FAQs

What to Use as *x* and *y*, and How to Interpret a Linear Model

Q: *In a problem where I must find a linear relationship between two quantities, which quantity do I use as x and which do I use as y?*

A: The key is to decide which of the two quantities is the independent variable, and which is the dependent variable. Then use the independent variable as x and the dependent variable as y. In other words, *y depends on x.*

Here are examples of phrases that convey this information, usually of the form *Find y [dependent variable] in terms of x [independent variable]:*

- Find the cost in terms of the number of items. y = Cost, x = # Items
- How does color depend on wavelength? y = Color, x = Wavelength

If no information is conveyed about which variable is intended to be independent, then you can use whichever is convenient.

Q: *How do I interpret a general linear model $y = mx + b$?*

A: The key to interpreting a linear model is to remember the units we use to measure m and b:

The slope m is measured in units of y per unit of x; the intercept b is measured in units of y.

For instance, if $y = 4.3x + 8.1$ and you know that x is measured in feet and y in kilograms, then you can already say, "y is 8.1 kilograms when $x = 0$ feet, and increases at a rate of 4.3 kilograms per foot" without even knowing anything more about the situation!

1.3 EXERCISES

▼ more advanced ◆ challenging

T indicates exercises that should be solved using technology

In Exercises 1–6, a table of values for a linear function is given. Fill in the missing value and calculate m in each case.

1.

x	−1	0	1
y	5	8	

2.

x	−1	0	1
y	−1	−3	

3.

x	2	3	5
$f(x)$	−1	−2	

4.

x	2	4	5
$f(x)$	−1	−2	

5.

x	−2	0	2
$f(x)$	4		10

6.

x	0	3	6
$f(x)$	−1		−5

In Exercises 7–10, first find f(0), if not supplied, and then find the equation of the given linear function.

7.

x	−2	0	2	4
$f(x)$	−1	−2	−3	−4

8.

x	−6	−3	0	3
$f(x)$	1	2	3	4

9.

x	−4	−3	−2	−1
$f(x)$	−1	−2	−3	−4

10.

x	1	2	3	4
$f(x)$	4	6	8	10

In each of Exercises 11–14, decide which of the two given functions is linear and find its equation. HINT [See Example 1.]

11.

x	0	1	2	3	4
$f(x)$	6	10	14	18	22
$g(x)$	8	10	12	16	22

12.

x	−10	0	10	20	30
$f(x)$	−1.5	0	1.5	2.5	3.5
$g(x)$	−9	−4	1	6	11

13.

x	0	3	6	10	15
$f(x)$	0	3	5	7	9
$g(x)$	−1	5	11	19	29

14.

x	0	3	5	6	9
$f(x)$	2	6	9	12	15
$g(x)$	−1	8	14	17	26

In Exercises 15–24, find the slope of the given line, if it is defined.

15. $y = -\frac{3}{2}x - 4$

16. $y = \frac{2x}{3} + 4$

17. $y = \frac{x+1}{6}$

18. $y = -\frac{2x-1}{3}$

19. $3x + 1 = 0$

20. $8x - 2y = 1$

21. $3y + 1 = 0$

22. $2x + 3 = 0$

23. $4x + 3y = 7$

24. $2y + 3 = 0$

In Exercises 25–38, graph the given equation. HINT [See Quick Examples on page 77.]

25. $y = 2x - 1$

26. $y = x - 3$

27. $y = -\frac{2}{3}x + 2$

28. $y = -\frac{1}{2}x + 3$

29. $y + \frac{1}{4}x = -4$

30. $y - \frac{1}{4}x = -2$

31. $7x - 2y = 7$

32. $2x - 3y = 1$

33. $3x = 8$

34. $2x = -7$

35. $6y = 9$

36. $3y = 4$

37. $2x = 3y$

38. $3x = -2y$

In Exercises 39–54, calculate the slope, if defined, of the straight line through the given pair of points. Try to do as many as you can without writing anything down except the answer. HINT [See Quick Examples on page 79.]

39. $(0, 0)$ and $(1, 2)$

40. $(0, 0)$ and $(-1, 2)$

41. $(-1, -2)$ and $(0, 0)$

42. $(2, 1)$ and $(0, 0)$

43. $(4, 3)$ and $(5, 1)$

44. $(4, 3)$ and $(4, 1)$

45. $(1, -1)$ and $(1, -2)$

46. $(-2, 2)$ and $(-1, -1)$

47. $(2, 3.5)$ and $(4, 6.5)$

48. $(10, -3.5)$ and $(0, -1.5)$

49. $(300, 20.2)$ and $(400, 11.2)$

50. $(1, -20.2)$ and $(2, 3.2)$

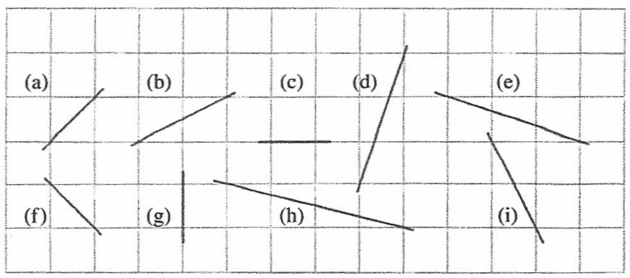
51. $(0, 1)$ and $\left(-\frac{1}{2}, \frac{3}{4}\right)$

52. $\left(\frac{1}{2}, 1\right)$ and $\left(-\frac{1}{2}, \frac{3}{4}\right)$

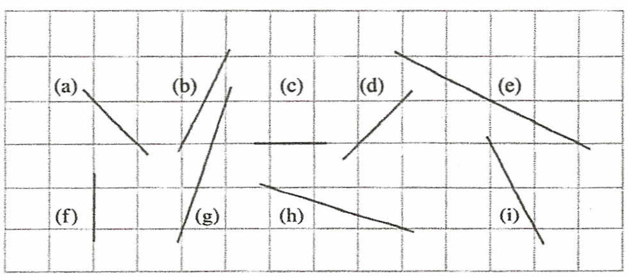
53. (a, b) and (c, d) $(a \neq c)$

54. (a, b) and (c, b) $(a \neq c)$

55. In the following figure, estimate the slopes of all line segments.



56. In the following figure, estimate the slopes of all line segments.



In Exercises 57–74, find a linear equation whose graph is the straight line with the given properties. HINT [See Example 2.]

57. Through $(1, 3)$ with slope 3

58. Through $(2, 1)$ with slope 2

59. Through $(1, -\frac{3}{4})$ with slope $\frac{1}{4}$

60. Through $(0, -\frac{1}{3})$ with slope $\frac{1}{3}$

61. Through $(20, -3.5)$ and increasing at a rate of 10 units of y per unit of x.

62. Through $(3.5, -10)$ and increasing at a rate of 1 unit of y per 2 units of x.

63. Through $(2, -4)$ and $(1, 1)$

64. Through $(1, -4)$ and $(-1, -1)$

65. Through $(1, -0.75)$ and $(0.5, 0.75)$

66. Through $(0.5, -0.75)$ and $(1, -3.75)$

67. Through $(6, 6)$ and parallel to the line $x + y = 4$

68. Through $(1/3, -1)$ and parallel to the line $3x - 4y = 8$

69. Through $(0.5, 5)$ and parallel to the line $4x - 2y = 11$

70. Through $(1/3, 0)$ and parallel to the line $6x - 2y = 11$

71. ▼ Through $(0, 0)$ and (p, q)

72. ▼ Through (p, q) and parallel to $y = rx + s$

73. ▼ Through (p, q) and (r, q) $(p \neq r)$

74. ▼ Through (p, q) and (r, s) $(p \neq r)$

APPLICATIONS

75. ***Cost*** The RideEm Bicycles factory can produce 100 bicycles in a day at a total cost of $10,500 and it can produce 120 bicycles in a day at a total cost of $11,000. What are the company's daily fixed costs, and what is the marginal cost per bicycle? HINT [See Example 3.]

76. ***Cost*** A soft-drink manufacturer can produce 1,000 cases of soda in a week at a total cost of $6,000, and 1,500 cases of soda at a total cost of $8,500. Find the manufacturer's weekly fixed costs and marginal cost per case of soda.

77. ***Cost: iPods*** It cost Apple approximately $800 to manufacture 5 30-gigabyte video iPods and $3,700 to manufacture 25.* Obtain the corresponding linear cost function. What was the cost to manufacture each additional iPod? Use the cost function to estimate the cost of manufacturing 100 iPods.

78. ***Cost: Xboxes*** If it costs Microsoft $4,500 to manufacture 8 Xbox 360s and $8,900 to manufacture 16,† obtain the corresponding linear cost function. What was the cost to manufacture each additional Xbox? Use the cost function to estimate the cost of manufacturing 50 Xboxes.

79. ***Demand*** Sales figures show that your company sold 1,960 pen sets each week when they were priced at $1/pen set and 1,800 pen sets each week when they were priced at $5/pen set. What is the linear demand function for your pen sets? HINT [See Example 4.]

80. ***Demand*** A large department store is prepared to buy 3,950 of your neon-colored shower curtains per month for $5 each, but only 3,700 shower curtains per month for $10 each. What is the linear demand function for your neon-colored shower curtains?

81. ***Demand for Cell Phones*** The following table shows worldwide sales of Nokia cell phones and their average wholesale prices in 2004:[27]

Quarter	*Second*	*Fourth*
Wholesale Price ($)	111	105
Sales (millions)	45.4	51.4

a. Use the data to obtain a linear demand function for (Nokia) cell phones, and use your demand equation to predict sales if Nokia lowered the price further to $103.

b. Fill in the blanks: For every ____ increase in price, sales of cell phones decrease by ___ units.

82. ***Demand for Cell Phones*** The following table shows projected worldwide sales of (all) cell phones and wholesale prices:[28]

Year	*2004*	*2008*
Wholesale Price ($)	100	80
Sales (millions)	600	800

a. Use the data to obtain a linear demand function for cell phones, and use your demand equation to predict sales if the price was set at $85.

b. Fill in the blanks: For every ____ increase in price, sales of cell phones decrease by ___ units.

83. ***Demand for Monorail Service, Las Vegas*** In 2005, the Las Vegas monorail charged $3 per ride and had an average ridership of about 28,000 per day. In December, 2005 the Las Vegas Monorail Company raised the fare to $5 per ride, and average ridership in 2006 plunged to around 19,000 per day.[29]

a. Use the given information to find a linear demand equation.

b. Give the units of measurement and interpretation of the slope.

c. What would be the effect on ridership of raising the fare to $6 per ride?

*Source for cost data: Manufacturing & Technology News, July 31, 2007, Volume 14, No. 14, www.manufacturingnews.com.

†Source for estimate of marginal cost: www.isuppli.com.

[27] Source: Embedded.com/Companyreports December 2004.

[28] Wholesale price projections are the authors'. Source for sales prediction: I-Stat/NDR December, 2004.

[29] Source: *New York Times,* February 10, 2007, p. A9.

84. ***Demand for Monorail Service, Mars*** The Utarek monorail, which links the three urbynes (or districts) of Utarek, Mars, charged Ƶ5 per ride[30] and sold about 14 million rides per day. When the Utarek City Council lowered the fare to Ƶ3 per ride, the number of rides increased to 18 million per day.

a. Use the given information to find a linear demand equation.
b. Give the units of measurement and interpretation of the slope.
c. What would be the effect on ridership of raising the fare to Ƶ10 per ride?

85. ***Equilibrium Price*** You can sell 90 pet chias per week if they are marked at \$1 each, but only 30 each week if they are marked at \$2/chia. Your chia supplier is prepared to sell you 20 chias each week if they are marked at \$1/chia, and 100 each week if they are marked at \$2 per chia.

a. Write down the associated linear demand and supply functions.
b. At what price should the chias be marked so that there is neither a surplus nor a shortage of chias? HINT [See Example 4.]

86. ***Equilibrium Price*** The demand for your college newspaper is 2,000 copies each week if the paper is given away free of charge, and drops to 1,000 each week if the charge is 10¢/copy. However, the university is prepared to supply only 600 copies per week free of charge, but will supply 1,400 each week at 20¢ per copy.

a. Write down the associated linear demand and supply functions.
b. At what price should the college newspapers be sold so that there is neither a surplus nor a shortage of papers?

87. ***Pasta Imports*** During the period 1990–2001, U.S. imports of pasta increased from 290 million pounds in 1990 ($t = 0$) by an average of 40 million pounds/year.[31]

a. Use these data to express q, the annual U.S. imports of pasta (in millions of pounds), as a linear function of t, the number of years since 1990.
b. Use your model to estimate U.S. pasta imports in 2005, assuming the import trend continued.

88. ***Mercury Imports*** During the period 2210–2220, Martian imports of mercury (from the planet of that name) increased from 550 million kg in 2210 ($t = 0$) by an average of 60 million kg/year.

a. Use these data to express h, the annual Martian imports of mercury (in millions of kilograms), as a linear function of t, the number of years since 2010.
b. Use your model to estimate Martian mercury imports in 2230, assuming the import trend continued.

89. ***Satellite Radio Subscriptions*** The number of Sirius Satellite Radio subscribers grew from 0.3 million in 2003 to 3.2 million in 2005.[32]

a. Use this information to find a linear model for the number N of subscribers (in millions) as a function of time t in years since 2000.
b. Give the units of measurement and interpretation of the slope.
c. Use the model from part (a) to predict the 2006 figure. (The actual 2006 figure was approximately 6 million subscribers.)

90. ***Freon Production*** The production of ozone-layer damaging Freon 22 (chlorodifluoromethane) in developing countries rose from 200 tons in 2004 to a projected 590 tons in 2010.[33]

a. Use this information to find a linear model for the amount F of Freon 22 (in tons) as a function of time t in years since 2000.
b. Give the units of measurement and interpretation of the slope.
c. Use the model from part (a) to estimate the 2008 figure and compare it with the actual projection of 400 tons.

91. ***Velocity*** The position of a model train, in feet along a railroad track, is given by

$$s(t) = 2.5t + 10$$

after t seconds.

a. How fast is the train moving?
b. Where is the train after 4 seconds?
c. When will it be 25 feet along the track?

92. ***Velocity*** The height of a falling sheet of paper, in feet from the ground, is given by

$$s(t) = -1.8t + 9$$

after t seconds.

a. What is the velocity of the sheet of paper?
b. How high is it after 4 seconds?
c. When will it reach the ground?

93. ▽ ***Fast Cars*** A police car was traveling down Ocean Parkway in a high speed chase from Jones Beach. It was at Jones Beach at exactly 10 pm ($t = 10$) and was at Oak Beach, 13 miles from Jones Beach, at exactly 10:06 pm.

a. How fast was the police car traveling? HINT [See Example 6.]
b. How far was the police car from Jones Beach at time t?

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[30] Ƶ designates Zonars, the official currency in Mars. See www.marsnext.com for details of the Mars colony, its commerce, and its culture.

[31] Data are rounded. Sources: Department of Commerce/*New York Times*, September 5, 1995, p. D4; International Trade Administration, March 2002, www.ita.doc.gov/.

[32] Figures are approximate. Source: Sirius Satellite Radio/*New York Times*, February 20, 2008, p. A1.

[33] Figures are approximate. Source: Lampert Kuijpers (Panel of the Montreal Protocol), National Bureau of Statistics in China, via CEIC DSata/*New York Times*, February 23, 2007, p. C1.

Substituting these values into the formula we get

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{n(\sum x^2) - (\sum x)^2} \cdot \sqrt{n(\sum y^2) - (\sum y)^2}}$$

$$= \frac{8(670) - (56)(88)}{\sqrt{8(560) - 56^2} \cdot \sqrt{8(986) - 88^2}}$$

$$\approx 0.982.$$

Thus, the fit is a fairly good one, that is, the original points lie nearly along a straight line, as can be confirmed from the graph in Example 2.

1.4 EXERCISES

▼ more advanced ◆ challenging

[T] indicates exercises that should be solved using technology

In Exercises 1–4, compute the sum-of-squares error (SSE) by hand for the given set of data and linear model. HINT [See Example 1.]

1. (1, 1), (2, 2), (3, 4); $y = x - 1$

2. (0, 1), (1, 1), (2, 2); $y = x + 1$

3. (0, −1), (1, 3), (4, 6), (5, 0); $y = -x + 2$

4. (2, 4), (6, 8), (8, 12), (10, 0); $y = 2x - 8$

[T] *In Exercises 5–8, use technology to compute the sum-of-squares error (SSE) for the given set of data and linear models. Indicate which linear model gives the better fit.*

5. (1, 1), (2, 2), (3, 4); **a.** $y = 1.5x - 1$ **b.** $y = 2x - 1.5$

6. (0, 1), (1, 1), (2, 2); **a.** $y = 0.4x + 1.1$ **b.** $y = 0.5x + 0.9$

7. (0, −1), (1, 3), (4, 6), (5, 0); **a.** $y = 0.3x + 1.1$ **b.** $y = 0.4x + 0.9$

8. (2, 4), (6, 8), (8, 12), (10, 0); **a.** $y = -0.1x + 7$ **b.** $y = -0.2x + 6$

Find the regression line associated with each set of points in Exercises 9–12. Graph the data and the best-fit line. (Round all coefficients to 4 decimal places.) HINT [See Example 2.]

9. (1, 1), (2, 2), (3, 4)

10. (0, 1), (1, 1), (2, 2)

11. (0, −1), (1, 3), (4, 6), (5, 0)

12. (2, 4), (6, 8), (8, 12), (10, 0)

In the next two exercises, use correlation coefficients to determine which of the given sets of data is best fit by its associated regression line and which is fit worst. Is it a perfect fit for any of the data sets? HINT [See Example 4.]

13. a. {(1, 3), (2, 4), (5, 6)}
b. {(0, −1), (2, 1), (3, 4)}
c. {(4, −3), (5, 5), (0, 0)}

14. a. {(1, 3), (−2, 9), (2, 1)}
b. {(0, 1), (1, 0), (2, 1)}
c. {(0, 0), (5, −5), (2, −2.1)}

APPLICATIONS

15. ***Worldwide Cell Phone Sales*** Following are forecasts of worldwide annual cell phone handset sales:[45]

Year x	3	5	7
Sales y (millions)	500	600	800

($x = 3$ represents 2003). Complete the following table and obtain the associated regression line. (Round coefficients to 2 decimal places.) HINT [See Example 3.]

	x	y	xy	x^2
	3	500		
	5	600		
	7	800		
Totals				

Use your regression equation to project the 2008 sales.

16. ***Investment in Gold*** Following are approximate values of the Amex Gold BUGS Index:[46]

Year x	0	4	7
Index y	50	250	470

[45] Source: In-StatMDR, www.in-stat.com/, July, 2004.

[46] BUGS stands for "basket of unhedged gold stocks." Figures are approximate. Sources: www.321gold.com, Bloomberg Financial Markets/*New York Times*, Sept 7, 2003, p. BU8, www.amex.com.

($x = 0$ represents 2000). Complete the following table and obtain the associated regression line. (Round coefficients to 2 decimal places.)

	x	y	xy	x^2
	0	50		
	4	250		
	7	470		
Totals				

Use your regression equation to estimate the 2005 index to the nearest whole number.

17. ***E-Commerce*** The following chart shows second quarter total retail e-commerce sales in the United States in 2000, 2004, and 2007 ($t = 0$ represents 2000):[47]

Year t	0	4	7
Sales ($ billion)	6	16	32

Find the regression line (round coefficients to two decimal places) and use it to estimate second quarter retail e-commerce sales in 2006.

18. ***Retail Inventories*** The following chart shows total January retail inventories in the United States in 2000, 2005, and 2007 ($t = 0$ represents 2000):[48]

Year t	0	5	7
Inventory ($ billion)	380	450	480

Find the regression line (round coefficients to two decimal places) and use it to estimate January retail inventories in 2004.

19. ***Oil Recovery*** The Texas Bureau of Economic Geology published a study on the economic impact of using carbon dioxide enhanced oil recovery (EOR) technology to extract additional oil from fields that have reached the end of their conventional economic life. The following table gives the approximate number of jobs for the citizens of Texas that would be created at various levels of recovery.[49]

Percent Recovery (%)	20	40	80	100
Jobs Created (millions)	3	6	9	15

Find the regression line and use it to estimate the number of jobs that would be created at a recovery level of 50%.

20. ***Oil Recovery*** (Refer to Exercise 19.) The following table gives the approximate economic value associated with various levels of oil recovery in Texas.[50]

Percent Recovery (%)	10	40	50	80
Economic Value ($ billions)	200	900	1000	2000

Find the regression line and use it to estimate the economic value associated with a recovery level of 70%.

21. ***Soybean Production*** The following table shows soybean production, in millions of tons, in Brazil's *Cerrados* region, as a function of the cultivated area, in millions of acres.[51]

Area (millions of acres)	25	30	32	40	52
Production (millions of tons)	15	25	30	40	60

a. Use technology to obtain the regression line, and to show a plot of the points together with the regression line. (Round coefficients to two decimal places.)
b. Interpret the slope of the regression line.

22. ***Trade with Taiwan*** The following table shows U.S. exports to Taiwan as a function of U.S. imports from Taiwan, based on trade figures in the period 1990–2003.[52]

Imports ($ billions)	22	24	27	35	25
Exports ($ billions)	12	15	20	25	17

a. Use technology to obtain the regression line, and to show a plot of the points together with the regression line. (Round coefficients to two decimal places.)
b. Interpret the slope of the regression line.

Exercises 23 and 24 are based on the following table comparing the number of registered automobiles, trucks, and motorcycles in Mexico for various years from 1980 to 2005.[53]

Year	**Automobiles (millions)**	**Trucks (millions)**	**Motorcycles (millions)**
1980	4.0	1.5	0.28
1985	5.3	2.1	0.25
1990	6.6	3.0	0.25
1995	7.5	3.6	0.13
2000	10.2	4.9	0.29
2005	14.7	7.1	0.61

[47] Figures are rounded. Source: US Census Bureau, www.census.gov, November 2007.
[48] Ibid.
[49] Source: "CO2–Enhanced Oil Recovery Resource Potential in Texas: Potential Positive Economic Impacts," Texas Bureau of Economic Geology, April 2004, www.rrc.state.tx.us/tepc/CO2-EOR_white_paper.pdf.
[50] Ibid.
[51] Source: Brazil Agriculture Ministry/*New York Times*, December 12, 2004, p. N32.
[52] Source: Taiwan Directorate General of Customs/*New York Times*, December 13, 2004, p. C7.
[53] Source: Instituto Nacional de Estadística y Geografía (INEGI), www.inegi.org.mx.