

## 7.4 Large sample ( $np \geq 5$ and $nq \geq 5$ ) hypothesis test for a population proportion, $p$

A drug currently on the market produces side-effects in 12% of its users. A new version of the drug is tested on 200 randomly selected patients, 14 of whom suffer side-effects. Does the sample provide sufficient evidence to conclude that the new drug is less likely to produce side-effects? (test using  $\alpha = .10$ )

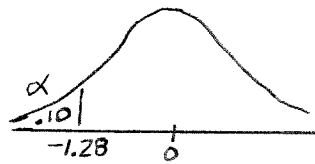
$$H_0: p = .12 \quad H_a: p < .12 \quad \left( \hat{p} = \frac{x}{n} = \frac{14}{200} = .07 \right)$$

Test Statistic:

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{.07 - .12}{\sqrt{\frac{.12(.88)}{200}}} = \frac{-.05}{.02298} = -2.18$$

Rejection Region:

$$Z < -1.28$$



$$\begin{aligned} \text{P-Value: } & P(Z < -2.18) \\ & = .0146 \quad (\text{calculator: } .0148) \end{aligned}$$

Decision:

Reject  $H_0$  (since  $-2.18 < -1.28$ )

$$\begin{aligned} \text{Decision: } & p\text{-value} < \alpha \\ & .0146 < .10 \quad \text{Reject } H_0 \end{aligned}$$

Conclusion:

There is sufficient evidence to conclude that the new version of the drug produces side-effects in less than 12% of its users.

Historically, 15% of a state's registered voters have been independents. In a recent sample of 500 of this state's registered voters, 17.2% are independents. Does the sample provide sufficient evidence to conclude that the proportion of the state's voters who are registered as independents has changed? (test using  $\alpha = .05$ )

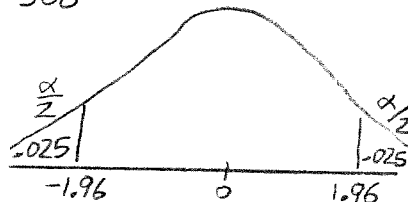
$$H_0: p = .15 \quad H_a: p \neq .15$$

Test Statistic:

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{.172 - .15}{\sqrt{\frac{.15(.85)}{500}}} = \frac{.022}{.01597} = 1.38$$

Rejection Region:

$$Z < -1.96 \text{ or } Z > 1.96$$



$$\begin{aligned} \text{P-Value: } & P(Z < -1.38) + P(Z > 1.38) \\ & = .0838 + .0838 \\ & = .1676 \quad (\text{calculator: } .1683) \end{aligned}$$

Decision:

Do not reject  $H_0$  (since  $-1.96 < 1.38 < 1.96$ )

$$\begin{aligned} \text{Decision: } & p\text{-value} \neq \alpha \\ & .1676 \neq .05 \quad \text{Do not reject } H_0 \end{aligned}$$

Conclusion:

There is not sufficient evidence to conclude that the proportion of the state's voters who are registered as independents has changed.

Homework: (section 7.4) Perform each hypothesis test by both approaches: rejection region and p-value.

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- 1) During the last year, 12% of drive-through orders at Loco-Taco were believed to be prepared incorrectly. In an effort to reduce this proportion, management retrains its workers. After the training, 400 drive-through orders are randomly sampled and 42 are found to be prepared incorrectly. Does the sample provide sufficient evidence to conclude that the proportion of Loco-Taco drive-through orders that are prepared incorrectly has been reduced? (test using  $\alpha = .10$ )
  - 2) Nationwide, 23% of a Sub shop's customers prefer turkey subs. The shop wants to open a store in a new city. They randomly sample 500 of the new city's residents (who eat sub sandwiches) and 101 of them prefer turkey subs. Does the sample provide sufficient evidence to conclude that the proportion of sub consumers who prefer turkey subs in the new city differs from the nationwide proportion? (test using  $\alpha = .05$ )
  - 3) Last year, only 35% of a state's voters supported a sales tax to fund mass transit. A lobbying effort has since been conducted to increase this proportion. In a recent sample of 600 voters from this state, 39% support the tax. Does the sample provide sufficient evidence to conclude that the proportion of the state's voters who support the tax has increased? (test using  $\alpha = .05$ )
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Answers:

1)  $H_0: p = .12$     $H_a: p < .12$

Test Statistic:  $z = -.92$

Rejection Region:  $Z < -1.28$

P-Value: .1788 (table)  
.1780 (calculator)

Decision: Do not reject  $H_0$   
(since  $-.92 \notin -1.28$ )

Decision:  $p\text{-value} \notin \alpha$ ,  
Do not reject  $H_0$

Conclusion: There is not sufficient evidence to conclude that the proportion of Loco-Taco drive-through orders that are prepared incorrectly is less than 12%.

2)  $H_0: p = .23$     $H_a: p \neq .23$

Test Statistic:  $z = -1.49$

Rejection Region:  $z < -1.96$  or  $z > 1.96$

P-Value: .1362 (table)  
.1368 (calculator)

Decision: Do not reject  $H_0$   
(since  $-1.96 < -1.49 < 1.96$ )

Decision:  $p\text{-value} \notin \alpha$   
Do not reject  $H_0$

Conclusion: There is not sufficient evidence to conclude that the proportion of sub consumers in the new city who prefer turkey subs differs from 23%.

3)  $H_0: p = .35$     $H_a: p > .35$

Test Statistic:  $z = 2.05$

Rejection Region:  $z > 1.645$

P-Value: .0202 (table)  
.0200 (calculator)

Decision: Reject  $H_0$   
(since  $2.05 > 1.645$ )

Decision:  $p\text{-value} < \alpha$   
Reject  $H_0$

Conclusion: There is sufficient evidence to conclude that the proportion of the state's voters who support the tax is greater than 35%.