Guided Notes

Section 10.5 Conic Sections

Summary

circles

$$(x-h)^2+(y-k)^2=r^2$$

Ellipse

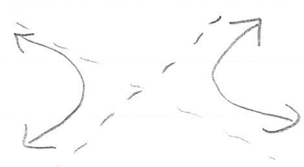
$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

major axis = larger of a or b.

Hyperbola

(on the + axis)

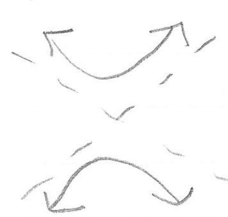
$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$$



+x

on x

$$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$$



+y

on y

parabola

$$y=a(x-h)^2+k$$

$$\frac{1}{a}(y-k)=(x-h)^2 \quad \text{or} \quad \frac{1}{a}(x-h)=(y-k)^2$$

$\boxed{a=4p}$

Let's explore with more detail

The big idea: they have the concept of distance in common

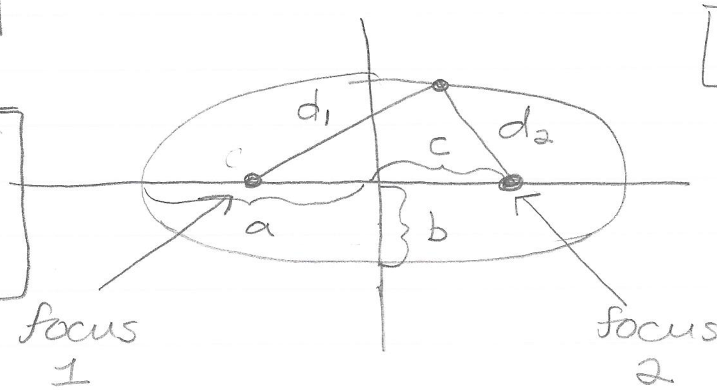
circles The set of all points that are equidistant from the center

formula $(x-h)^2 + (y-k)^2 = r^2$

Ellipse

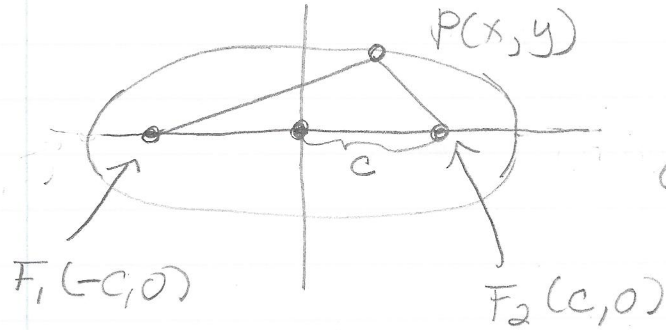
$\boxed{d_1 + d_2 = \text{constant}}$

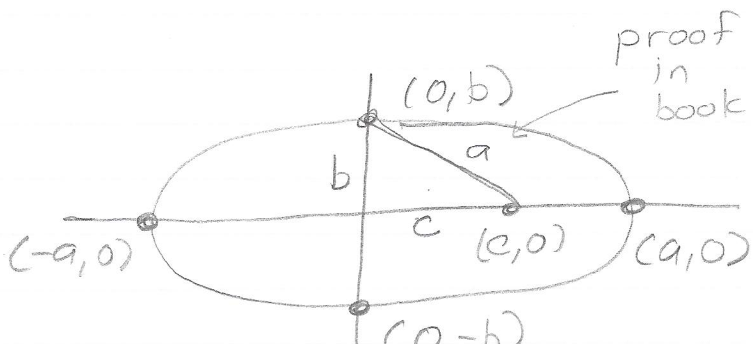
a = major axis
b = minor axis



c = distance from center to a focus point

alternate views





centered at (0, 0)

$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1} \quad a \geq b > 0$$

$$\boxed{\text{foci} = (\pm c, 0)}$$

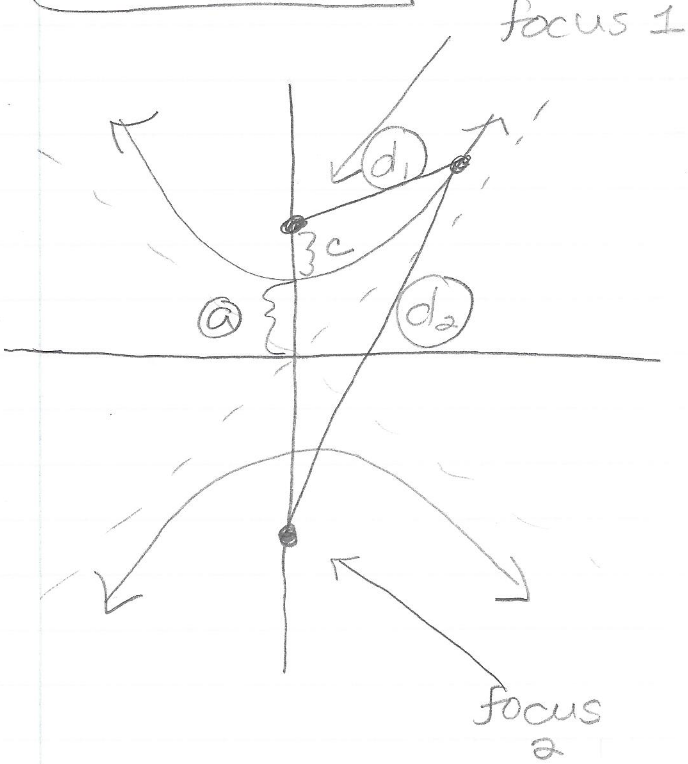
$$b^2 + c^2 = a^2$$

$$\boxed{c^2 = a^2 - b^2}$$

$$\boxed{c = \sqrt{a^2 - b^2}}$$

$$c = \sqrt{\left(\begin{smallmatrix}\text{larger}\\ \text{denom}\end{smallmatrix}\right)^2 - \left(\begin{smallmatrix}\text{smaller}\\ \text{denom}\end{smallmatrix}\right)^2}$$

Hyperbola



$$\boxed{\begin{array}{c} d_2 - d_1 \\ \text{constant} \end{array}}$$

for the hyperbola

$c = \sqrt{a^2 + b^2}$

$$\boxed{\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1}$$

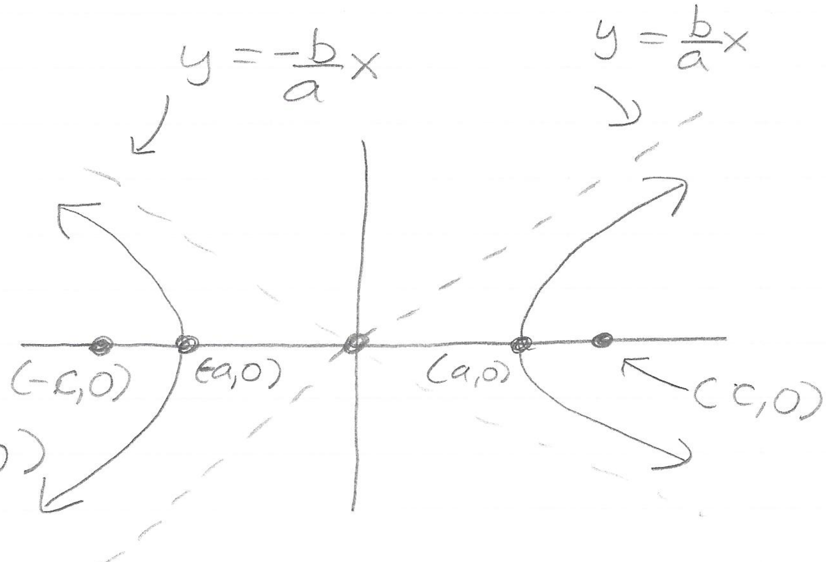
foci: $(\pm c, 0)$

$c^2 = a^2 + b^2$

vertices: $(\pm a, 0)$

(on the x)

+

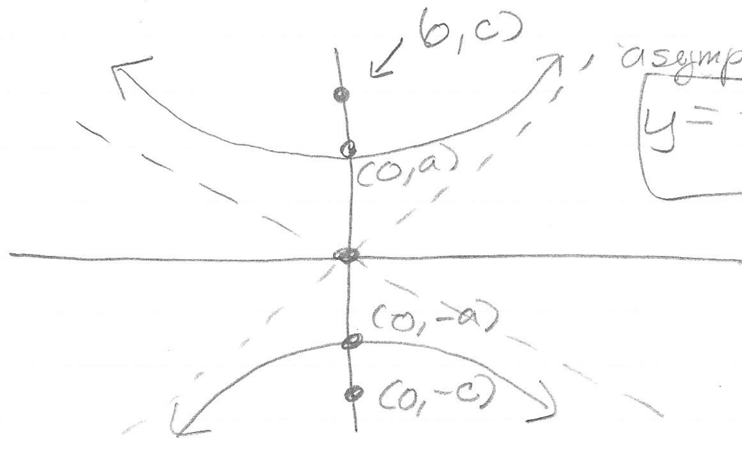


$$\boxed{\frac{y^2}{a^2} - \frac{x^2}{b} = 1}$$

foci: $(0, \pm c)$

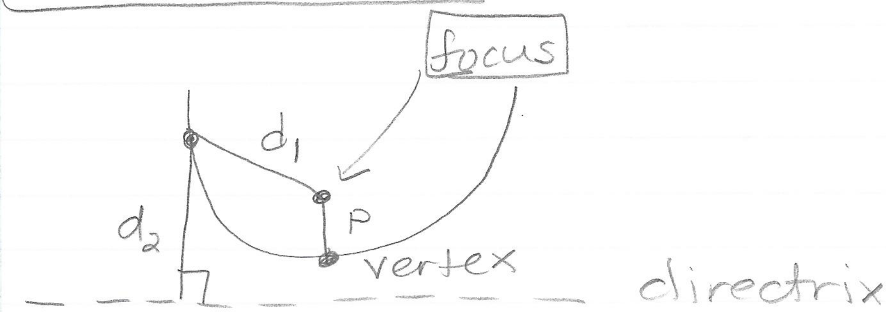
$c^2 = a^2 + b^2$

vertices: $(0, \pm a)$



$\boxed{y = \pm \frac{a}{b} x}$

The parabola



constant
$d_1 = d_2$

p = distance between the vertex and the focus.

$$\boxed{(x-h)^2 = 4p\,(y-k)}$$

Example

Graph the ellipse

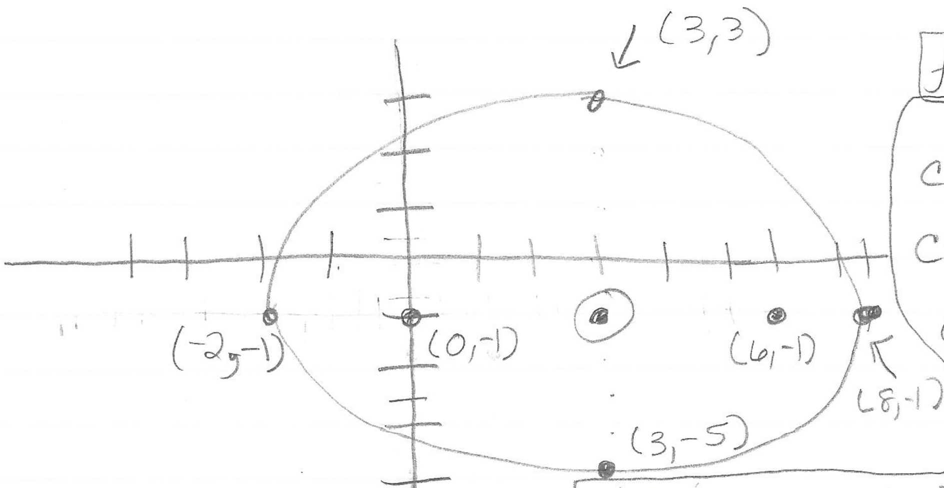
$$\frac{(x-3)^2}{25} + \frac{(y+1)^2}{16} = 1$$

foci are on the major axis

$\boxed{a=5}$ major axis (vertices)

$\boxed{b=4}$

center $(3,-1)$



$\boxed{\text{focus} = c}$

$c = \sqrt{a^2-b^2}$

$c = \sqrt{25-16}$

$c = \sqrt{9} = 3$

so the foci are 3 away from the center

$(3+3,-1) = (6,-1)$

$(3-3,-1) = (0,-1)$

Example hyperbola

$$\frac{(x-1)^2}{16} - \frac{(y+2)^2}{9} = 1$$

$a=4$ $b=3$

* vertices from here

$\boxed{(x-1)^2=16}$

$x-1=\pm 4$

$x = 1 \pm 4$ $x = 5, -3$

$\boxed{\text{center } (1,-2)}$

Recall

$x^2 - y^2 = 1$

opens on + axis.

focus & vertices on x-axis

note

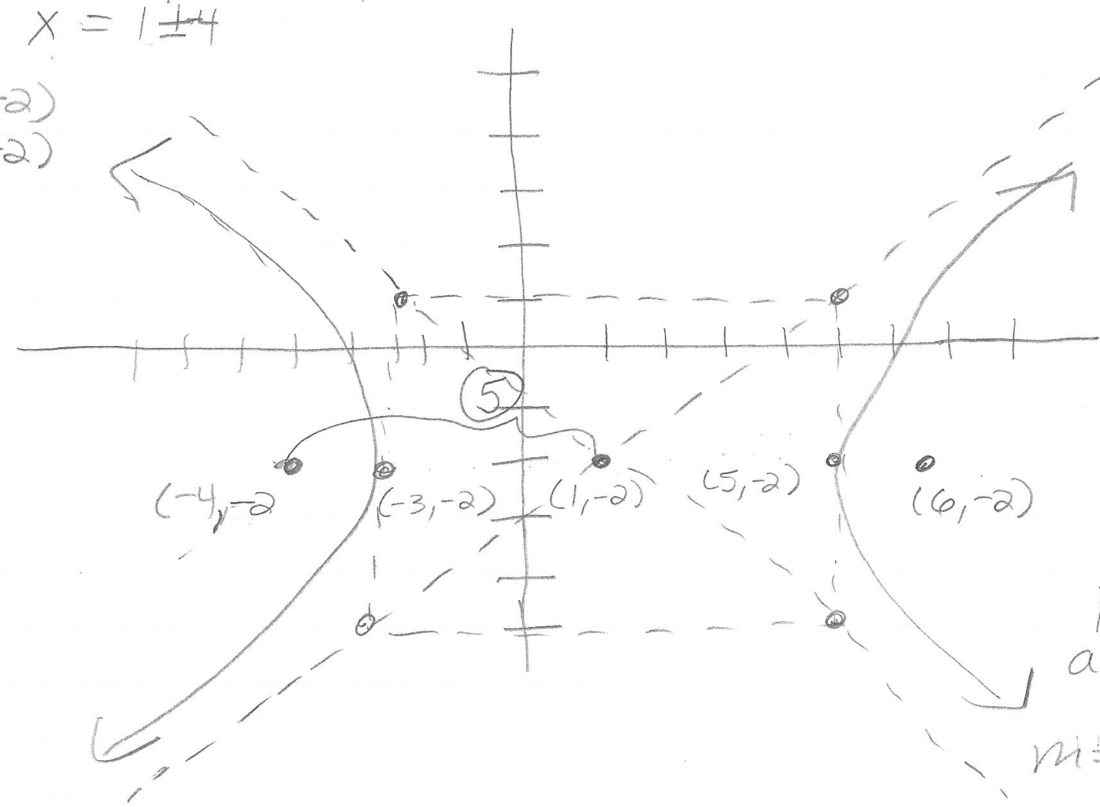
$x=0$

$y^2=-1$

but $y^2 \neq -1$

for this one vertices = $(1,0)$ $(-1,0)$

(5, -2)
(-3, -2)



x-intercepts may not be accurate on this graph

note that hyperbolas are bounded by linear asymptotes

$m = \pm\sqrt{\frac{y\,(\text{denom})}{x\,(\text{denom})}}$

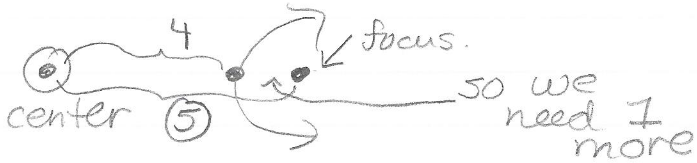
$m = \pm\sqrt{\frac{9}{16}} = \pm\frac{3}{4}$

What are the foci?

$c = \sqrt{a^2+b^2} = \sqrt{16+9} = \sqrt{25} = 5$

$y - -2 = \pm\frac{3}{4}(x-1)$

Equations of asymptotes

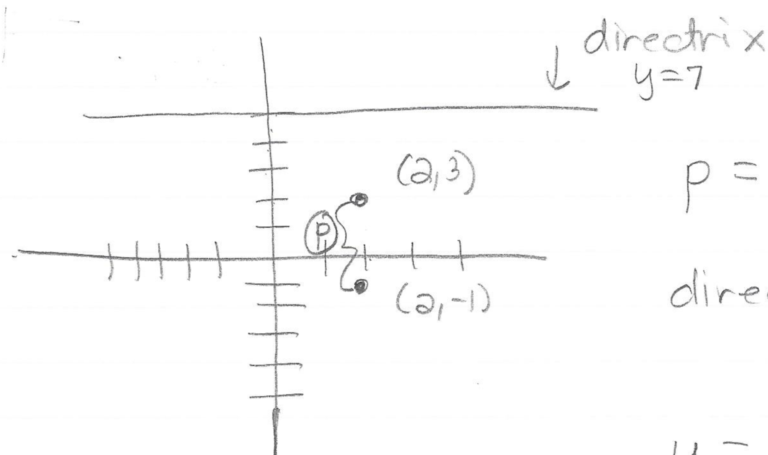


Example | Parabola

Find the equation of the parabola with focus $(2,-1)$ and vertex $(2,3)$. Graph the parabola

$$(x-h)^2 = 4p\,(y-k)$$

vertical
opens down



$p = -1-3 = -4$

directrix = 4 units above the vertex

$y = 3+4 = 7$

$$(x-2)^2 = 4(-4)(y-3)$$

$$\boxed{(x-2)^2 = -16(y-3)}$$

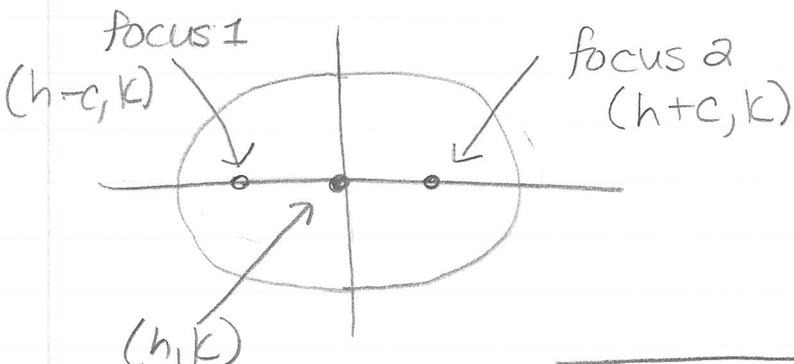
Summary of formulas for shifted conic sections

Ellipses

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

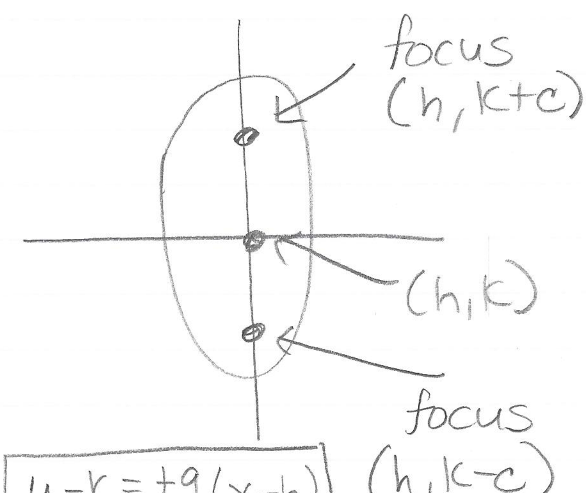
Horizontal major axis

foci $(h \pm c, k)$



Vertical major axis

foci $(h, k \pm c)$

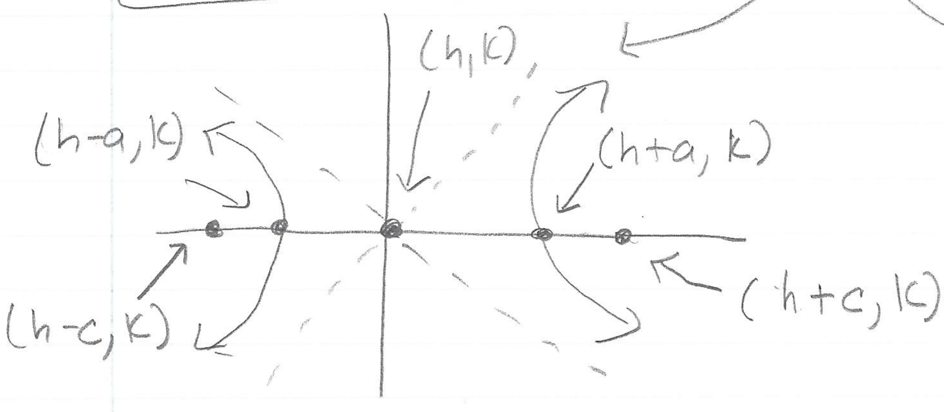


Hyperbolas

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

asymptotes

$$y-k = \pm\frac{b}{a}(x-h)$$



$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

$$y-k = \pm\frac{a}{b}(x-h)$$