TRUTH TABLES

Negation

$\sim p$

not p

p	$\sim p$
T	F
F	T

opposite truth value

Conjuction

$p \wedge q$

p and q

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

true only when both p and q are true

Disjunction

$p \vee q$

p or q

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

true if either p is true, or q is true, or both are true

Conditional

$p \rightarrow q$

if p, then q

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

true except when p is true and q is false

Biconditional

$p \leftrightarrow q$

p if and only if q

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

true only when p and q have the same truth value

Dominance of Connectives

Evaluate the least dominant connective first.
Evaluate the most dominant connective last.
(but parentheses take priority)

Least dominant to most dominant: $\sim$ $\quad \wedge, \vee \quad \rightarrow \quad \leftrightarrow$

Truth tables will now be constructed for several examples:

1) $p \wedge \sim q$

p	q	$\sim q$	$p \wedge \sim q$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

2) $\sim (p \vee q)$

p	q	$p \vee q$	$\sim (p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

3) $\sim p \rightarrow q$

p	q	$\sim p$	q	$\sim p \rightarrow q$
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	F	F

Note: It is not necessary to rewrite a column (q in this case), but in a more complex case (like the next) it tends to make it easier to determine truth values for the conditional.

4) $\sim (p \wedge q) \rightarrow r$

p	q	r	$p \wedge q$	$\sim (p \wedge q)$	r	$\sim (p \wedge q) \rightarrow r$
T	T	T	T	F	T	T
T	T	F	T	F	F	T
T	F	T	F	T	T	T
T	F	F	F	T	F	F
F	T	T	F	T	T	T
F	T	F	F	T	F	F
F	F	T	F	T	T	T
F	F	F	F	T	F	F

5) $(p \vee \sim q) \leftrightarrow (\sim q \wedge r)$

p	q	r	$\sim q$	$p \vee \sim q$	$\sim q \wedge r$	$(p \vee \sim q) \leftrightarrow (\sim q \wedge r)$
T	T	T	F	T	F	F
T	T	F	F	T	F	F
T	F	T	T	T	T	T
T	F	F	T	T	F	F
F	T	T	F	F	F	T
F	T	F	F	F	F	T
F	F	T	T	T	T	T
F	F	F	T	T	F	F

NEGATIONS

Statement	Negation
All A is/are B	Some A is/are not B
Some A is/are B	All A is/are not B (No A is/are B)
No A is/are B	Some A is/are B

Statement	Negation
p and q	Not p or not q
p or q	Not p and not q
If p, then q	p and not q

Examples:

Statement	Negation
All pizza is tasty.	Some pizza is not tasty.
Some cats are black.	All cats are not black. (No cats are black.)
No dogs are white.	Some dogs are white.
The cat is black and the dog is white.	The cat is not black or the dog is not white.
The cat is black or the dog is white.	The cat is not black and the dog is not white.
If the cat is black, then the dog is white.	The cat is black and the dog is not white.

VARIATIONS OF THE CONDITIONAL

Conditional:	$p \rightarrow q$	If p, then q
Converse:	$q \rightarrow p$	If q, then p
Inverse:	$\sim p \rightarrow \sim q$	If not p, then not q
Contrapositive:	$\sim q \rightarrow \sim p$	If not q, then not p

<u>Examples</u>:

Conditional:	If I am in Orlando, then I am in Florida.
Converse:	If I am in Florida, then I am in Orlando.
Inverse:	If I am not in Orlando, then I am not in Florida.
Contrapositive:	If I am not in Florida, then I am not in Orlando.

Note that a conditional statement and its contrapositive are logically equivalent.
(In the case above, note that the conditional statement and its contrapositive are both true.)

Statements logically equivalent to $p \rightarrow q$

Logically equivalent statements		Examples:
$p \rightarrow q$	If p, then q	If I am in Orlando, then I am in Florida.
$\sim q \rightarrow \sim p$	If not q, then not p	If I am not in Florida, then I am not in Orlando.
$\sim p \vee q$	Not p or q	I am not in Orlando or I am in Florida.

(Note that all three statements above are true.)

SYMBOLIC ARGUMENTS

Symbolic arguments are arguments that use connectives (and, or, not, if-then, and if and only if). An example of a symbolic argument is shown below. Note that it is first expressed in words, and then expressed in symbols.

If Yogi is a bear, then Yogi likes picnic baskets.
<u>Yogi is a bear.</u>
∴ Yogi likes picnic baskets.

(∴ is read as "therefore")

The argument expressed in symbols is shown below:

$$\begin{array}{l} p \rightarrow q \\ \underline{p \qquad\;\;} \\ \therefore\; q \end{array}$$

where p = Yogi is a bear.
q = Yogi likes picnic baskets.

In this argument, the first two lines are premises and the last line is the conclusion.

<u>An argument is valid</u> when its conclusion necessarily follows from its premises (meaning that if the premises are true, then the conclusion must be true).

<u>An argument is invalid</u> when its conclusion does not necessarily follow from its premises (meaning that if the premises are true then the conclusion is not necessarily true).

To determine if a symbolic argument is valid or invalid, we can always use a truth table (demonstrated on the next page) or we can simply compare the argument's form to a standard form. Standard forms of valid and invalid arguments (verifiable by using truth tables) are shown below.

<u>STANDARD FORMS OF ARGUMENTS</u>

	Law of Detachment	Law of Contraposition	Law of Syllogism	Disjunctive Syllogism
Valid Arguments	$\begin{array}{l} p \rightarrow q \\ \underline{p \qquad\;} \\ \therefore\; q \end{array}$	$\begin{array}{l} p \rightarrow q \\ \underline{\sim q \qquad} \\ \therefore\; \sim p \end{array}$	$\begin{array}{l} p \rightarrow q \\ \underline{q \rightarrow r \quad} \\ \therefore\; p \rightarrow r \end{array}$	$\begin{array}{l} p \vee q \\ \underline{\sim p \qquad} \\ \therefore\; q \end{array}$

	Fallacy of the Converse	Fallacy of the Inverse
Invalid Arguments	$\begin{array}{l} p \rightarrow q \\ \underline{q \qquad\;} \\ \therefore\; p \end{array}$	$\begin{array}{l} p \rightarrow q \\ \underline{\sim p \qquad} \\ \therefore\; \sim q \end{array}$

As an example, the argument below will be written in symbolic form and then compared to one of the standard forms in order to determine if it is valid or invalid.

(argument:in words)
If I study, then I'll be more confident.
<u>If I'm more confident, then I'll be more likely to pass</u>.
∴ If I study, then I'll be more likely to pass.

(argument:in symbols)

$$\begin{array}{l} p \rightarrow q \\ \underline{q \rightarrow r \quad} \\ \therefore\; p \rightarrow r \end{array}$$

The argument's symbolic form matches that of one of the valid standard forms above, therefore the argument is valid.

If an argument's symbolic form does not match one of the standard forms shown previously, we can use a truth table to determine if the argument is valid or invalid. The general procedure is outlined below:

P1 P2 ⋮ Pn ∴ C	In the argument at the left, let P1, P2, ...Pn be premises and let C be the conclusion. We form a symbolic conditional statement (as shown below) where the antecedent is the premises joined by and connectives, and the consequent is the conclusion. $[P1 \wedge P2 \wedge \ldots Pn] \rightarrow C$ Then we construct a truth table for this statement. <u>If the final column of the truth table is all Ts (trues), then the argument is valid</u>. <u>If the final column of the truth table is not all Ts, then the argument is invalid</u>.

We will first use this method to verify that one of the standard valid forms is valid, and then we will work with an argument of nonstandard form.

Use a truth table to verify that the argument form shown at the right is valid.

$p \vee q$
$\sim p$
$\therefore q$

So we must construct a truth table for $[(p \vee q) \wedge \sim p] \rightarrow q$ and have the final column be all Ts.

p	q	$p \vee q$	$\sim p$	$[(p \vee q) \wedge \sim p]$	q	$[(p \vee q) \wedge \sim p] \rightarrow q$
T	T	T	F	F	T	T
T	F	T	F	F	F	T
F	T	T	T	T	T	T
F	F	F	T	F	F	T

The truth table's final column is all Ts (trues), so the argument is valid.

Use a truth table to determine if the argument below is valid or invalid.

(argument in words)
If I win Lotto, then I will be happy.
If I am happy, then people will love me.
I did not win Lotto.
∴ People will not love me.

(argument:in symbols)
$p \rightarrow q$
$q \rightarrow r$
$\sim p$
$\therefore \sim r$

Construct a truth table for $[(p \rightarrow q) \wedge (q \rightarrow r) \wedge \sim p] \rightarrow \sim r$

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$\sim p$	$[(p \rightarrow q) \wedge (q \rightarrow r) \wedge \sim p]$	$\sim r$	$[(p \rightarrow q) \wedge (q \rightarrow r) \wedge \sim p] \rightarrow \sim r$
T	T	T	T	T	F	F	F	T
T	T	F	T	F	F	F	T	T
T	F	T	F	T	F	F	F	T
T	F	F	F	T	F	F	T	T
F	T	T	T	T	T	T	F	F
F	T	F	T	F	T	F	T	T
F	F	T	T	T	T	T	F	F
F	F	F	T	T	T	T	T	T

The truth table's final column is not all Ts, so the argument is invalid.

SYLLOGISTIC ARGUMENTS

Syllogistic arguments (or syllogisms) are arguments that use quantifiers (all, some, and none). Examples of syllogisms are shown below.

All dogs bark.
No cats are dogs.
$\therefore$ No cats bark.

Some birds fly.
All eagles fly.
$\therefore$ Some birds are eagles.

($\therefore$ is read as "therefore")

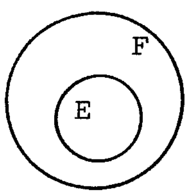
In each argument, the first two lines are premises and the last line is the conclusion.

An argument is valid when its conclusion necessarily follows from its premises (meaning that if the premises are true, then the conclusion must be true).

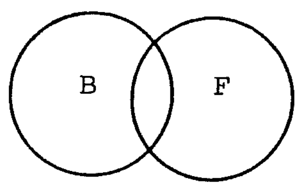
An argument is invalid when its conclusion does not necessarily follow from its premises (meaning that if the premises are true, then the conclusion is not necessarily true).

To determine if a syllogistic argument is valid or invalid we will use Euler diagrams. An Euler diagram (or circle diagram) is a visual device for displaying the relationship between sets. Examples of Euler diagrams involving quantifiers (all, some, and none) are shown below.

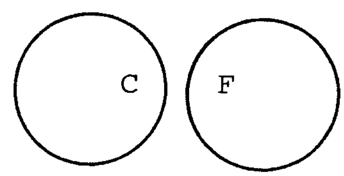
All eagles fly
(Eagles fly)



Some birds fly



No cats fly



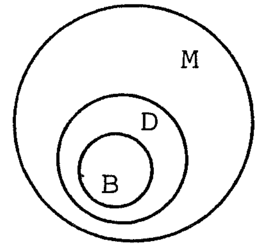
To show that an argument is valid using Euler diagrams, it must be shown that for any diagram satisfying the premises, the conclusion must follow. (This is usually easier than it seems since often the number of different ways to construct a diagram is restricted).

To show that an argument is invalid using Euler diagrams, we need only show one diagram that satisfies the premises, but for which the conclusion does not necessarily follow (Note that regardless of how many possible diagrams would force the conclusion, to show that an argument is invalid we need to only show a single diagram for which the conclusion does not follow).

Also, be careful not to confuse the validity of an argument with the truth of its conclusion.
Valid arguments can have false conclusions.
(because they are based on a false premise)
Invalid arguments can have true conclusions.
(but the conclusion does not necessarily follow solely from the premises)

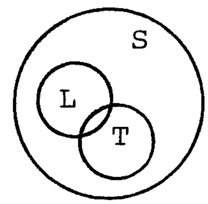
What follows are examples of the use of Euler diagrams to determine the validity of syllogistic arguments (and instructive comments are included).

All beagles are dogs.
All dogs are mammals.
∴ All beagles are mammals.



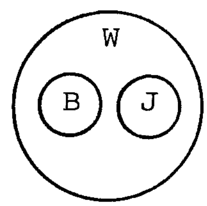
This argument is valid.
(Note that there is only one way to draw the diagram)

All linebackers are strong.
Some teenagers are linebackers.
∴ Some teenagers are strong.



This argument is valid.
(Note that regardless of how the T circle is drawn, whether entirely or partially contained within the S circle, since some element of T is an element of L, some element of T must also be an element of S)

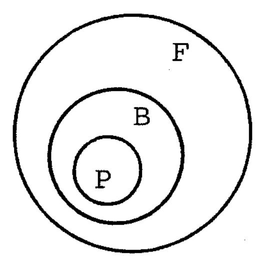
All birds have wings.
No jets are birds.
∴ No jets have wings.



This argument is invalid. (our diagram satisfies the premises, but the conclusion doesn't follow). Note that there are different ways the diagram can be drawn (some in which J and W do not intersect), but that to show that the argument is invalid, we need to show only a single diagram which satisfies the premises, but for which the conclusion doesn't follow.

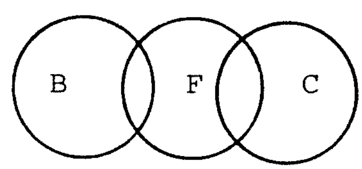
Also, note that while the conclusion is true, the argument is invalid since the conclusion does not follow solely from the premises.

All birds fly.
All penguins are birds.
∴ All penguins fly.



This argument is valid.
(But note that its conclusion is false because it is based on the false premise that all birds fly)

Some birds fly.
No cats are birds.
∴ No cats fly.



This argument is invalid. (once again we have a true conclusion, but not one that follows solely from the premises) Note that our diagram satisfies the premises, but the conclusion does not follow.