Calculus for Business MAC 2233
Review for final
Name___________________________________

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Solve the problem.

1) Compute the average rate of change of the function over the indicated interval.
$f(x) = x^2 + 3x \quad [3, 4]$

2) Determine: $\lim\limits_{x \to -3} \dfrac{x^2 - 2x - 15}{x + 3}$

3) Use the definition of the derivative to compute a formula for the derivative of the given function. Use the formula to determine the slope of the line tangent to the graph of the function at the indicated value.
$f(x) = x^2 + 5x$ at $x = 4$

4) Determine the derivative of $f(x) = (4x - 3)(3x^3 - x^2 + 1)$.

5) Determine an equation of the line tangent to the graph of the following equation at the indicated point.
$f(x) = (9x^2 + 64)^{1/2}$; $(2, 10)$

6) The Wally Widget Company determines that the daily cost in dollars of producing x widgets is given by $C(x) = 845.62 + 30.35x$.
i) What are the fixed and variable costs for producing the widgets?
ii) Evaluate C(157) and interpret.
iii) How many widgets can be produced for 16,870.42?
iv) What is the marginal cost for the widgets?
v) If the current daily production level is 707 widgets, what is the cost of producing the 708th widget?

7) The number of bacteria in a colony after t hours is given by $g(t) = t^2 + 12t + 1200$, $0 \le t \le 24$ where t is the number of hours since the colony was established and g(t) represents the number of bacteria.
i) Evaluate g(4) and interpret.
ii) Determine the average rate of change in the increase of the colony's population for $t = 4$ to $t = 8$ and interpret. Round answer to the nearest hundredth, if necessary.

8) Given: $C(x) = 10x + 240$; $p(x) = 27$
Assume the cost function C and the price–demand function p are in dollars.
i) Determine the revenue function R and the profit function P.
ii) Determine the marginal cost function MC and the marginal profit function MP.

9) Use u–substitution to evaluate.
$\int 8t\sqrt[3]{4t^2 - 9}\, dt$

10) Evaluate $\displaystyle\int_0^2 \frac{x^2}{(5 + 2x^3)^2}\, dx$. Do not change the limits of integration.

Find the integral.

11) $\int \frac{5x^4\,dx}{(5 + x^5)^6}$

Find the area between the curves.

12) $y = 2x - x^2$, $y = 2x - 4$

Solve the problem.

13) Determine the area of the region bounded by $f(x) = x + 6$ and $g(x) = -x^2 + 2$ on the interval $[-2, 2]$.

14) Determine the derivative of $f(x) = \ln(x^5 - e^{2x})$.

15) Given: $C(x) = -0.004x^3 + 8x + 130$; $p(x) = -0.005x + 16$

Assume the cost function C and the price–demand function p are in dollars.

i) Determine the revenue function R and the profit function P.

ii) Determine the marginal cost function MC and the marginal profit function MP.

Find the indicated absolute extremum as well as all values of x where it occurs on the specified domain.

16) $f(x) = 3x^4 + 16x^3 + 24x^2 + 32$; $[-3, 1]$

Maximum

Solve the problem.

17) Determine the derivative of $f(x) = \frac{5}{(2x - 3)^4}$.

18) For the function $f(x) = \frac{2}{3}x^3 - 2x^2 - 6x$

i) Determine the intervals where the function is increasing and where it is decreasing.

ii) Determine the relative extrema.

iii) Determine the intervals where the graph of the function is concave up and where it is concave down.

iv) Determine any inflection points.

19) Determine the derivative of $f(x) = \frac{x^6}{(1 + x^3)^3}$.

20) If the price charged for a bolt is p cents, then x thousand bolts will be sold in a certain hardware store, where $p = 48 - \frac{x}{16}$. How many bolts must be sold to maximize revenue?

1) 10
2) -8
3) $f'(x) = 2x + 5$; $f'(4) = 13$
4) $f'(x) = 48x^3 - 39x^2 + 6x + 4$
5) $y = \frac{9}{5}x + \frac{32}{5}$
6) i) fixed costs: \$845.62 ; variable costs: \$30.35 per widget
   ii) $C(157) = 5610.57$; The cost to produce 157 widgets is \$5610.57 .
   iii) 528 widgets
   iv) 30.35
   v) 30.35
7) i) $g(4) = 1264$; After 4 hours, there will be 1264 bacteria in the colony.
   ii) 24; From 4 to 8 hours after the colony is started, the colony will be growing at an average rate of 24 bacteria per hour.
8) i) $R(x) = 27x$; $P(x) = 17x - 240$
   ii) $MC(x) = 10$; $MP(x) = 17$
9) $\frac{3(4t^2 - 9)^{4/3}}{4} + C$
10) $\frac{8}{315}$
11) $-\frac{1}{5(5 + x^5)^5} + C$
12) $\frac{32}{3}$
13) $\frac{64}{3}$
14) $f'(x) = \frac{5x^4 - 2e^{2x}}{x^5 - e^{2x}}$
15) i) $R(x) = -0.005x^2 + 16x$; $P(x) = 0.004x^3 - 0.005x^2 + 8x - 130$
   ii) $MC(x) = -0.012x^2 + 8$; $MP(x) = 0.012x^2 - 0.01x + 8$
16) 75 at $x = 1$
17) $f'(x) = \frac{-40}{(2x - 3)^5}$
18) i) f is increasing on $(-\infty, -1) \cup (3, \infty)$
   f is decreasing on $(-1, 3)$
   ii) f has a relative maximum at $(-1, \frac{10}{3})$
   f has a relative minimum at $(3, -18)$
   iii) f is concave up on $(1, \infty)$
   f is concave down on $(-\infty, 1)$
   iv) f has an inflection point at $(1, -\frac{22}{3})$
19) $f'(x) = \frac{3x^5(2 - x^3)}{(1 + x^3)^4}$
20) 384 thousand bolts