

5 Assess Your Understanding

Are You Prepared?' Answers are given at the end of these exercises. If you get a wrong answer, read the pages listed in red.

1. Graph the inequality: $x \geq -2$. (pp. 17–26)

2. **True or False** $-5 > -3$ (pp. 17–26) False

Concepts and Vocabulary

3. If each side of an inequality is multiplied by a(n) negative number, then the sense of the inequality symbol is reversed.

4. A(n) closed interval, denoted $[a, b]$, consists of all real numbers x for which $a \leq x \leq b$.

5. The Multiplication Properties state that the sense, or direction, of an inequality remains the same if each side is multiplied by a positive number, while the direction is reversed if each side is multiplied by a negative number.

In Problems 6–9, assume that $a < b$ and $c < 0$.

6. **True or False** $a + c < b + c$ True

7. **True or False** $a - c < b - c$ True

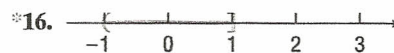
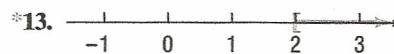
8. **True or False** $ac > bc$ True

9. **True or False** $\frac{a}{c} < \frac{b}{c}$ False

10. **True or False** The square of any real number is always nonnegative. True

Building

Problems 11–16, express the graph shown in blue using interval notation. Also express each as an inequality involving x .



Problems 17–22, an inequality is given. Write the inequality obtained by:

17. Adding 3 to each side of the given inequality.

18. Subtracting 5 from each side of the given inequality.

19. Multiplying each side of the given inequality by 3.

20. Multiplying each side of the given inequality by -2 .

21. $3 < 5$ *18. $2 > 1$ *19. $4 > -3$ *20. $-3 > -5$ *21. $2x + 1 < 2$ *22. $1 - 2x > 5$

Problems 23–30, write each inequality using interval notation, and illustrate each inequality using the real number line.

23. $0 \leq x \leq 4$ *24. $-1 < x < 5$ *25. $4 \leq x < 6$ *26. $-2 < x < 0$

27. $x \geq 4$ *28. $x \leq 5$ *29. $x < -4$ *30. $x > 1$

Problems 31–38, write each interval as an inequality involving x , and illustrate each inequality using the real number line.

31. $[-2, 5]$ *32. $(1, 2)$ *33. $(-3, -2)$ *34. $[0, 1)$

35. $(-4, \infty)$ *36. $(-\infty, 2]$ *37. $(-\infty, -3)$ *38. $(-8, \infty)$

Problems 39–52, fill in the blank with the correct inequality symbol.

39. If $x < 5$, then $x - 5$ < 0.

40. If $x < -4$, then $x + 4$ < 0.

41. If $x > -4$, then $x + 4$ > 0.

42. If $x > 6$, then $x - 6$ > 0.

43. If $x \geq -4$, then $3x$ \geq -12 .

44. If $x \leq 3$, then $2x$ \leq 6.

45. If $x > 6$, then $-2x$ < -12 .

46. If $x > -2$, then $-4x$ < 8.

47. If $x \geq 5$, then $-4x$ \leq -20 .

48. If $x \leq -4$, then $-3x$ \geq 12.

49. If $2x > 6$, then x > 3.

50. If $3x \leq 12$, then x \leq 4.

51. If $-\frac{1}{2}x \leq 3$, then x \geq -6 .

52. If $-\frac{1}{4}x > 1$, then x < -4 .

For restrictions, answers to these exercises may be found in the Answers in the back of the book.

In Problems 53–88, solve each inequality. Express your answer using set notation or interval notation. Graph the solution set.

*53. $x + 1 < 5$

*54. $x - 6 < 1$

*55. $1 - 2x \leq 3$

*56. $2 - 3x \leq 5$

*57. $3x - 7 > 2$

*58. $2x + 5 > 1$

*59. $3x - 1 \geq 3 + x$

*60. $2x - 2 \geq 3 + x$

*61. $-2(x + 3) < 8$

*62. $-3(1 - x) < 12$

*63. $4 - 3(1 - x) \leq 3$

*64. $8 - 4(2 - x) \leq -2x$

*65. $\frac{1}{2}(x - 4) > x + 8$

*66. $3x + 4 > \frac{1}{3}(x - 2)$

*67. $\frac{x}{2} \geq 1 - \frac{x}{4}$

*68. $\frac{x}{3} \geq 2 + \frac{x}{6}$

*69. $0 \leq 2x - 6 \leq 4$

*70. $4 \leq 2x + 2 \leq 10$

*71. $-5 \leq 4 - 3x \leq 2$

*72. $-3 \leq 3 - 2x \leq 9$

*73. $-3 < \frac{2x - 1}{4} < 0$

*74. $0 < \frac{3x + 2}{2} < 4$

*75. $1 < 1 - \frac{1}{2}x < 4$

*76. $0 < 1 - \frac{1}{3}x < 1$

*77. $(x + 2)(x - 3) > (x - 1)(x + 1)$

*78. $(x - 1)(x + 1) > (x - 3)(x + 4)$

*79. $x(4x + 3) \leq (2x + 1)^2$

*80. $x(9x - 5) \leq (3x - 1)^2$

*81. $\frac{1}{2} \leq \frac{x + 1}{3} < \frac{3}{4}$

*82. $\frac{1}{3} < \frac{x + 1}{2} \leq \frac{2}{3}$

*83. $(4x + 2)^{-1} < 0$

*84. $(2x - 1)^{-1} > 0$

*85. $0 < \frac{2}{x} < \frac{3}{5}$

*86. $0 < \frac{4}{x} < \frac{2}{3}$

*87. $0 < (2x - 4)^{-1} < \frac{1}{2}$

*88. $0 < (3x + 6)^{-1} < \frac{1}{3}$

Applications and Extensions

In Problems 89–98, find a and b .

89. If $-1 < x < 1$, then $a < x + 4 < b$. $a = 3$, $b = 5$

90. If $-3 < x < 2$, then $a < x - 6 < b$. $a = -9$, $b = -4$

*91. If $2 < x < 3$, then $a < -4x < b$. $a = -12$, $b = -8$

92. If $-4 < x < 0$, then $a < \frac{1}{2}x < b$. $a = -2$, $b = 0$

93. If $0 < x < 4$, then $a < 2x + 3 < b$. $a = 3$, $b = 11$

94. If $-3 < x < 3$, then $a < 1 - 2x < b$. $a = -5$, $b = 7$

95. If $-3 < x < 0$, then $a < \frac{1}{x + 4} < b$. $a = \frac{1}{4}$, $b = 1$

96. If $2 < x < 4$, then $a < \frac{1}{x - 6} < b$. $a = -\frac{1}{2}$, $b = -\frac{1}{4}$

97. If $6 < 3x < 12$, then $a < x^2 < b$. $a = 4$, $b = 16$

98. If $0 < 2x < 6$, then $a < x^2 < b$. $a = 0$, $b = 9$

*99. What is the domain of the variable in the expression $\sqrt{3x + 6}$?

100. What is the domain of the variable in the expression $\sqrt{8 + 2x}$? $\{x \mid x \geq -4\}$

101. A young adult may be defined as someone older than 21, but less than 30 years of age. Express this statement using inequalities. $21 < \text{Age} < 30$

*102. Middle-aged may be defined as being 40 or more and less than 60. Express this statement using inequalities.

103. **Life Expectancy** The Social Security Administration determined that an average 30-year-old male in 2005 could expect to live at least 46.60 more years and an average

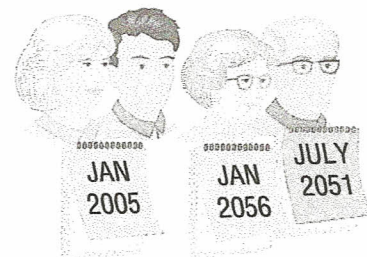
30-year-old female in 2005 could expect to live at least 51.05 more years.

*104. (a) To what age can an average 30-year-old male expect to live? Express your answer as an inequality.

(b) To what age can an average 30-year-old female expect to live? Express your answer as an inequality.

(c) Who can expect to live longer, a male or a female? By how many years?

Source: Social Security Administration, Period Life Table, 2005



104. **General Chemistry** For a certain ideal gas, the volume V (in cubic centimeters) equals 20 times the temperature T (in degrees Celsius). If the temperature varies from 80° to 120° C inclusive, what is the corresponding range of the volume of the gas? From 1600 to 2400 cm^3 , inclusive

*105. **Real Estate** A real estate agent agrees to sell an apartment complex according to the following commission schedule: \$45,000 plus 25% of the selling price in excess of \$900,000. Assuming that the complex will sell at some price between \$900,000 and \$1,100,000 inclusive, over what range

Skill Building

Problems 7–34, solve each equation.

7. $|2x| = 6$ $\{-3, 3\}$

8. $|3x| = 12$ $\{-4, 4\}$

9. $|2x + 3| = 5$ $\{-4, 1\}$

10. $|3x - 1| = 2$ $\left\{-\frac{1}{3}, 1\right\}$

11. $|1 - 4t| + 8 = 13$ $\left\{-1, \frac{3}{2}\right\}$

12. $|1 - 2z| + 6 = 9$ $\{-1, 2\}$

13. $|-2x| = |8|$ $\{-4, 4\}$

14. $|-x| = |1|$ $\{-1, 1\}$

15. $-2|x| = 4$ $\{2\}$

16. $|3|x| = 9$ $\{3\}$

17. $\frac{2}{3}|x| = 9$ $\left\{-\frac{27}{2}, \frac{27}{2}\right\}$

18. $\frac{3}{4}|x| = 9$ $\{-12, 12\}$

19. $\frac{x}{3} + \frac{2}{5} = 2$ $\left\{-\frac{36}{5}, \frac{24}{5}\right\}$

20. $\left|\frac{x}{2} - \frac{1}{3}\right| = 1$ $\left\{-\frac{4}{3}, \frac{8}{3}\right\}$

*21. $|u - 2| = -\frac{1}{2}$

*22. $|2 - v| = -1$

23. $4 - |2x| = 3$ $\left\{-\frac{1}{2}, \frac{1}{2}\right\}$

24. $5 - \left|\frac{1}{2}x\right| = 3$ $\{-4, 4\}$

25. $|x^2 - 9| = 0$ $\{-3, 3\}$

26. $|x^2 - 16| = 0$ $\{-4, 4\}$

27. $|x^2 - 2x| = 3$ $\{-1, 3\}$

28. $|x^2 + x| = 12$ $\{-4, 3\}$

*29. $|x^2 + x - 1| = 1$

*30. $|x^2 + 3x - 2| = 2$

31. $\left|\frac{3x - 2}{2x - 3}\right| = 2$ $\left\{\frac{8}{7}, 4\right\}$

32. $\left|\frac{2x + 1}{3x + 4}\right| = 1$ $\{-3, -1\}$

*33. $|x^2 + 3x| = |x^2 - 2x|$

*34. $|x^2 - 2x| = |x^2 + 6x|$

Problems 35–62, solve each inequality. Express your answer using set notation or interval notation. Graph the solution set.

35. $|2x| < 8$

*36. $|3x| < 15$

*37. $|3x| > 12$

*38. $|2x| > 6$

39. $|x - 2| + 2 < 3$

*40. $|x + 4| + 3 < 5$

*41. $|3t - 2| \leq 4$

*42. $|2u + 5| \leq 7$

43. $|2x - 3| \geq 2$

*44. $|3x + 4| \geq 2$

*45. $|1 - 4x| - 7 < -2$

*46. $|1 - 2x| - 4 < -1$

47. $|1 - 2x| > 3$

*48. $|2 - 3x| > 1$

*49. $|-4x| + |-5| \leq 1$

*50. $|-x| - |4| \leq 2$

51. $|-2x| > |-3|$

*52. $|-x - 2| \geq 1$

*53. $-|2x - 1| \geq -3$

*54. $-|1 - 2x| \geq -3$

55. $|2x| < -1$

*56. $|3x| \geq 0$

*57. $|5x| \geq -1$

*58. $|6x| < -2$

59. $\left|\frac{2x + 3}{3} - \frac{1}{2}\right| < 1$

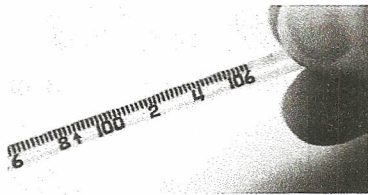
*60. $3 - |x + 1| < \frac{1}{2}$

*61. $5 + |x - 1| > \frac{1}{2}$

*62. $\left|\frac{2x - 3}{2} + \frac{1}{3}\right| > 1$

Applications and Extensions

63. **Body Temperature** “Normal” human body temperature is 98.6°F . If a temperature x that differs from normal by at least 1.5° is considered unhealthy, write the condition for an unhealthy temperature x as an inequality involving an absolute value, and solve for x .
 $|x - 98.6| \geq 1.5$;
 $x \leq 97.1$ or $x \geq 100.1$



64. **Household Voltage** In the United States, normal household voltage is 110 volts. However, it is not uncommon for actual voltage to differ from normal voltage by at most 5 volts. Express this situation as an inequality involving an absolute value. Use x as the actual voltage and solve for x .
65. **Reading Books** A Gallup poll conducted May 20–22, 2005, found that Americans read an average of 13.4 books per year. Gallup is 99% confident that the result from this poll is off by fewer than 1.35 books from the actual average x . Express this situation as an inequality involving absolute

value, and solve the inequality for x to determine the interval in which the actual average is likely to fall.

Note: In statistics, this interval is called a 99% **confidence interval**.

66. **Speed of Sound** According to data from the Hill Aerospace Museum (Hill Air Force Base, Utah), the speed of sound varies depending on altitude, barometric pressure, and temperature. For example, at 20,000 feet, 13.75 inches of mercury, and -12.3°F , the speed of sound is about 707 miles per hour, but the speed can vary from this result by as much as 55 miles per hour as conditions change.
- (a) Express this situation as an inequality involving an absolute value.
- (b) Using x for the speed of sound, solve for x to find an interval for the speed of sound.
- *67. Express the fact that x differs from 3 by less than $\frac{1}{2}$ as an inequality involving an absolute value. Solve for x .
- *68. Express the fact that x differs from -4 by less than 1 as an inequality involving an absolute value. Solve for x .
- *69. Express the fact that x differs from -3 by more than 2 as an inequality involving an absolute value. Solve for x .
- *70. Express the fact that x differs from 2 by more than 3 as an inequality involving an absolute value. Solve for x .

Space restrictions, answers to these exercises may be found in the Answers in the back of the book.