frac{5}{\frac{11}{15}}$

61.  $\frac{3.2}{0.3} = \frac{x}{1.4}$

Round to the nearest tenth.

62.  $\frac{1.8}{z} = \frac{2.5}{8.4}$

Round to the nearest tenth.

63.  $\frac{z}{5.2} = \frac{0.08}{6}$

Round to the nearest hundredth.

64.  $\frac{4.25}{6.03} = \frac{5}{y}$

Round to the nearest hundredth.

65.  $\frac{7}{18} = \frac{x}{5}$

Round to the nearest tenth.

66.  $\frac{17}{x} = \frac{9}{4}$

Round to the nearest thousandth.

67.  $\frac{43}{17} = \frac{8}{z}$

Round to the nearest thousandth.

68.  $\frac{x}{12} = \frac{18}{7}$

Round to the nearest hundredth.

## Review

Insert  $<$  or  $>$  to form a true statement. See Sections 4.7 and 5.1.

69.  $8.01 < 8.1$

70.  $7.26 < 7.026$

71.  $2\frac{1}{2} < 2\frac{1}{3}$

72.  $9\frac{1}{5} < 9\frac{1}{4}$

Simplify each fraction. See Section 4.2.

73.  $\frac{75}{125}$

74.  $\frac{11y}{99y}$

75.  $\frac{12x}{42}$

76.  $\frac{28y^2}{42y^3}$

## Concept Extensions

Use the numbers in each proportion to write two other true proportions. See the first Concept Check in this section.

77.  $\frac{9}{15} = \frac{3}{5}$

78.  $\frac{1}{4} = \frac{5}{20}$

79.  $\frac{6}{18} = \frac{1}{3}$

80.  $\frac{2}{7} = \frac{4}{14}$

81. If the proportion  $\frac{a}{b} = \frac{c}{d}$  is a true proportion, write two other true proportions using the same letters.

82. Write a true proportion.



83. Explain the difference between a ratio and a proportion.

84. Explain how to find the unknown number in a proportion such as  $\frac{n}{18} = \frac{12}{8}$ .

For each proportion, solve for the variable.

$$85. \frac{x}{7} = \frac{0}{8}$$

$$86. \frac{0}{2} = \frac{y}{3.5}$$

$$87. \frac{z}{1150} = \frac{588}{483}$$

$$88. \frac{585}{x} = \frac{117}{474}$$

$$89. \frac{222}{1515} = \frac{37}{y}$$

$$90. \frac{1425}{1062} = \frac{z}{177}$$



## THE BIGGER PICTURE Operations on Sets of Numbers and Solving Equations

Continue your outline from Sections 1.6, 1.7, 2.4, 3.3, 4.3, 4.7, and 5.4. Suggestions are once again written to help you complete this part of your outline. Notice that this part of the outline has to do with solving a certain type of equation, proportions.

### I. Operations on Sets of Numbers

#### A. Whole Numbers

1. Add or Subtract (Section 1.3)
2. Multiply or Divide (Sections 1.5, 1.6)
3. Exponent (Section 1.7)
4. Order of Operations (Section 1.7)

#### B. Integers

1. Add (Section 2.2)
2. Subtract (Section 2.3)
3. Multiply or Divide (Section 2.4)

#### C. Fractions

1. Simplify (Section 4.2)
2. Multiply (Section 4.3)
3. Divide (Section 4.3)
4. Add or Subtract (Sections 4.4, 4.5)

#### D. Decimals

1. Add or Subtract (Section 5.2)
2. Multiply (Section 5.3)
3. Divide (Section 5.4)

### II. Solving Equations

#### A. Equations in General (Section 3.3)

**B. Proportions:** Set cross products equal to each other. Then solve.

$$\frac{14}{3} = \frac{2}{n}, \text{ or } 14 \cdot n = 3 \cdot 2, \text{ or } 14n = 6, \text{ or } n = \frac{6}{14} = \frac{3}{7}$$

Perform the indicated operations.

$$1. \frac{7}{20} - \frac{1}{10}$$

$$2. \frac{7}{20} \cdot \frac{1}{10}$$

$$3. \frac{7}{20} \div \frac{1}{10}$$

$$4. \frac{7}{20} + \frac{1}{10}$$

$$5. 7.6 + 0.02$$

$$6. 7.6(0.02)$$

For each proportion, find the unknown number,  $n$ .

$$7. \frac{4}{n} = \frac{50}{100}$$

$$8. \frac{60}{10} = \frac{15}{n}$$

$$9. \frac{n}{0.8} = \frac{0.06}{12}$$

$$10. \frac{7}{8} = \frac{n}{5}$$

$$\frac{1}{4} = \frac{n}{6}$$

# INTEGRATED REVIEW

## Sections 6.1–6.2

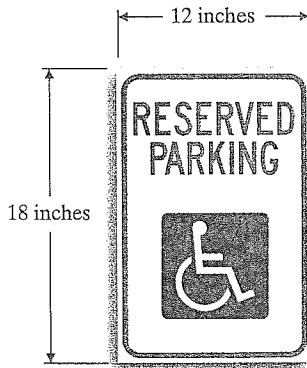
### Ratio, Rate, and Proportion

Write each ratio as a ratio of whole numbers using fractional notation. Write the fraction in simplest form.

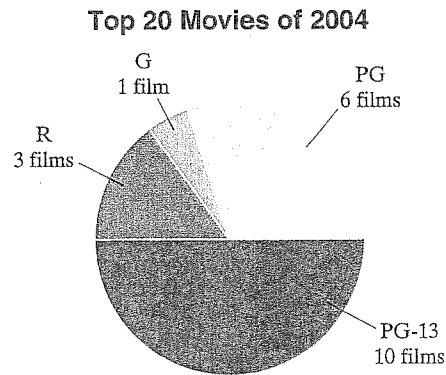
- |                           |                         |                        |                                     |
|---------------------------|-------------------------|------------------------|-------------------------------------|
| 1. 27 to 30               | 2. 18 to 50             | 3. 9.4 to 10           | 4. 3.2 to 9.2                       |
| 5. 8.65 to 6.95           | 6. 3.6 to 4.2           | 7. $\frac{7}{2}$ to 13 | 8. $1\frac{2}{3}$ to $2\frac{3}{4}$ |
| 9. 16 inches to 24 inches | 10. 5 hours to 40 hours |                        |                                     |

Find the ratio described in each problem.

11. Find the ratio of the width (shorter side) to the length (longer side) of the sign below.



12. The circle graph below shows how the top 20 movies of 2004 were rated. Use this graph to answer the questions.
- How many top 20 movies were rated PG-13?
  - Find the ratio of top 20 PG-13 movies to total movies for that year.



Source: Internet search and Bryan Gay

Write each rate as a fraction in simplest form.

- |   |                                       |
|---|---------------------------------------|
| 13. 4 professors for every 20 graduate assistants | 14. 6 lights every 20 feet            |
| 15. 100 U.S. Senators for 50 states               | 16. 5 teachers for every 140 students |
| 17. 21 inches every 7 seconds                     | 18. \$40 every 5 hours                |

### Answers

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34. \_\_\_\_\_
35. \_\_\_\_\_
36. \_\_\_\_\_
37. \_\_\_\_\_
38. \_\_\_\_\_

19. 76 households with computers for every 100 households

20. 538 electoral votes for 50 states

*Write each rate as a unit rate.*

21. 560 feet in 4 seconds

22. 195 miles in 3 hours

23. 63 employees per 3 fax lines

24. 85 phone calls for 5 teenagers

25. 156 miles per 6 gallons

26. 112 teachers for 7 computers

27. 8125 books for 1250 college students

28. 2310 pounds for 14 adults

*Write each unit price and decide which is the better buy. Round to 3 decimal places.*

29. Cat food:  
8 pounds for \$2.16  
18 pounds for \$4.99

30. Paper plates:  
100 for \$1.98  
500 for \$8.99

31. Microwave popcorn:  
3 packs for \$2.39  
8 packs for \$5.99

32. AA batteries:  
4 for \$3.69  
10 for \$9.89

*Determine whether each proportion is true.*

33.  $\frac{7}{4} = \frac{5}{3}$

34.  $\frac{8.2}{2} = \frac{16.4}{4}$

*Solve for the variable.*

35.  $\frac{5}{3} = \frac{40}{x}$

36.  $\frac{y}{10} = \frac{13}{4}$

37.  $\frac{6}{11} = \frac{z}{5}$

38.  $\frac{21}{x} = \frac{7}{3}$

# 6.3 PROPORTIONS AND PROBLEM SOLVING

## Objective

**A** Solve Problems by Writing Proportions.

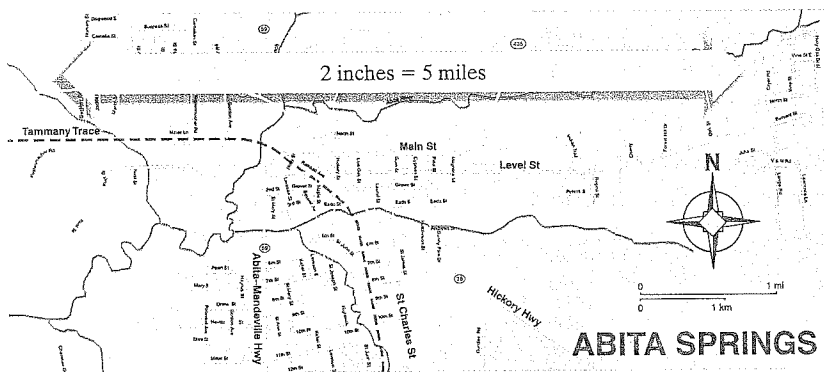
### Objective **A** Solving Problems by Writing Proportions

Writing proportions is a powerful tool for solving problems in almost every field, including business, chemistry, biology, health sciences, and engineering, as well as in daily life. Given a specified ratio (or rate) of two quantities, a proportion can be used to determine an unknown quantity.

In this section, we use the same problem solving steps that we have used earlier in this text.

#### **EXAMPLE 1** Determining Distances from a Map

On a chamber of commerce map of Abita Springs, 5 miles corresponds to 2 inches. How many miles correspond to 7 inches?

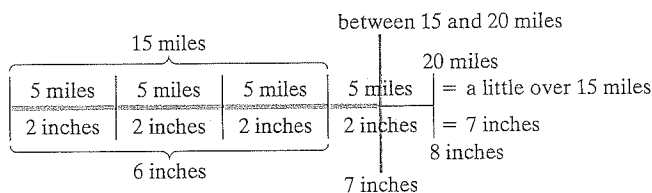


#### PRACTICE PROBLEM 1

On an architect's blueprint, 1 inch corresponds to 4 feet. How long is a wall represented by a  $4\frac{1}{4}$ -inch line on the blueprint?

#### Solution:

1. UNDERSTAND. Read and reread the problem. You may want to draw a diagram.



From the diagram we can see that a reasonable solution should be between 15 and 20 miles.

2. TRANSLATE. We will let  $x$  be our unknown number. Since 5 miles corresponds to 2 inches as  $x$  miles corresponds to 7 inches, we have the proportion
 
$$\begin{array}{l} \text{miles} \rightarrow \frac{5}{2} = \frac{x}{7} \leftarrow \text{miles} \\ \text{inches} \rightarrow \end{array}$$
3. SOLVE: In earlier sections, we estimated to obtain a reasonable answer. Notice we did this in Step 1 above.

$$\frac{5}{2} = \frac{x}{7}$$

$5 \cdot 7 = 2 \cdot x$  Set the cross products equal to each other.

$35 = 2x$  Multiply.

$\frac{35}{2} = \frac{2x}{2}$  Divide both sides by 2.

$17\frac{1}{2} = x$  or  $x = 17.5$  Simplify.

#### Answer

1. 17 ft

Continued on next page

4. **INTERPRET.** Check your work. This result is reasonable since it is between 15 and 20 miles. State your conclusion: 7 inches corresponds to 17.5 miles.

**Work Practice Problem 1**

**Helpful Hint**

We can also solve Example 1 by writing the proportion

$$\frac{2 \text{ inches}}{5 \text{ miles}} = \frac{7 \text{ inches}}{x \text{ miles}}$$

Although other proportions may be used to solve Example 1, we will solve by writing proportions so that the numerators have the same unit measures and the denominators have the same unit measures.

**PRACTICE PROBLEM 2**

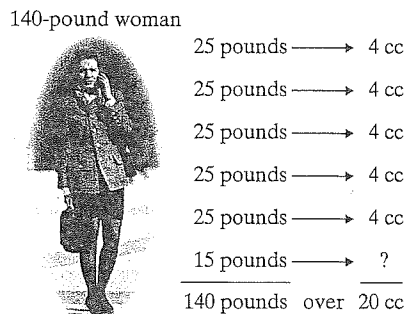
An auto mechanic recommends that 5 ounces of isopropyl alcohol be mixed with a tankful of gas (16 gallons) to increase the octane of the gasoline for better engine performance. At this rate, how many gallons of gas can be treated with a 8-ounce bottle of alcohol?

**EXAMPLE 2 Finding Medicine Dosage**

The standard dose of an antibiotic is 4 cc (cubic centimeters) for every 25 pounds (lb) of body weight. At this rate, find the standard dose for a 140-lb woman.

**Solution:**

1. **UNDERSTAND.** Read and reread the problem. You may want to draw a diagram to estimate a reasonable solution.



From the diagram, we can see that a reasonable solution is a little over 20 cc.

2. **TRANSLATE.** We will let  $x$  be the unknown number. From the problem, we know that 4 cc is to 25 pounds as  $x$  cc is to 140 pounds, or

$$\begin{array}{ccc} \text{cubic centimeters} & \rightarrow & 4 & = & \frac{x}{140} & \leftarrow & \text{cubic centimeters} \\ \text{pounds} & \rightarrow & 25 & = & 140 & \leftarrow & \text{pounds} \end{array}$$

3. **SOLVE:**

$$\frac{4}{25} = \frac{x}{140}$$

$$4 \cdot 140 = 25 \cdot x$$

Set the cross products equal to each other.

$$560 = 25x$$

Multiply.

$$\frac{560}{25} = \frac{25x}{25}$$

Divide both sides by 25.

$$22\frac{2}{5} = x \text{ or } x = 22.4 \quad \text{Simplify.}$$

4. **INTERPRET.** Check your work. This result is reasonable since it is a little over 20 cc. State your conclusion: The standard dose for a 140-lb woman is 22.4 cc.

**Work Practice Problem 2**

**Answer**

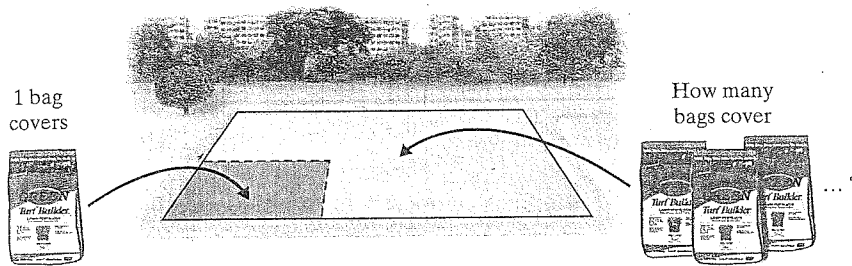
2.  $25\frac{3}{5}$  or 25.6 gal

**EXAMPLE 3** Calculating Supplies Needed to Fertilize a Lawn

A 50-pound bag of fertilizer covers 2400 square feet of lawn. How many bags of fertilizer are needed to cover a town square containing 15,360 square feet of lawn? Round the answer up to the nearest whole bag.

**Solution:**

1. **UNDERSTAND.** Read and reread the problem. Draw a picture.



Since one bag covers 2400 square feet, let's see how many 2400s there are in 15,360. We will estimate. The number 15,360 rounded to the nearest thousand is 15,000 and 2400 rounded to the nearest thousand is 2000. Then

$$15,000 \div 2000 = 7\frac{1}{2} \text{ or } 7.5.$$

2. **TRANSLATE.** We'll let  $x$  be the unknown number. From the problem, we know that 1 bag is to 2400 square feet as  $x$  bags is to 15,360 square feet.

$$\begin{array}{lcl} \text{bags} & \rightarrow & \frac{1}{2400} = \frac{x}{15,360} \leftarrow \text{bags} \\ \text{square feet} & \rightarrow & \end{array}$$

3. **SOLVE:**

$$\frac{1}{2400} = \frac{x}{15,360}$$

$$1 \cdot 15,360 = 2400 \cdot x \quad \text{Set the cross products equal to each other.}$$

$$15,360 = 2400 \cdot x \quad \text{Multiply.}$$

$$\frac{15,360}{2400} = \frac{2400x}{2400} \quad \text{Divide both sides by 2400.}$$

$$6.4 = x \quad \text{Simplify.}$$

4. **INTERPRET.** Check that replacing  $x$  with 6.4 makes the proportion true. Is the answer reasonable? Yes, since it's close to  $7\frac{1}{2}$  or 7.5. Since we must buy whole bags of fertilizer, 7 bags are needed. State your conclusion: To cover 15,360 square feet of lawn, 7 bags are needed.

**Work Practice Problem 3**

✓ **Concept Check** You are told that 12 ounces of ground coffee will brew enough coffee to serve 20 people. How could you estimate how much ground coffee will be needed to serve 95 people?

**PRACTICE PROBLEM 3**

If a gallon of paint covers 450 square feet, how many gallons are needed to paint a retaining wall that is 270 feet long and 11 feet high? Round the answer up to the nearest whole gallon.

**Answer**

3. 7 gal

✓ **Concept Check Answer**

Find how much will be needed for 100 people ( $20 \times 5$ ) by multiplying 12 ounces by 5, which is 60 ounces.

# 6.3 EXERCISE SET

## FOR EXTRA HELP



Student Solutions Manual



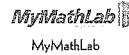
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Objective **A** Solve. See Examples 1 through 3. An NBA basketball player averages 45 baskets for every 100 attempts.

1. If he attempted 800 field goals, how many baskets did he make?
2. If he completed 225 baskets, how many did he attempt?

It takes a word processor 30 minutes to word process and spell check 4 pages.

3. Find how long it takes her to word process and spell check 22 pages.
4. Find how many pages she can word process and spell check in 4.5 hours.

University Law School accepts 2 out of every 7 applicants.

5. If the school accepted 180 students, find how many applications they received.
6. If the school accepted 150 students, find how many applications were received.

On an architect's blueprint, 1 inch corresponds to 8 feet.

7. Find the length of a wall represented by a line  $2\frac{7}{8}$  inches long on the blueprint.
8. Find the length of a wall represented by a line  $5\frac{1}{4}$  inches on the blueprint.

A human-factors expert recommends that there be at least 9 square feet of floor space in a college classroom for every student in the class.

9. Find the minimum floor space that 30 students require.
10. Due to a lack of space, a university converts a 21-by-15-foot conference room into a classroom. Find the maximum number of students the room can accommodate.

A Honda Civic Hybrid car averages 627 miles on a 12.3 gallon tank of gas.

11. Manuel Lopez is planning a 1250-mile vacation trip in his Honda Civic Hybrid. Find how many gallons of gas he can expect to burn. Round to the nearest gallon.
12. Ramona Hatch has enough money to put 6.9 gallons of gas in her Honda Civic Hybrid. She is planning on driving home from college for the weekend. If her home is 290 miles away, should she make it home before she runs out of gas?

The scale on an Italian map states that 1 centimeter corresponds to 30 kilometers.

13. Find how far apart Milan and Rome are if their corresponding points on the map are 15 centimeters apart.
14. On the map, a small Italian village is located 0.4 centimeter from the Mediterranean Sea. Find the actual distance.



A bag of Scott fertilizer covers 3000 square feet of lawn.

15. Find how many bags of fertilizer should be purchased to cover a rectangular lawn 260 feet by 180 feet.

A Cubs baseball player gets 3 hits in every 8 times at bat.

17. If this Cubs player comes up to bat 40 times in the World Series, find how many hits he would be expected to get.

A survey reveals that 2 out of 3 people prefer Coke to Pepsi.

19. In a room of 40 people, how many people are likely to prefer Coke? Round the answer to the nearest person.

A self-tanning lotion advertises that a 3-oz bottle will provide four applications.

21. Jen Haddad found a great deal on a 14-oz bottle of the self-tanning lotion she had been using. Based on the advertising claims, how many applications of the self-tanner should Jen expect? Round down to the whole number.

The school's computer lab goes through 5 reams of printer paper every 3 weeks.

23. Find out how long a case of printer paper is likely to last (a case of paper holds 8 reams of paper). Round to the nearest week.

A recipe for pancakes calls for 2 cups flour and  $1\frac{1}{2}$  cup milk to make a serving for four people.

25. Ming has plenty of flour, but only 4 cups milk. How many servings can he make?

27. A 16-oz grande Tazo Black Iced Tea at Starbucks has 80 calories. How many calories are there in a 24-oz venti Tazo Black Iced Tea? (Source: Starbucks Coffee Company)

29. Mosquitos are annoying insects. To eliminate mosquito larvae, a certain granular substance can be applied to standing water in a ratio of 1 tsp per 25 sq ft of standing water.

- At this rate, find how many teaspoons of granules must be used for 450 square feet.
- If 3 tsp = 1 tbsp, how many tablespoons of granule must be used?

- △ 16. Find how many bags of fertilizer should be purchased to cover a square lawn measuring 160 feet on each side.

18. At this rate, if he got 12 hits, find how many times he batted.

20. In a college class of 36 students, find how many students are likely to prefer Pepsi.

22. The Community College thespians need fake tans for a play they are doing. If the play has a cast of 35, how many ounces of self-tanning lotion should the cast purchase? Round up to the next whole number ounces.

24. How many cases of printer paper should be purchased to last the entire semester of 15 weeks? Round to the next case.

26. The swim team has a weekly breakfast after early practice. How much flour will it take to make pancakes for 18 swimmers?

28. A 16-oz nonfat Caramel Macchiato at Starbucks has 220 calories. How many calories are there in a 12-oz nonfat Caramel Macchiato? (Source: Starbucks Coffee Company)

30. Another type of mosquito control is liquid, where 3 oz of pesticide is mixed with 100 oz of water. This mixture is sprayed on roadsides to control mosquito breeding grounds hidden by tall grass.

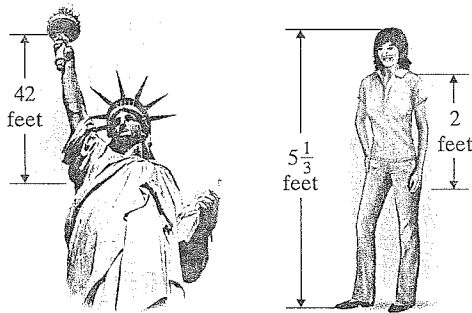
- If one mixture of water with this pesticide can treat 150 feet of roadway, how many ounces of pesticide are needed to treat one mile? (Hint: 1 mile = 5280 feet)
- If 8 liquid ounces equals one cup, write your answer to part a in cups. Round to the nearest cup.



31. The daily supply of oxygen for one person is provided by 625 square feet of lawn. A total of 3750 square feet of lawn would provide the daily supply of oxygen for how many people? (Source: Professional Lawn Care Association of America)

33. A student would like to estimate the height of the Statue of Liberty in New York City's harbor. The length of the Statue of Liberty's right arm is 42 feet. The student's right arm is 2 feet long and her height is  $5\frac{1}{3}$  feet. Use this information to estimate the

height of the Statue of Liberty. How close is your estimate to the statue's actual height of 111 feet, 1 inch from heel to top of head? (Source: National Park Service)



35. There are 72 milligrams of cholesterol in a 3.5 ounce serving of lobster. How much cholesterol is in 5 ounces of lobster? Round to the nearest tenth of a milligram. (Source: The National Institute of Health)
37. Trump World Tower in New York City is 881 feet tall and contains 72 stories. The Empire State Building contains 102 stories. If the Empire State Building has the same number of feet per floor as the Trump World Tower, approximate its height rounded to the nearest foot. (Source: skyscrapers.com)
39. Medication is prescribed in 7 out of every 10 hospital emergency room visits that involve an injury. If a large urban hospital had 620 emergency room visits involving an injury in the past month, how many of these visits would you expect included a prescription for medication? (Source: National Center for Health Statistics)
41. One out of three American adults has worked in the restaurant industry at some point during his or her life. In an office of 84 workers, how many of these people would you expect to have worked in the restaurant industry at some point? (Source: National Restaurant Association)

32. In the United States, approximately 71 million of the 200 million cars and light trucks in service have driver-side air bags. In a parking lot containing 800 cars and light trucks, how many would be expected to have driver-side air bags? (Source: Insurance Institute for Highway Safety)

34. The length of the Statue of Liberty's index finger is 8 feet while the height to the top of the head is about 111 feet. Suppose your measurements are proportionally the same as this statue and your height is 5 feet.
- Use this information to find the proposed length of your index finger. Give an exact measurement and then a decimal rounded to the nearest hundredth.
  - Measure your index finger and write it as decimal in feet rounded to the nearest hundredth. How close is the length of your index finger to the answer to a? Explain why.

36. There are 76 milligrams of cholesterol in a 3-ounce serving of skinless chicken. How much cholesterol is in 8 ounces of chicken? (Source: USDA)
38. Two out of every 5 men blame their poor eating habits on too much fast food. In a room of 40 men, how many would you expect to blame their not eating well on fast food? (Source: Healthy Choice Mixed Grills survey)
40. Currently in the American population of people aged 65 years old and older, there are 145 women for every 100 men. In a nursing home with 280 male residents over the age of 65, how many female residents over the age of 65 would be expected? (Source: U.S. Bureau of the Census)
42. One pound of firmly packed brown sugar yields  $2\frac{1}{4}$  cups. How many pounds of brown sugar will be required in a recipe that calls for 6 cups of firmly packed brown sugar? (Source: Based on data from Family Circle magazine)

When making homemade ice cream in a hand-cranked freezer, the tub containing the ice cream mix is surrounded by a brine (water/salt) solution. To freeze the ice cream mix rapidly so that smooth and creamy ice cream results, the brine solution should combine crushed ice and rock salt in a ratio of 5 to 1. Use this for Exercises 43 and 44. (Source: White Mountain Freezers, The Rival Company)

43. A small ice cream freezer requires 12 cups of crushed ice. How much rock salt should be mixed with the ice to create the necessary brine solution?
44. A large ice cream freezer requires  $18\frac{3}{4}$  cups of crushed ice. How much rock salt will be needed?
45. The gas/oil ratio for a certain chainsaw is 50 to 1.
- How much oil (in gallons) should be mixed with 5 gallons of gasoline?
  - If 1 gallon equals 128 fluid ounces, write the answer to part a in fluid ounces. Round to the nearest whole ounce.
46. The gas/oil ratio for a certain tractor mower is 20 to 1.
- How much oil (in gallons) should be mixed with 10 gallons of gas?
  - If 1 gallon equals 4 quarts, write the answer to part a in quarts.
47. The adult daily dosage for a certain medicine is 150 mg (milligrams) of medicine for every 20 pounds of body weight.
- At this rate, find the daily dose for a man who weighs 275 pounds.
  - If the man is to receive 500 mg of this medicine every 8 hours, is he receiving the proper dosage?
48. The adult daily dosage for a certain medicine is 80 mg (milligrams) for every 25 pounds of body weight.
- At this rate, find the daily dose for a woman who weighs 190 pounds.
  - If she is to receive this medicine every 6 hours, find the amount to be given every 6 hours.

## Review

Find the prime factorization of each number. See Section 4.2.

49. 200

50. 300

51. 32

52. 81

## Concept Extensions

As we have seen earlier, proportions are often used in medicine dosage calculations. The exercises below have to do with liquid drug preparations, where the weight of the drug is contained in a volume of solution. The description of mg and ml below will help. We will study metric units further in Chapter 9.

mg means milligrams (A paper clip weighs about a gram. A milligram is about the weight of  $\frac{1}{1000}$  of a paper clip.)

ml means milliliter (A liter is about a quart. A milliliter is about the amount of liquid in  $\frac{1}{1000}$  of a quart.)

One way to solve the applications below is to set up the proportion  $\frac{\text{mg}}{\text{ml}} = \frac{\text{mg}}{\text{ml}}$ .

A solution strength of 15 mg of medicine in 1 ml of solution is available.

53. If a patient needs 12 mg of medicine, how many ml do you administer?
54. If a patient needs 33 mg of medicine, how many ml do you administer?

A solution strength of 8 mg of medicine in 1 ml of solution is available.

55. If a patient needs 10 mg of medicine, how many ml do you administer?
56. If a patient needs 6 mg of medicine, how many ml do you administer?

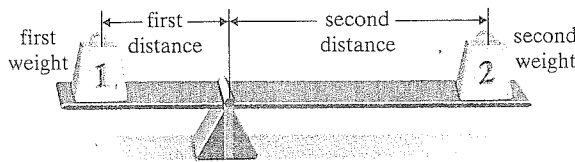
Estimate the following. See the Concept Check in this section.

57. It takes 1.5 cups of milk to make 11 muffins. Estimate the amount of milk needed to make 8 dozen muffins. Explain your calculation.

58. A favorite chocolate chip recipe calls for  $2\frac{1}{2}$  cups of flour to make 2 dozen cookies. Estimate the amount of flour needed to make 50 cookies. Explain your calculation.

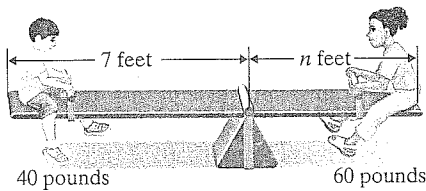
A board such as the one pictured below will balance if the following proportion is true:

$$\frac{\text{first weight}}{\text{second distance}} = \frac{\text{second weight}}{\text{first distance}}$$

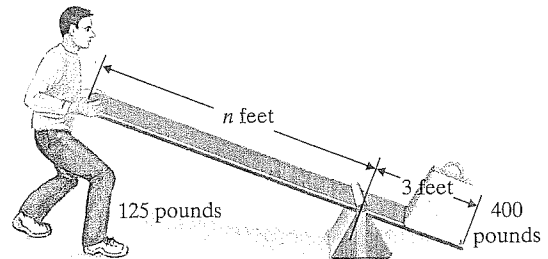


Use this proportion to solve Exercises 59 and 60.

59. Find the distance  $n$  that will allow the board to balance.



60. Find the length  $n$  needed to lift the weight below.



61. Describe a situation in which writing a proportion might solve a problem related to driving a car.



**STUDY SKILLS BUILDER**

**How Are You Doing?**

If you haven't done so yet, take a few moments and think about how you are doing in this course. Are you working toward your goal of successfully completing this course? Is your performance on homework, quizzes, and tests satisfactory? If not, you might want to see your instructor to see if he/she has any suggestions on how you can improve your performance. Reread Section 1.1 for ideas on places to get help with your mathematics course.

Answer the following.

1. List any textbook supplements you are using to help you through this course.

2. List any campus resources you are using to help you through this course.
3. Write a short paragraph describing how you are doing in your mathematics course.
4. If improvement is needed, list ways that you can work toward improving your situation as described in Exercise 3.