Chapter 7
LaPlace Transforms

Section 7.1

Definition of LaPlace Transform
Basic definitions of common transforms

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt$$

$\mathcal{L}\{0\} = 0$

$\mathcal{L}\{1\} = 1/s$

$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$

$\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$

$\mathcal{L}\{t\} = \frac{1}{s^2}$

$\mathcal{L}\{t^n e^{at}\} = \frac{n!}{(s-a)^{n+1}}$

$\mathcal{L}\{\sin kt\} = \frac{k}{s^2+k^2}$

$\mathcal{L}\{\cos kt\} = \frac{s}{s^2+k^2}$

$\mathcal{L}\{\sinh kt\} = \frac{k}{s^2-k^2}$

$\mathcal{L}\{\cosh kt\} = \frac{s}{s^2-k^2}$

Use the definition to find the following LaPlace Transforms:

1) $\mathcal{L}\{t^3\}$
2) $\mathcal{L}\{e^{4t}\}$
3) $\mathcal{L}\{e^{-2t}\}$
4) $\mathcal{L}\{te^{3t}\}$
5) $\mathcal{L}\{\sin 2t\}$
6) $\mathcal{L}\{3\cos 4t\}$

Check each answer using the formulas.

Final Solutions

① $\mathcal{L}\{t^3\}$ $(n=3)$ $\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$

$= \frac{3!}{s^4} = \boxed{\frac{6}{s^4}}$

② $\mathcal{L}\{e^{4t}\}$ $(a=4)$ $\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$

$= \boxed{\frac{1}{s-4}}$

③ $\mathcal{L}\{e^{-2t}\}$ $(a=-2)$

$= \boxed{\frac{1}{a+2}}$

④ $\mathcal{L}\{te^{3t}\} = \boxed{\frac{1}{(s-3)^2}}$

$n=1$ $\frac{1!}{s^2}$

⑤ $\mathcal{L}\{\sin 2t\}$ $k=2$ $\mathcal{L}\{\sin kt\} = \frac{k}{s^2+k^2}$

$= \boxed{\frac{2}{s^2+4}}$

⑥ $\mathcal{L}\{3\cos 4t\}$ $k=4$ $\mathcal{L}\{\cos kt\} = \frac{s}{s^2+k^2}$

$= 3 \cdot \frac{s}{s^2+16}$

$= \boxed{\frac{3s}{s^2+16}}$