SETS: BASIC CONCEPTS AND SUBSETS

A set is a well-defined collection. [Note: we will often use capital letters to label sets]
examples: The collection of U.S. presidents who have served two full terms
[The above collection IS a set because it is well-defined (membership can be clearly determined)]
The collection of successful U.S. presidents
[The above collection IS NOT a set because it is not well-defined (membership is subject to opinion)]

Representing a set: Sets are represented by word description, roster form, or set-builder notation.
Representation of the set of numbers 2, 3, 5, 7 by each of the three methods is shown below.
Word description: The set of one-digit prime numbers
Roster form: { 2, 3, 5, 7 }
Set-builder notation: { x | x is a one-digit prime number }

An **empty set** is a set with no elements.
(null set) [symbolized by { } or ∅]
example: The set of odd numbers that are divisible by 2.

[items in a set are called elements or members]
Caution: { ∅ } is NOT an empty set

Notation for set membership: ∈ (is an element of) ∉ (is not an element of)
examples: For the set { 1, 2, 3 } we can say that $3 \in \{1, 2, 3\}$, but $5 \notin \{1, 2, 3\}$

The **cardinal number of a set** is the number of elements in the set. [symbolized by n(A)]
examples: For the set A = { 4, 5, 6 }, n(A) = 3 For the set B = { 7, 8, 8 }, n(B) = 2

Sets are **equivalent** if they have the same number of elements.
example: If A = { 2, 4, 6 } and B = { 1, 3, 5 }, then set A is equivalent to set B [note that n(A) = n(B) = 3]
Sets are **equal** if they contain exactly the same elements.
example: If A = { 3, 2, 5 } and B = { 5, 3, 2 }, then A = B

Equal sets must also be equivalent, but equivalent sets are not necessarily equal.
examples: A = { 7, 8 } and B = { 8, 7 } are equal (same elements) and also equivalent (same number of elements).
However, C = { 2 } and D = { 6 } are equivalent (same number of elements), but not equal (not the same elements).

Set A is a **subset** of set B if every element in A is also in B. [symbolized by $A \subseteq B$]
examples: For A = { 2, 3 }, B = { 2, 3, 4 }, and C = { 2, 7 }, $A \subseteq B$, but $C \nsubseteq B$ since $7 \notin \{2, 3, 4\}$.

Set A is a **proper subset** of set B if A is a subset of B, but $A \neq B$ [symbolized by $A \subset B$]
example: For A = { 2, 3 }, B = { 2, 3, 4 }, and D = { 4, 2, 3}, $A \subset B$, but $D \not\subset B$ since D = B.

To avoid confusing the symbols ⊆ (subset) and ⊂ (proper subset), note that they function like ≤ and <

both symbols apply	only one symbol applies
$4 \leq 5$ and $4 < 5$	$5 \leq 5$, but $5 \nless 5$
$\{2, 3\} \subseteq \{2, 3, 4\}$, and $\{2, 3\} \subset \{2, 3, 4\}$	$\{2, 3\} \subseteq \{2, 3\}$, but $\{2, 3\} \not\subset \{2, 3\}$

A set with n elements has 2^n subsets.
example: { a, b, c } has $2^3 = (2)(2)(2) = 8$ subsets.
The subsets are; { }, { a }, { b }, { c }, { a, b }, { a, c }, { b, c }, { a, b, c }

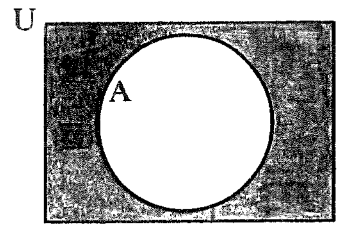
While the empty set is a subset of any set, no set is a proper subset of itself. This leads to the following result:
A set with n elements has $2^n - 1$ proper subsets.

SET OPERATIONS AND VENN DIAGRAMS

operation: Venn diagram:

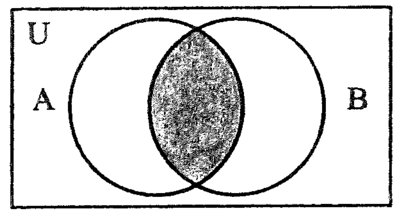
COMPLEMENT

A' = The set of all elements that are not in set A, but that are in the universal set.



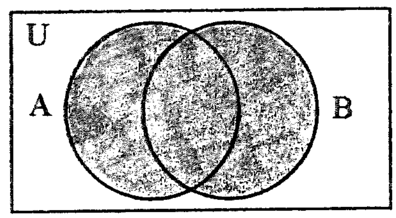
INTERSECTION

$A \cap B$ = The set of all elements that are common to sets A and B.

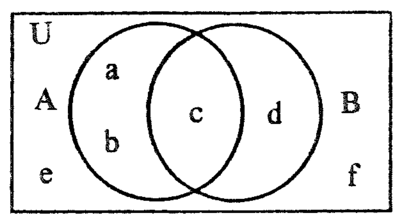


UNION

$A \cup B$ = The set of all elements that are in set A, or in set B, or in both sets.



Given: $U = \{ a, b, c, d, e, f \}$
$A = \{ a, b, c \}$
$B = \{c, d \}$



We will find the following sets:

1) $A' = \{ a, b, c \}'$
$= \{d, e, f \}$

2) $A \cap B$
$= \{a, b, c \} \cap \{c, d \}$
$= \{ c \}$

3) $A \cup B$
$= \{ a, b, c \} \cup \{ c, d \}$
$= \{ a, b, c, d \}$

4) $(A \cap B)'$
$= (\{a, b, c \} \cap \{ c, d \})'$
$= \{ c \}'$
$= \{ a, b, d, e, f \}$

5) $A' \cup B'$
$= \{ a, b, c \}' \cup \{ c, d \}'$
$= \{ d, e, f \} \cup \{ a, b, e, f \}$
$= \{ a, b, d, e, f \}$

It is not a coincidence that the answers to the last two examples are the same. It is a result of what are called De Morgan's Laws.

De Morgan's Laws
$(A \cap B)' = A' \cup B'$
$(A \cup B)' = A' \cap B'$

Note: You may have observed that the five examples above may have been easier to do by simply inspecting the accompanying Venn diagram.
Therefore, how to construct a Venn diagram for two sets will be demonstrated.

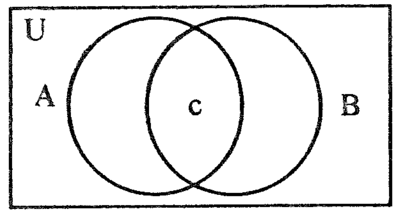
CONSTRUCTING A VENN DIAGRAM FOR TWO SETS

For the sets shown at the right, how to construct a Venn diagram will be demonstrated.

$U = \{ a, b, c, d, e, f \}$
$A = \{ a, b, c \}$
$B = \{ c, d \}$

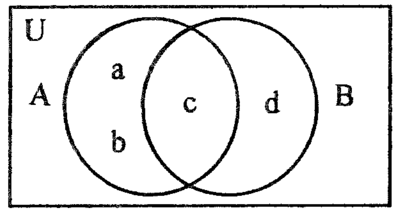
Step 1: Identify elements belonging to both sets.

$$A \cap B = \{ c \}$$



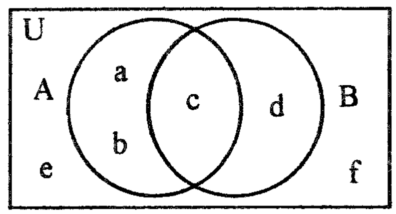
Step 2: Identify elements belonging to only one of the sets.

only to A: $A \cap B' = \{ a, b \}$
only to B: $B \cap A' = \{ d \}$



Step 3: Identify elements belonging to neither set.

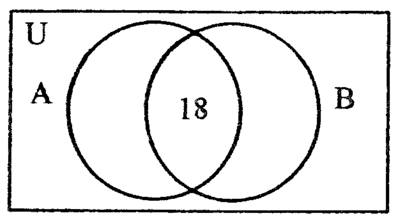
$$A' \cap B' = (A \cup B)' = \{ e, f \}$$



SURVEY PROBLEM INVOLVING TWO SETS (entries in Venn diagram represent counts)

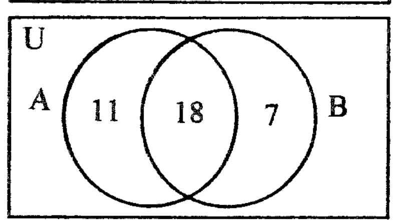
A survey of 50 science students reveals the following:
29 students are studying Anatomy
25 students are studying Biology
18 students are studying both Anatomy and Biology

Step 1: $n(A \cap B) = 18$ students studying Anatomy and Biology

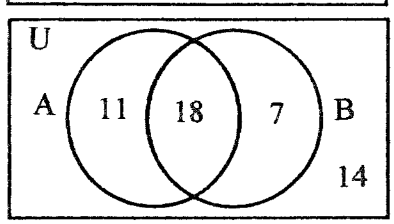


Step 2: $n(A \cap B') = 29 - 18 = 11$ students studying only Anatomy

$n(B \cap A') = 25 - 18 = 7$ students studying only Biology



Step 3: $n(A' \cap B') = n(A \cup B)'$
$= 50 - (11 + 18 + 7)$
$= 50 - 36 = 14$ students studying neither subject



With the Venn diagram, it is easier to answer a question such as, how many students are studying only one of these subjects?
(answer is 11 + 7 = 18)

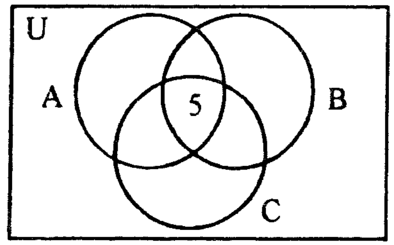
SURVEY PROBLEM INVOLVING THREE SETS

A survey of 70 students reveals the following;
38 students are studying Anatomy
31 students are studying Biology
24 students are studying Chemistry
15 students are studying Anatomy and Biology
13 students are studying Anatomy and Chemistry
7 students are studying Biology and Chemistry
5 students are studying all three subjects.

We proceed (in similar fashion to the two set example) to construct a Venn diagram.

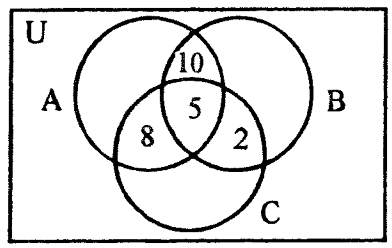
Step 1: Students studying all three subjects

$n(A \cap B \cap C) = 5$



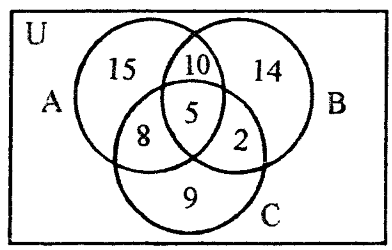
Step 2: Students studying only two of the subjects

Anatomy and Biology, but not Chemistry:
$n(A \cap B \cap C') = 15 - 5 = 10$
Anatomy and Chemistry, but not Biology:
$n(A \cap C \cap B') = 13 - 5 = 8$
Biology and Chemistry, but not Anatomy:
$n(B \cap C \cap A') = 7 - 5 = 2$



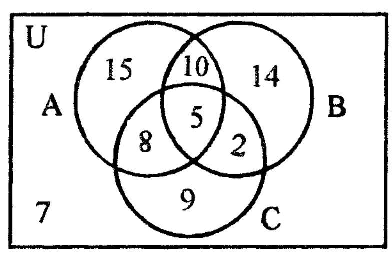
Step 3: Students studying only one of the subjects

Only Anatomy:
$n(A \cap B' \cap C') = 38 - (10+5+8) = 15$
Only Biology:
$n(B \cap A' \cap C') = 31 - (10+5+2) = 14$
Only Chemistry:
$n(C \cap A' \cap B') = 24 - (8+5+2) = 9$



Step 4: Students studying none of the subjects

$n(A' \cap B' \cap C')$
$= n(A \cup B \cup C)'$
$= 70 - (15+10+14+8+5+2+9) = 7$



We now use the completed Venn diagram to answer several questions:
How many of these students are studying ....
1) Anatomy and Biology, but not Chemistry? 10
2) Anatomy or Biology, but not Chemistry? 15 + 10 + 14 = 39
3) Only Biology? 14
4) only one of these subjects? 15 + 14 + 9 = 38
5) only two of these subjects? 10 + 8 + 2 = 20
6) none of these subjects? 7