Out of 50 points

Test 1 ~~Fall 2017, Version 1~~
Student Name: Key

Show all work for partial credit. Use EXACT answers unless otherwise directed.

5

1. Solve the radical equation for x. $\sqrt{27-x} = -1 + \sqrt{2x+3}$

If there is no solution, enter "NS".

$$\left(\sqrt{27-x}\right)^2 = \left(-1+\sqrt{2x+3}\right)^2$$

$$27 - x = 1 - 2\sqrt{2x+3} + 2x + 3$$

$$23 - 3x = -2\sqrt{2x+3}$$

$$\left(\frac{3x-23}{2}\right)^2 = \left(\sqrt{2x+3}\right)^2$$

$$\frac{9x^2 - 138x + 529}{4} = 2x + 3$$

$$9x^2 - 138x + 529 = 8x + 12$$

$$9x^2 - 146x + 517 = 0$$

$$a = 9 \quad b = -146 \quad c = 517$$

$$x = \frac{-(-146) \pm \sqrt{(-146)^2 - 4(9)(517)}}{2(9)}$$

$$x = \frac{146 \pm \sqrt{21316 - 18612}}{18}$$

$$x = \frac{146 \pm \sqrt{2704}}{18}$$

$$x = \frac{146 \pm 52}{18}$$

$$x = 11 \quad \cancel{\frac{47}{9}}$$

check

5

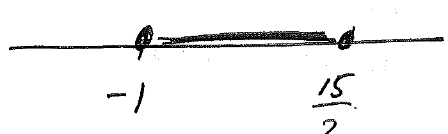
2. Solve the polynomial inequality and express the solution set in interval notation.

$$2t^2 - 15 \leq 13t$$

$$2t^2 - 13t - 15 \leq 0$$

$$(2t - 15)(t + 1) \leq 0$$

$$t = \frac{15}{2},\ -1$$



$$\left[-1, \frac{15}{2}\right]$$

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3. Solve the inequality and express the solution in interval notation.

5

$$|7 - 5x| \geq 2$$

$$7 - 5x \geq 2 \ \cup \ 7 - 5x \leq -2$$

$$-5x \geq -5 \ \cup \ -5x \leq -9$$

$$x \leq +1 \ \cup \ x \geq \frac{9}{5}$$

$$(-\infty, +1] \cup [\tfrac{9}{5}, \infty)$$

4. Find the difference quotient $\dfrac{f(x+h) - f(x)}{h}$ for the following function.

5

$$f(x) = \frac{x+5}{x-8}$$

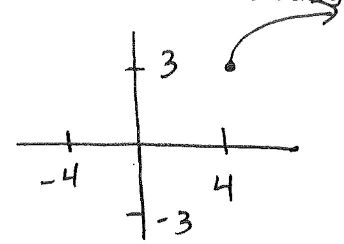
$$DQ = \frac{\dfrac{x+h+5}{x+h-8} - \dfrac{x+5}{x-8}}{h} = \frac{(x+h+5)(x-8) - (x+5)(x+h-8)}{(x+h-8)(x-8)h}$$

$$DQ = \frac{x^2 + xh + 5x - 8x - 8h - 40 - (x^2 + xh - 8x + 5x + 5h - 40)}{(x+h-8)(x-8)h}$$

$$DQ = \frac{\cancel{x^2} + \cancel{xh} + \cancel{5x} - \cancel{8x} - 8h - \cancel{40} - \cancel{x^2} - \cancel{xh} + \cancel{8x} - \cancel{5x} - 5h + \cancel{40}}{(x+h-8)(x-8)h}$$

$$DQ = \frac{-13\cancel{h}}{(x+h-8)(x-8)\cancel{h}} = \frac{-13}{(x+h-8)(x-8)}$$

5. The function f is one-to-one. Find its inverse and determine the domain and range of both f and f^{-1}. $f(x) = \sqrt{x-4} + 3$



$$x = \sqrt{y-4} + 3$$

3

$$(x-3)^2 = \left(\sqrt{y-4}\right)^2$$

$$(x-3)^2 = y-4$$

$$(x-3)^2 + 4 = y$$

$f^{-1} = (x-3)^2 + 4$

1 Domain (f) : $x-4 \geq 0$; $x \geq 4$; $[4, \infty)$

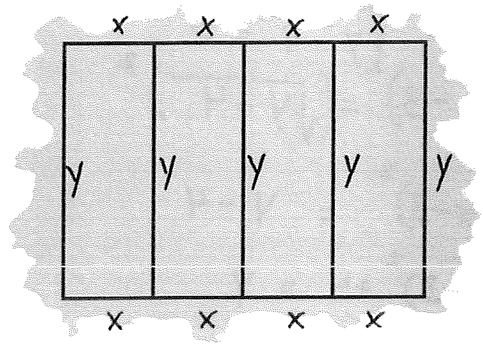
1 Range (f) : $[3, \infty)$

1 Domain (f^{-1}) : $[3, \infty)$

1 Range (f^{-1}) : $[4, \infty)$

6. Ranching. A rancher has 50,000 linear feet of fencing and wants to enclose a rectangular field and then divide it into four equal pastures with three internal fences parallel to one of the rectangular sides. What is the maximum area of each pasture?

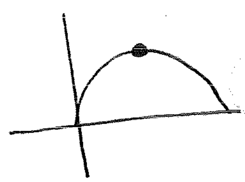
5



$$\begin{cases} 50{,}000 = 5y + 8x \\ \max = xy \end{cases} \Rightarrow y = \frac{50{,}000 - 8x}{5}$$

$$\max = x\left(\frac{50{,}000 - 8x}{5}\right) = \frac{50{,}000x - 8x^2}{5}$$

$$\max = 10{,}000x - \frac{8}{5}x^2$$



$$\frac{-b}{2a} = -\frac{10{,}000}{2\left(-\frac{8}{5}\right)} = \frac{10{,}000(5)}{2(8)}$$

$$= \frac{50{,}000}{16} = \frac{12500}{4} = 3125$$

$$\max = 10{,}000(3125) - \frac{8}{5}(3125)^2 = 15{,}625{,}000$$

7. Consider the polynomial function

$$f(x) = x^2(x-2)^3(x+3)^2 \quad = 1x^7 + \ldots$$

$x = 0, 2, -3$

(a) List each real zero, its multiplicity, and determine whether the graph touches or crosses at each x -intercept (zero).

6

Zero	Multiplicity	Touches or Crosses
$x =$ -3	2	touches
$x =$ 0	2	touches
$x =$ 2	3	crosses

(b) Find the y -intercept and a few points on the graph.

y -intercept = (0,0)

$y = (-2)^2(-2-2)^3(-2+3)^2$
$4 \quad (-4)^3 \quad (1)^2$
-256

$y = (-1)^2(-1-2)^3(-1+3)^2$
$(1) \quad (-3)^3 \quad (2)^2$
-108

$y = (1)^2(1-2)^3(1+3)^2$
$(1)(-1)^3(4)^2 = -16$

3

x	$f(x)$
-3	0
-2	-256
-1	-108
0	0
1	-16
2	0

(c) Determine the end behavior of $f(x)$.

$\nearrow$

$\swarrow$

1

To the left, the graph falls without bound.

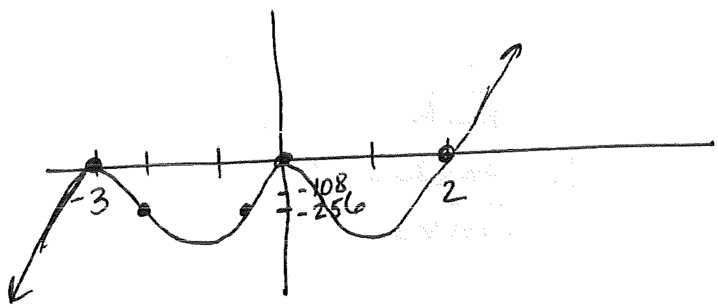
1

To the right, the graph rises without bound.

(d) Sketch the graph of $f(x)$.

Make sure you plot each x -intercept, the y -intercept, and any points from part **(b)** that appear in the window below. If the same point is both an x -intercept and a y -intercept, plot it as an x -intercept.

7



8. Factor the polynomial as a product of linear factors given that x=3 and x=5/2 are zeros.

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$$P(x) = 2x^5 - 13x^4 + 24x^3 - 34x^2 + 150x - 225$$

$$\begin{array}{c|cccccc} 3 & 2 & -13 & 24 & -34 & 150 & -225 \\ & \downarrow & 6 & -21 & 9 & -75 & 225 \\ \hline & 2 & -7 & 3 & -25 & 75 & 0 \end{array}$$

$P(x) = (x-3)(2x^4 - 7x^3 + 3x^2 - 25x + 75)$

$$\begin{array}{c|ccccc} \frac{5}{2} & 2 & -7 & 3 & -25 & 75 \\ & \downarrow & 5 & -5 & -5 & -75 \\ \hline & 2 & -2 & -2 & -30 & 0 \end{array}$$

$P(x) = (x-3)(x-\frac{5}{2})(2x^3 - 2x^2 - 2x - 30)$

$P(x) = 2(x-3)(x-\frac{5}{2})(x^3 - x^2 - x - 15)$

$P(x) = (x-3)(2x-5)(x^3 - x^2 - x - 15)$

$$\begin{array}{c|cccc} 3 & 1 & -1 & -1 & -15 \\ & \downarrow & 3 & 6 & 15 \\ \hline & 1 & 2 & 5 & 0 \end{array}$$

$P(x) = (x-3)^2(2x-5)(x^2 + 2x + 5)$

$a = 1 \quad b = 2 \quad c = 5$

$x = \dfrac{-2 \pm \sqrt{2^2 - 4(1)(5)}}{2(1)}$

$x = \dfrac{-2 \pm \sqrt{4 - 20}}{2}$

$x = \dfrac{-2 \pm \sqrt{-16}}{2} = \dfrac{-2 \pm 4i}{2}$

$= -1 \pm 2i$

$P(x) = (x-3)^2(2x-5)(x+1+2i)(x+1-2i)$