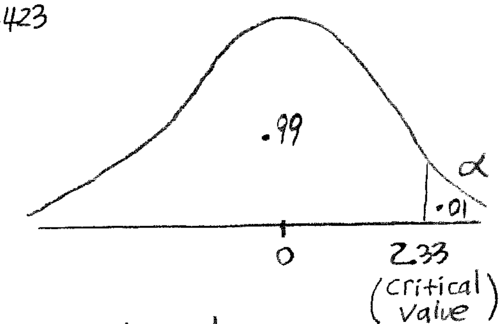
Large-sample, ($n \geq 30$), hypothesis test for a population mean, μ

Attempting to keep pace with its competitors in the super-size sandwich race, Burgerworld introduces the "Belt-Buster" burger, advertised at 1200 calories. Nutrition watchdogs are outraged, claiming that the calorie content of this burger is even higher than advertised. Analysis of a random sample of 45 Belt-Buster burgers shows a mean of 1213 calories and a standard deviation of 29.8 calories. Does the sample provide sufficient evidence to conclude that the mean calorie content of the Belt-Buster burger exceeds 1200 calories?
(Test using α = .01)

H_0: $\mu = 1200$ H_a: $\mu > 1200$

Test Statistic: $z = \dfrac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \approx \dfrac{1213 - 1200}{\frac{29.8}{\sqrt{45}}} = \dfrac{13}{4.4423} = 2.93$



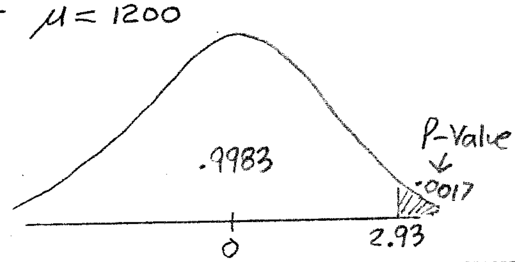
Rejection Region:
$z > 2.33$

Decision: Reject H_0 (since $2.93 > 2.33$)

Conclusion: There is sufficient evidence to conclude that the mean calorie content of the Belt-Buster burger is greater than 1200.

Find the p-value for this hypothesis test and use it to make the decision about H_0.

P-value $= P(\bar{x} \geq 1213)$ if $\mu = 1200$
$= P(z > 2.93)$
$= 1 - .9983$
$= .0017$



p-value $< \alpha$
$.0017 < .01$
Reject H_0

State what a Type I error would be (in the context of this problem).
Rejecting H_0 when H_0 is true.
(Concluding H_a when H_a is false)
Concluding that the mean calorie content of the Belt-Buster burger is greater than 1200 when in fact it is not greater than 1200.

State what a Type II error would be (in the context of this problem).
Not rejecting H_0 when H_0 is false.
(Not concluding H_a when H_a is true)
Not concluding that the mean calorie content of the Belt-Buster burger is greater than 1200 when in fact it is greater than 1200.

Large sample, ($n \geq 30$), hypothesis test for a population mean, μ

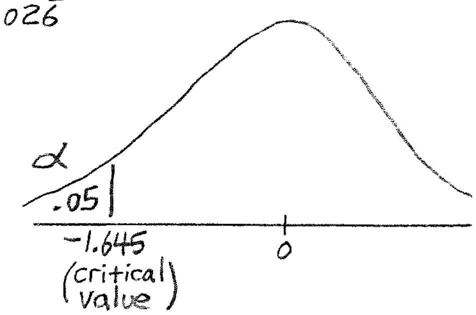
The makers of Ittie-Bittie-Kittie cat food have received numerous complaints that their five pound bags of cat food are underweight. An independent inspector obtains a random sample of 36 bags of this brand and weighs the contents. The sampled bags have a mean weight of 4.98 pounds and a standard deviation of 0.16 pounds. Does the sample provide sufficient evidence to conclude that the mean weight of Ittie-Bittie-Kittie's five pound bags of cat food is less than advertised?
(Test using $\alpha = .05$)

H_0: $\mu = 5$ $\qquad$ H_a: $\mu < 5$

Test Statistic: $z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \approx \dfrac{4.98 - 5}{.16/\sqrt{36}} = \dfrac{-.02}{.02\bar{6}} = -.75$

Rejection Region:

$z < -1.645$



Decision: Do not reject H_0 (since $-.75 \nless -1.645$)

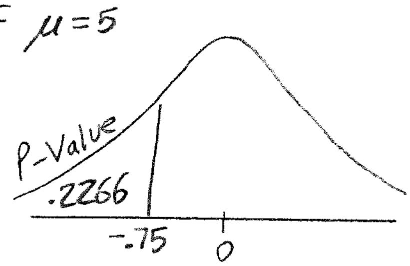
Conclusion: There is not sufficient evidence to conclude that the mean weight of Ittie-Bittie-Kittie's five pound bags of cat food is less than 5 pounds.

Find the p-value for this hypothesis test and use it to make the decision about H_0.

$\underline{P\text{-Value}} = P(\bar{x} \leq 4.98)$ if $\mu = 5$

$= P(z < -.75)$

$= .2266$



p-value $\nless \alpha$

$.2266 \nless .05$

Do not reject H_0

State what a Type I error would be (in the context of this problem).

Rejecting H_0 when H_0 is true.

(Concluding H_a when H_a is false)

Concluding that the mean weight of Ittie-Bittie-Kittie's five pound bags of cat food is less than 5 pounds when in fact it is not less than 5 pounds.

State what a Type II error would be (in the context of this problem).

Not rejecting H_0 when H_0 is false.

(Not concluding H_a when H_a is true)

Not concluding that the mean weight of Ittie-Bittie-Kittie's five pound bags of cat food is less than 5 pounds when in fact it is less than 5 pounds.

Large sample, ($n \geq 30$), hypothesis test for a population mean, μ

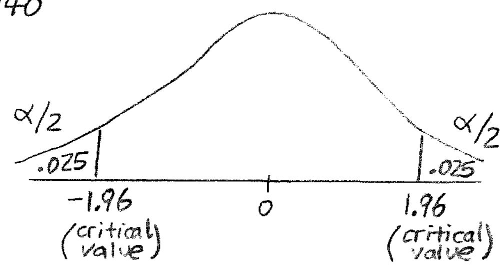
Fab-Fizz cola wants to insure that its filling machine is dispensing an average of 12 ounces of soda into its 12 ounce cans. They obtain a random sample of 40 cans and measure the volume of soda in the cans. The sampled cans contain a mean soda volume of 12.06 ounces with a standard deviation of 0.13 ounces. Does the sample provide sufficient evidence to conclude that the mean soda volume dispensed by Fab-Fizz's filling machine differs from 12 ounces?
(Test using $\alpha = .05$)

H_0: $\mu = 12$ H_a: $\mu \neq 12$

Test Statistic: $z = \dfrac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \approx \dfrac{12.06 - 12}{.13/\sqrt{40}} = \dfrac{.06}{.020555} = 2.92$

Rejection Region:

$z < -1.96$ or $z > 1.96$

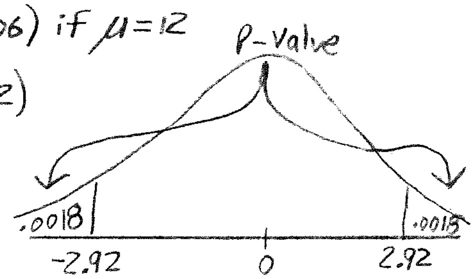


Decision: Reject H_0 (since $2.92 > 1.96$)

Conclusion: There is sufficient evidence to conclude that the mean soda volume dispensed by Fab-Fizz's filling machine differs from 12 ounces.

Find the p-value for this hypothesis test and use it to make the decision about H_0.

$$\underline{\text{P-Value}} = P(\bar{x} \leq 11.94) + P(\bar{x} \geq 12.06) \text{ if } \mu = 12$$
$$= P(z < -2.92) + P(z > 2.92)$$
$$= .0018 + .0018$$
$$= .0036$$



p-value $< \alpha$
$.0036 < .05$

Reject H_0

State what a Type I error would be (in the context of this problem).

Rejecting H_0 when H_0 is true.
(Concluding H_a when H_a is False)
Concluding that the mean soda volume dispensed by Fab-Fizz's filling machine differs from 12 ounces when in fact it does not differ.

State what a Type II error would be (in the context of this problem).

Not rejecting H_0 when H_0 is false.
(Not concluding H_a when H_a is true)
Not concluding that the mean soda volume dispensed by Fab-Fizz's filling machine differs from 12 ounces when in fact it does differ.

Homework problems: Large-sample hypothesis test for a population mean, μ

For each problem, perform the hypothesis test using both approaches (rejection region & p-value), and if an error was made state (in the context of the problem) what the error would have been.

1) The makers of Captain-Salty's fish sticks claim that their fish sticks contain an average of 600 milligrams of sodium. However, they have been engulfed by a tidal wave of complaints, suggesting that the mean sodium content is greater than 600 mg. A government investigator obtains a random sample of 50 Captain-Salty's fish sticks. Analysis reveals that the sampled fish sticks have a mean sodium content of 606.5 mg and a standard deviation of 24.5 mg. Does the sample provide sufficient evidence to conclude that mean sodium content of Captain-Salty's fish sticks is greater than advertised? (Test using $\alpha = .025$).
2) Historically, the mean waiting time for drive-through orders at Lucky Clucky Chicken has been 4.2 minutes. Management implements a program to reduce the average waiting time. After the program's implementation, they examined 60 randomly sampled drive-through orders, having a mean waiting time of 3.72 minutes and a standard deviation of 1.85 minutes. Does the sample provide sufficient evidence to conclude that the program has reduced the mean waiting time for drive-through orders at Lucky Clucky? (Test using $\alpha = .10$)
3) According to the last census, the average age of a city's residents was 34.5 years. The city wants to determine if this has changed, so they recently obtain a random sample of 400 of their residents. The sampled residents have a mean age of 34.7 years and a standard deviation of 12.4 years. Does the sample provide sufficient evidence to conclude that the mean age of the city's residents has changed? (Test using $\alpha = .10$)

Answers: **1)** H_0: $\mu = 600$ H_a: $\mu > 600$

Test Statistic: $z = 1.88$

Rejection region: $z > 1.96$

Decision: Do not reject H_0

Conclusion: There is not sufficient evidence to conclude that the mean sodium content of of Captain-Salty's fish sticks is greater than 600 mg

P-value: .0301(table), .0303(calculator)

Decision: Do not reject H_0 (since .0303 is not less than .025)

If an error, Type II error. Not concluding that the mean sodium content of Captain-Salty's fish sticks is greater than 600 mg when in fact it is greater than 600 mg.

2) H_0: $\mu = 4.2$ H_a: $\mu < 4.2$

Test Statistic: $z = -2.01$

Rejection region: $z < -1.28$

Decision: Reject H_0

Conclusion: There is sufficient evidence to conclude that the mean waiting time for drive-through orders at Lucky Clucky is less than 4.2 minutes.

P-Value: .0222

Decision: Reject H_0 (since .0222 is less than .10)

If an error, Type I error. Concluding that the mean waiting time for drive-though orders at Lucky Clucky is less than 4.2 minutes when in fact it is not less than 4.2 minutes.

3) H_0: $\mu = 34.5$ H_a: $\mu \neq 34.5$

Test Statistic: $z = .32$

Rejection region: $z < -1.645$ or $z > 1.645$

Decision: Do not reject H_0

Conclusion: There is not sufficient evidence to conclude that the mean age of the city's residents is different than 34.5 years.

P-Value: .749(table), .747(calculator)

Decision: Do not reject H_0 (since .747 is not less than .10)

If an error, Type II error. Not concluding that the mean age of the city's residents is different than 34.5 years when in fact it is different than 34.5 years.