(10.1) Chi-square goodness-of-fit test

An experiment is conducted to determine if a consumer preference exists among five cola brands. Each of 100 randomly selected cola drinkers is given five unmarked glasses of cola (one of each brand), tastes each brand (in random order) and then states their preferred brand. Use the results (shown below) to determine if there is sufficient evidence to conclude that a consumer preference among the five cola brands exists. (test using $\alpha = .10$)

Brand:	A	B	C	D	E
Votes:	15	29	16	14	26

Ho: $p_1 = p_2 = p_3 = p_4 = p_5 = \frac{1}{5}$ (no preference exists)

Ha: not all $p_i = \frac{1}{5}$ (a preference exists)

	A	B	C	D	E
O	15	29	16	14	26
E	20	20	20	20	20

O = observed frequencies

E = expected frequencies

$E_i = np_i = 100(\frac{1}{5}) = 20$

K = number of categories (cells)

Test Statistic:

$$\chi^2 = \sum \frac{(O-E)^2}{E} = \frac{(15-20)^2}{20} + \frac{(29-20)^2}{20} + \frac{(16-20)^2}{20} + \frac{(14-20)^2}{20} + \frac{(26-20)^2}{20}$$

$$= 1.25 + 4.05 + .80 + 1.80 + 1.80$$

$$= 9.7$$

Rejection Region: $\chi^2 > 7.779$ $\left(\begin{array}{l}\alpha = .10 \\ d.f. = K-1 = 5-1 = 4\end{array}\right)$

Decision: Reject Ho (since $9.7 > 7.779$)

Conclusion: There is sufficient evidence to conclude that a consumer preference among the five cola brands exists.

Ten years ago, Smileytown's voters were distributed among various political parties as follows: 42% democrat, 38% republican, 12% independent, and 8% from other parties. A recent sample of 500 Smileytown voters contained 213 democrats, 184 republicans, 67 independents, and 36 from other parties. Does the sample provide sufficient evidence to conclude that Smileytown's distribution of voters among various political parties has changed? (test using $\alpha = .05$)

Ho: $p_1 = .42,\ p_2 = .38,\ p_3 = .12,\ p_4 = .08$ (distribution is unchanged)

Ha: not all p_i equal their hypothesized values (distribution has changed)

	D	R	I	Other
O	213	184	67	36
E	210	190	60	40
	↑ .42(500)	↑ .38(500)	↑ .12(500)	↑ .08(500)

Test Statistic:

$$\chi^2 = \sum \frac{(O-E)^2}{E} = \frac{(213-210)^2}{210} + \frac{(184-190)^2}{190} + \frac{(67-60)^2}{60} + \frac{(36-40)^2}{40}$$

$$= .043 + .189 + .817 + .400$$

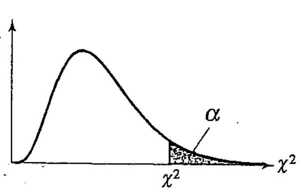
$$= 1.449$$

Rejection Region: $\chi^2 > 7.815$ $\left(\begin{array}{l}\alpha = .05 \\ d.f. = K-1 = 4-1 = 3\end{array}\right)$

Decision: Do not reject Ho (since $1.449 \not> 7.815$)

Conclusion: There is not sufficient evidence to conclude that Smileytown's distribution of voters among various political parties has changed.

Chi-Square Distribution



Right tail

Degrees of freedom	α 0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
1	—	—	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.071	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188