(8.1) Large-sample hypothesis test for the difference of two population means (independent samples)

A nutritionist wants to determine if two energy drinks, Hyper-Hippo and Loco-Lion, have different mean caffeine levels. Cans of each brand are randomly sampled and analyzed (results summarized below). Do the samples provide sufficient evidence to conclude that the mean caffeine level of Hyper-Hippo differs from that of Loco-Lion? (test using α = .05)

	Hyper-Hippo	Loco-Lion
n	35	30
$\bar{x}$	87.1	85.9
s	2.7	2.4

(in milligrams)

H_0: $\mu_1 = \mu_2$ $(\mu_1 - \mu_2 = 0)$

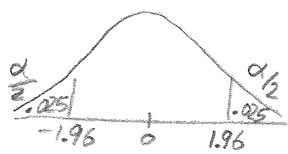
H_a: * $\mu_1 \neq \mu_2$ $(\mu_1 - \mu_2 \neq 0)$

Test Statistic:

$$Z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \approx \frac{(87.1 - 85.9) - (0)}{\sqrt{\frac{(2.7)^2}{35} + \frac{(2.4)^2}{30}}} = \frac{1.2}{.63268137} = 1.90^{*}$$

Rejection Region:

$Z < -1.96$ or $Z > 1.96$



Decision: Do not reject H_0 (since 1.90 is not in the rejection region)

Conclusion:
There is not sufficient evidence to conclude that the mean caffeine level of Hyper-Hippo differs from that of Loco-Lion

P-Value: $* P(Z < -1.90) + P(Z > 1.90)$
$= .0287 + .0287$
$= .0574$

Decision:
p-value $\not< \alpha$
$.0574 \not< .05$, do not reject H_0

To compare the mean weights of hamburger patties from two fast food chains, Burger-Buddy and Burger-World, random samples of 50 patties are obtained from each chain. The sampled Burger-Buddy patties have a mean weight of 3.032 ounces and a standard deviation of .071 ounces. The sampled Burger-World patties have a mean weight of 2.987 ounces and a standard deviation of .083 ounces. Do the samples provide sufficient evidence to conclude that the mean weight of Burger-Buddy hamburger patties is greater than those from Burger-World? (test using α = .04)

H_0: $\mu_1 = \mu_2$ $(\mu_1 - \mu_2 = 0)$

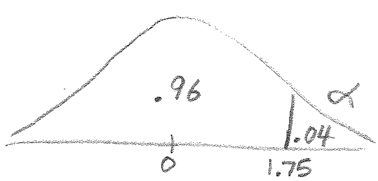
H_a: * $\mu_1 > \mu_2$ $(\mu_1 - \mu_2 > 0)$

Test Statistic:

$$Z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \approx \frac{(3.032 - 2.987) - (0)}{\sqrt{\frac{(.071)^2}{50} + \frac{(.083)^2}{50}}} = \frac{.045}{.01544668} = 2.91^{*}$$

Rejection Region:

$Z > 1.75$



Decision: Reject H_0 (since $2.91 > 1.75$)

Conclusion:
There is sufficient evidence to conclude that the mean weight of Burger-Buddy hamburger patties is greater than those from Burger-World.

P-Value: $* P(Z > 2.91)$
$= 1 - .9982$
$= .0018$

Decision:
p-value $< \alpha$
$.0018 < .04$, reject H_0

(8.4) Large-sample hypothesis test for the difference of two population proportions

Two tutoring programs, ABC and XYZ, claim to be effective in helping high school seniors pass a college entrance exam. A random sample of 250 high school seniors are enrolled in ABC's program, and 157 of them pass the entrance exam. A random sample of 200 high school seniors are enrolled in XYZ's program, and 107 of them pass the entrance exam. Do the samples provide sufficient evidence to conclude that the proportion of seniors who pass the entrance exam using program ABC is higher than for those using program XYZ? (test using $\alpha = .04$)

$$\hat{p}_1 = \frac{X_1}{n_1} = \frac{157}{250} = .628$$

$$\hat{p}_2 = \frac{X_2}{n_2} = \frac{107}{200} = .535$$

$$\bar{p} = \frac{X_1 + X_2}{n_1 + n_2} = \frac{157+107}{250+200} = \frac{264}{450} = .58\bar{6}$$

$$\bar{q} = 1 - \bar{p} = 1 - .58\bar{6} = .41\bar{3}$$

H₀: $p_1 = p_2$ $\quad (p_1 - p_2 = 0)$

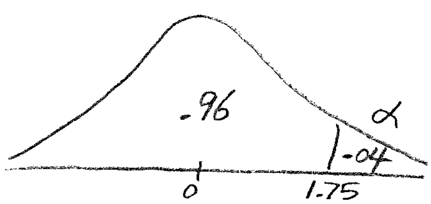
Hₐ: $p_1 \overset{*}{>} p_2$ $\quad (p_1 - p_2 > 0)$

Test Statistic:

$$Z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\bar{p}\bar{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{(.628 - .535) - (0)}{\sqrt{(.58\bar{6})(.41\bar{3})\left(\frac{1}{250} + \frac{1}{200}\right)}} = \frac{.093}{.046716164} = 1.99 *$$

Rejection Region:

$Z > 1.75$



Decision: Reject H_0 (since $1.99 > 1.75$)

P-Value: $* \; P(Z > 1.99)$
$= 1 - .9767$
$= .0233$

Decision: p-value $< \alpha$
$.0233 < .04$, reject H_0

Conclusion:

There is sufficient evidence to conclude that the proportion of seniors who pass the entrance exam using program ABC is higher than for those using program XYZ.

A company wants to compare the proportion of defective parts that are produced by its West and East factories. Five hundred parts are randomly sampled from each factory. From the West factory, 8% of the sampled parts are defective. From the East factory, 6% of the sampled parts are defective. Do the samples provide sufficient evidence to conclude that the proportions of defective parts produced by the West and East factories differ? (test using $\alpha = .02$)

$n_1 = 500 \quad \hat{p}_1 = .08 \quad X_1 = n_1\hat{p}_1 = 500(.08) = 40$

$n_2 = 500 \quad \hat{p}_2 = .06 \quad X_2 = n_2\hat{p}_2 = 500(.06) = 30$

$$\bar{p} = \frac{X_1 + X_2}{n_1 + n_2} = \frac{40 + 30}{500 + 500} = \frac{70}{1000} = .07$$

$$\bar{q} = 1 - \bar{p} = 1 - .07 = .93$$

H₀: $p_1 = p_2$ $\quad (p_1 - p_2 = 0)$

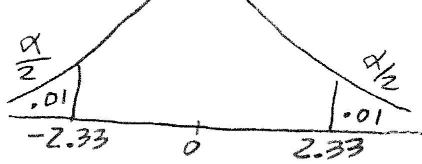
Hₐ: $p_1 \overset{*}{\neq} p_2$ $\quad (p_1 - p_2 \neq 0)$

Test Statistic:

$$Z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\bar{p}\bar{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{(.08 - .06) - (0)}{\sqrt{(.07)(.93)\left(\frac{1}{500} + \frac{1}{500}\right)}} = \frac{.02}{.016137} = 1.24 *$$

Rejection Region:

$Z < -2.33$ or $Z > 2.33$



Decision:
Do not reject H_0 (since $-2.33 < 1.24 < 2.33$)

P-Value: $* \; P(Z < -1.24) + P(Z > 1.24)$
$= .1075 + .1075$
$= .215$

Decision: p-value $\not< \alpha$
$.215 \not< .02$, do not reject H_0

Conclusion:

There is not sufficient evidence to conclude that the proportions of defective parts produced by the West and East factories differ.

Homework: sections 8.1 and 8.4 Perform each hypothesis test by using both approaches (rejection region & p-value)

1) To compare the mean weights of hamburger patties from two fast food chains, Burger-Buddy and Burger-World, random samples of 50 patties are obtained from each chain. The sampled Burger-Buddy patties have a mean weight of 3.032 ounces and a standard deviation of .071 ounces. The sampled Burger-World patties have a mean weight of 2.987 ounces and a standard deviation of .083 ounces. Do the samples provide sufficient evidence to conclude that the mean weight of Burger-Buddy hamburger patties is greater than those from Burger-World? (test using $\alpha = .03$)

2) To determine if bottles of soda from two brands, Ultra-Fizz and Funny-Foam, differ in mean soda volume, random samples of bottles of each brand are obtained and analyzed (results summarized at the right). Do the samples provide sufficient evidence to conclude that the mean soda volumes differ for the two brands. (test using $\alpha = .04$)

	Ultra-Fizz	Funny-Foam
n	33	34
$\bar{x}$	47.8	48.1
s	0.43	0.59

3) A drug company wants to compare the proportion of patients who suffer side-effects from two pain relief medications, formula A and formula B. Two hundred fifty patients are randomly selected to use formula A, and 24 of these patients suffer side-effects. Two hundred patients are randomly selected to use formula B, and 32 of these patients suffer side-effects. Do the samples provide sufficient evidence to conclude that patients using formula A are less likely to suffer side-effects than patients using formula B? (test using $\alpha = .10$)

4) Presidential candidate, Will E. Taxus, wants to compare the proportion of men who support his candidacy versus the proportion of women who do so. In a random sample of 400 men, 27% support him, and in a random sample of 400 women, 29% support him. Do the samples provide sufficient evidence to conclude that the proportions of men and women who support Will E. Taxus differ? (test using $\alpha = .05$)

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1) Ho: $\mu_1 = \mu_2$ Ha: $\mu_1 > \mu_2$
T.S. $Z = 2.91$
R.R. $Z > 1.88$
Decision: Reject Ho (since $2.91 > 1.88$)
Conclusion: There is sufficient evidence to conclude that the mean weight of Burger-Buddy hamburger patties is greater than those from Burger-World.
P-Value: .0018
Decision: Reject Ho (since p-value $< \alpha$)

2) Ho: $\mu_1 = \mu_2$ Ha: $\mu_1 \neq \mu_2$
T.S. $Z = -2.38$
R.R. $Z < -2.05$ or $Z > 2.05$
Decision: Reject Ho (since $-2.38 < -2.05$)
Conclusion: There is sufficient evidence to conclude that the mean soda volumes differ for the two brands.
P-Value: .0174 (calculator: .0171)
Decision: Reject Ho (since p-value $< \alpha$)

3) Ho: $p_1 = p_2$ Ha: $p_1 < p_2$
T.S. $Z = -2.04$
R.R. $Z < -1.28$
Decision: Reject Ho (since $-2.04 < -1.28$)
Conclusion: There is sufficient evidence to conclude that patients using formula A are less likely to suffer side-effects than patients using formula B.
P-Value: .0207 (calculator: .0205)
Decision: Reject Ho (since p-value $< \alpha$)

4) Ho: $p_1 = p_2$ Ha: $p_1 \neq p_2$
T.S. $Z = -.63$
R.R. $Z < -1.96$ or $Z > 1.96$
Decision: Do not reject Ho (since $-1.96 < -.63 < 1.96$)
Conclusion: There is not sufficient evidence to conclude that the proportions of men and women who support Will. E. Taxus differ.
P-Value: .5286 (calculator: .5287)
Decision: Do not reject Ho (since p-value $\nless \alpha$)