Simplifying Radicals

When simplifying radicals, you are looking for "perfect groups". The size of the group is equal to the index of the radical

Ex $\sqrt{\ } = \sqrt[2]{\ }$ groups of 2

$\sqrt[3]{\ }$ groups of 3

$\sqrt[4]{\ }$ groups of 4

Example 1 Simplify $\sqrt{45x^3y^5z^4}$ groups of 2

$$\begin{array}{r|l} 3 & 45 \\ 3 & 15 \\ & 5 \end{array}$$

$3 \cdot 3 \cdot 5 \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y \cdot z \cdot z \cdot z \cdot z$

$= \boxed{3xy^2z^2\sqrt{5xy}}$

Example 2 Simplify $\sqrt[3]{240x^5y^7z^4w^2}$

groups of 3

$$\begin{array}{r|l} 2 & 240 \\ 2 & 120 \\ 2 & 60 \\ 2 & 30 \\ 3 & 15 \\ & 5 \end{array}$$

$x \cdot x \cdot x \cdot x \cdot x$

$y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y$

$z \cdot z \cdot z \cdot z$

$w \cdot w$

$$\boxed{2xy^2z\sqrt[3]{30x^2yzw^2}}$$

Simplifying Radicals (continued)

Example 3 Simplify $\sqrt[4]{240x^3y^{10}z^6}$

$2 \,|\, 240$
$2 \,|\, 120$
$2 \,|\, 60$
$2 \,|\, 30$
$3 \,|\, 15$
5

$x \cdot x \cdot x \cdot$

$y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \; y \cdot y$

$z \cdot z \cdot z \cdot z \; z \cdot z$

$= 2y^2z \sqrt[4]{3 \cdot 5x^3y^2z^2}$

$= \boxed{2y^2z \sqrt[4]{15x^3y^2z^2}}$

Example 4 Simplify $\sqrt[5]{320x^7y^5z^{10}}$

$2 \,|\, 320$
$2 \,|\, 160$
$2 \,|\, 80$
$2 \,|\, 40$
$2 \,|\, 20$
$2 \,|\, 10$
5

$x \cdot x \cdot x \cdot x \, x \; x \; x$

$y \; y \; y \; y \; y$

$z \; z \; z \; z \; z \; z \; z \; z \; z \; z$

$= \boxed{2xyz^2 \sqrt[5]{10x^2}}$

The shorter way to simplify the radicals is to divide the index into each power. The remainder must stay inside the radical. (underneath)

$3 \,|\, 5 = 1$ ← 1 group of 3
$\underline{3}$
2 ← 2 left over

$\sqrt[3]{x^5y^6z^2} =$

$\boxed{x y^2 \sqrt{x^2z^2}}$

Adding & Subtracting Radicals

Only like terms can be added together. Imagine each radical is just an x or y. Always simplify first.

Example 1 $\quad 2\sqrt{5} + 3\sqrt{2} - 4\sqrt{5} + 4\sqrt{2}$

$*\quad ☺ \quad * \quad ☺$

$2-4$	$3+4$
$\sqrt{5}$	$\sqrt{2}$

$$\boxed{-2\sqrt{5} + 7\sqrt{2}}$$

Example 2 $\quad \sqrt[3]{54y^5} + 4\sqrt[3]{2y^2}$

$2\underline{|54}$
$3\underline{|27}$
$3\underline{|9}$
3

$3y\sqrt[3]{2y^2} + 4\sqrt[3]{2y^2}$

$$\boxed{(3y+4)\sqrt[3]{2y^2}}$$

Example 3

$3\cdot3\cdot5$

$$7\sqrt{80x^3y} - 2x\sqrt{125xy} + 3\sqrt{45x^3y}$$

$$7\cdot2\cdot2\cdot x\sqrt{5xy} - 2x(5)\sqrt{5xy} + 3\cdot3\cdot x\sqrt{5xy}$$

$2\underline{|80}$ $\quad 5\underline{|125}$
$2\underline{|40}$ $\quad 5\underline{|25}$
$2\underline{|20}$ $\quad 5$
$2\underline{|10}$
5

$$(28x - 10x - 9x)\sqrt{5xy}$$

$$\boxed{9x\sqrt{5xy}}$$

Multiplying Radicals
(acts like distribution / FOIL)

When the index of each radical being multiplied is the same, just multiply the parts inside the radicals. You can simplify either at the beginning or the end of this process.

Example 1

$$\sqrt{3}(\sqrt{2}+3) = \boxed{\sqrt{6}+3\sqrt{3}}$$

Example 2

$$(\sqrt{3}-2)(\sqrt{5}-4)$$
$$= \boxed{\sqrt{15}-4\sqrt{3}-2\sqrt{5}+8}$$

combine like terms when you can.

Example 3

$$(2\sqrt{2}-\sqrt{3})^2$$ FOIL!!

$$= (2\sqrt{2}-\sqrt{3})(2\sqrt{2}-\sqrt{3})$$
$$= 4(2)-2\sqrt{6}-2\sqrt{6}+9$$
$$= 8-4\sqrt{6}+9$$
$$= \boxed{17-4\sqrt{6}}$$

Dividing Fractions | Rationalizing Denominators

The object of rationalizing is to eliminate the radical from the denominator. To do so, you must multiply the numerator and denominator by the same quantity. (a quantity that supplies what is missing).

Example 1 — Rationalize $\frac{2x}{\sqrt[3]{2xy^2}}$

$$\frac{2x}{\sqrt[3]{2xy^2}} \cdot \frac{\sqrt[3]{4x^2y}}{\sqrt[3]{4x^2y}} = \frac{\cancel{2x}\sqrt[3]{4x^2y}}{\cancel{2x}y} = \boxed{\frac{\sqrt[3]{4x^2y}}{y}}$$

need groups of 3

have: 2 · x · y · y

need: 2 · 2 x · x y

Example 2 — Rationalize $\frac{5}{\sqrt{18xy^3}}$

$$\frac{5}{\sqrt{18xy^3}} = \frac{5}{3y\sqrt{2xy}} \cdot \frac{\sqrt{2xy}}{\sqrt{2xy}} = \frac{5\sqrt{2xy}}{3y(2xy)} = \boxed{\frac{5\sqrt{2xy}}{6xy^2}}$$

$2 (3 \cdot 3) x (y \cdot y) y$

(simplify first if you can)

have: 2 x y

need: 2 x y

Rationalizing Continued

If a denominator contains addition or subtraction, you must multiply by the conjugate

	conjugate
$3+2i$	$3-2i$
$5-4i$	$5+4i$
$-4+5i$	$-4-5i$

Example 1 Rationalize $\frac{3}{2+\sqrt{5}}$

$$\frac{3}{(2+\sqrt{5})}\cdot\frac{(2-\sqrt{5})}{(2-\sqrt{5})} = \frac{6-3\sqrt{5}}{\underbrace{4-\cancel{2\sqrt{5}}+\cancel{2\sqrt{5}}-5}_{-1}}$$

$$= \frac{6-3\sqrt{5}}{-1} = \boxed{-6+3\sqrt{5}}$$

Example 2 Rationalize $\frac{5x}{2\sqrt{3}-\sqrt{7}}$

$$\frac{5x}{(2\sqrt{3}-\sqrt{7})}\cdot\frac{(2\sqrt{3}+\sqrt{7})}{(2\sqrt{3}+\sqrt{7})} = \frac{10x\sqrt{3}+5x\sqrt{7}}{4(3)+\cancel{2\sqrt{21}}-\cancel{2\sqrt{21}}-7}$$

$$= \frac{10x\sqrt{3}+5x\sqrt{7}}{5}$$

$$= \boxed{2x\sqrt{3}+x\sqrt{7}}$$