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SOLUTIONS TO SEC 8.3

DAILY ASSIGNMENT PROBLEMS

$$\textcircled{26} \quad \frac{-11x+6}{5x^2-4x-2} = \frac{A}{5x+6} + \frac{B}{x-2} = \frac{A(x-2)+B(5x+6)}{(5x+6)(x-2)}$$

$$= \frac{(A+5B)x + (-2A+6B)}{(5x+6)(x-2)}$$

$$\Rightarrow \begin{cases} A+5B = -1 & \textcircled{1} \\ -2A+6B = 6 & \textcircled{2} \end{cases}$$

$$\textcircled{1} \times 2 \Rightarrow 2A + 10B = -2 \quad \textcircled{3}$$

$$\textcircled{2} + \textcircled{3} \Rightarrow 16B = 4 \Rightarrow B = \frac{1}{4}$$

$$\textcircled{1} \Rightarrow A + 5\left(\frac{1}{4}\right) = -1 \Rightarrow A = -\frac{9}{4}$$

$$\boxed{\frac{-11x+6}{5x^2-4x-2} = \frac{-\frac{9}{4}}{5x+6} + \frac{\frac{1}{4}}{x-2}}$$

$$\textcircled{28} \quad \frac{x^2+24x-12}{x^3-4x} = \frac{x^2+24x-12}{x(x-2)(x+2)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+2}$$

$$= \frac{A(x-2)(x+2) + Bx(x+2) + Cx(x-2)}{x(x-2)(x+2)}$$

$$= \frac{A(x^2-4) + Bx^2 + 2Bx + Cx^2 - 2Cx}{x(x-2)(x+2)}$$

$$= \frac{(A+B+C)x^2 + (2B-2C)x - 4A}{x(x-2)(x+2)}$$

$$\Rightarrow \begin{cases} A+B+C = 1 \\ 2B-2C = 24 \\ -4A = -12 \end{cases} \Rightarrow \begin{cases} A = 3 \\ B+C = -2 \\ B-C = 12 \end{cases}$$

$$\Rightarrow \begin{cases} B+C = -2 \\ B-C = 12 \end{cases} \Rightarrow \begin{cases} 2B = 10 \\ B = 5 \end{cases} \Rightarrow C = -7$$

$$\therefore \boxed{\frac{x^2+24x-12}{x^3-4x} = \frac{3}{x} + \frac{5}{x-2} - \frac{7}{x+2}}$$

(2)

(30)

$$\begin{aligned} \frac{-2x^2 - 7x + 28}{x^3 - 4x^2 + 4x} &= \frac{-2x^2 - 7x + 28}{x(x-2)^2} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{(x-2)^2} \\ &= \frac{A(x-2)^2 + Bx(x-2) + Cx}{x(x-2)^2} \\ &= \frac{A(x^2 - 4x + 4) + Bx^2 - 2Bx + Cx}{x(x-2)^2} \\ &= \frac{(A+B)x^2 + (-4A - 2B + C)x + 4A}{x(x-2)^2} \end{aligned}$$

$$\begin{aligned} \Rightarrow \quad A+B &= -2 \\ -4A - 2B + C &= -7 \\ 4A &= 28 \Rightarrow A=7 \quad 7+B = -2 \Rightarrow B=-9 \\ -4(7) - 2(-9) + C &= -7 \Rightarrow C=3 \end{aligned}$$

$$\boxed{\frac{-2x^2 - 7x + 28}{x^3 - 4x^2 + 4x} = \frac{7}{x} - \frac{9}{x-2} + \frac{3}{(x-2)^2}}$$

(32)

$$\frac{2x^3 - 2x^2 + 6x}{x^4 + 4x^2 + 3} = \frac{2x^3 - 2x^2 + 6x}{(x^2+1)(x^2+3)} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2+3}$$

$$\begin{aligned} &= \frac{(Ax+B)(x^2+3) + (Cx+D)(x^2+1)}{(x^2+1)(x^2+3)} \\ &= \frac{Ax^3 + 3Ax + Bx^2 + 3B + Cx^3 + Dx^2 + Cx + D}{(x^2+1)(x^2+3)} \\ &= \frac{(A+C)x^3 + (B+D)x^2 + (3A+C)x + 3B+D}{(x^2+1)(x^2+3)} \end{aligned}$$

$$\left. \begin{aligned} A+C &= 2 \quad \textcircled{1} \\ B+D &= -2 \quad \textcircled{2} \\ 3A+C &= 6 \quad \textcircled{3} \\ 3B+D &= 0 \quad \textcircled{4} \end{aligned} \right\} \Rightarrow A=2, C=0, B=1, D=-3$$

$$\Rightarrow \boxed{\frac{2x^3 - 2x^2 + 6x}{x^4 + 4x^2 + 3} = \frac{2x+1}{x^2+1} + \frac{-3}{x^2+3}}$$

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$$\frac{3x^2+4x-1}{x^3-1} = \frac{3x^2+4x-1}{(x-1)(x^2+x+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1}$$

$$= \frac{Ax^2+Ax+A+Bx^2+Bx+C}{(x-1)(x^2+x+1)}$$

$$= \frac{(A+B)x^2 + (A-B+C)x + A-C}{(x-1)(x^2+x+1)}$$

$$\Rightarrow \begin{cases} A+B=3 & \text{--- (1)} \\ A-B+C=4 & \text{--- (2)} \\ A-C=-1 & \text{--- (3)} \end{cases} \Rightarrow \begin{cases} A+B=3 \\ 2A-B=3 \end{cases}$$

$$\Rightarrow \frac{3A=6}{\text{--- (4)}} \Rightarrow \begin{cases} A=2 \\ B=1 \\ C=3 \end{cases}$$

$$\Rightarrow \frac{3x^2+4x-1}{x^3-1} = \frac{2}{x-1} + \frac{x+3}{x^2+x+1}$$

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$$f(x) = \frac{-3x^4-11x^2+x-12}{x^5-4x^3+4x} = \frac{-3x^4-11x^2+x-12}{x(x^2-2)^2}$$

$$= \frac{A}{x} + \frac{Bx+C}{x^2-2} + \frac{Dx+E}{(x^2-2)^2}$$

$$= \frac{A(x^2-2)^2 + (Bx+C)x(x^2-2) + (Dx+E)x}{x(x^2-2)^2}$$

$$= \frac{A(x^4-4x^2+4) + x(Bx^3-2Bx+Cx^2-2C) + Dx^2+Ex}{x(x^2-2)^2}$$

$$= \frac{(A+B)x^4 + Cx^3 + (-4A-2B+D)x^2 + (-2C+E)x + 4A}{x(x^2-2)^2}$$

$$\Rightarrow \begin{cases} A+B=-3 \\ C=0 \\ -4A-2B+D=-11 \\ -2C+E=1 \\ 4A=-12 \end{cases} \Rightarrow \begin{cases} C=0 \\ E=1 \\ A=-3 \\ B=0 \\ D=-13 \end{cases} \Rightarrow f(x) = \frac{-3}{x} - \frac{23x-1}{(x^2-2)^2}$$

(4)

Q5

$$\frac{2x^3 - x + 6}{x^4 - 2x^3 + 3x^2} = \frac{2x^3 - x + 6}{x^2(x^2 - 2x + 3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx + D}{x^2 - 2x + 3}$$

$$= \frac{Ax(x^2 - 2x + 3) + B(x^2 - 2x + 3) + x^2(Cx + D)}{x^2(x^2 - 2x + 3)}$$

$$= \frac{(A+C)x^3 + (-2A+B+D)x^2 + (3A-2B)x + 3B}{x^2(x^2 - 2x + 3)}$$

$A+C = 2$ (1)	(4) $\Rightarrow B = 2$
$-2A+B+D = 0$ (2)	(3) $\Rightarrow A = 1$
$3A-2B = -1$ (3)	(1) $\Rightarrow C = 1$
$3B = 6$ (4)	(2) $\Rightarrow D = 0$

$$\frac{2x^3 - x + 6}{x^4 - 2x^3 + 3x^2} = \frac{1}{x} + \frac{2}{x^2} + \frac{x}{x^2 - 2x + 3}$$