

TRIGONOMETRIC IDENTITIES (FOR MAC1114)

1) a) $\csc x = \frac{1}{\sin x}$ b) $\sec x = \frac{1}{\cos x}$ c) $\cot x = \frac{1}{\tan x}$ d) $\tan x = \frac{\sin x}{\cos x}$ e) $\cot x = \frac{\cos x}{\sin x}$

2) a) $\sin(-x) = -\sin x$ b) $\cos(-x) = \cos x$ c) $\tan(-x) = -\tan x$

3) a) $\sin\left(\frac{\pi}{2} - x\right) = \cos x$ b) $\sec\left(\frac{\pi}{2} - x\right) = \csc x$ c) $\tan\left(\frac{\pi}{2} - x\right) = \cot x$

d) $\cos\left(\frac{\pi}{2} - x\right) = \sin x$ e) $\csc\left(\frac{\pi}{2} - x\right) = \sec x$ f) $\cot\left(\frac{\pi}{2} - x\right) = \tan x$

4) a) $\sin^2 x + \cos^2 x = 1$ b) $\tan^2 x + 1 = \sec^2 x$ c) $\cot^2 x + 1 = \csc^2 x$

Angle	0°	$\frac{\pi}{6}$ $=30^\circ$	$\frac{\pi}{4}=45^\circ$	$\frac{\pi}{3}=60^\circ$	$\frac{\pi}{2}=90^\circ$
Sin x	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
Cos x	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Tan x	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undef

5) a) $\sin(x+y) = \sin x \cos y + \cos x \sin y$ b) $\sin(x-y) = \sin x \cos y - \cos x \sin y$

c) $\cos(x+y) = \cos x \cos y - \sin x \sin y$ d) $\cos(x-y) = \cos x \cos y + \sin x \sin y$

e) $\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$ f) $\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

6) a) $\sin x \cos y = \frac{1}{2} [\sin(x+y) + \sin(x-y)]$ b) $\cos x \sin y = \frac{1}{2} [\sin(x+y) - \sin(x-y)]$

c) $\sin x \sin y = \frac{1}{2} [\cos(x-y) - \cos(x+y)]$ d) $\cos x \cos y = \frac{1}{2} [\cos(x+y) + \cos(x-y)]$

7) a) $\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$ b) $\sin x - \sin y = 2 \cos \frac{x+y}{2} \sin \frac{x-y}{2}$

c) $\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$ d) $\cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}$

8) a) $\sin 2x = 2 \sin x \cos x$ b) $\cos 2x = \begin{cases} \cos^2 x - \sin^2 x \\ 1 - 2 \sin^2 x \\ 2 \cos^2 x - 1 \end{cases}$ c) $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$

9) a) $\sin \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{2}}$ b) $\cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{2}}$ c) $\tan \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}} = \frac{\sin x}{1 + \cos x}$

10) a) Law of sines: $\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$ b) Laws of cosines: $\begin{cases} a^2 = b^2 + c^2 - 2bc \cos \alpha \\ b^2 = a^2 + c^2 - 2ac \cos \beta \\ c^2 = a^2 + b^2 - 2ab \cos \gamma \end{cases}$