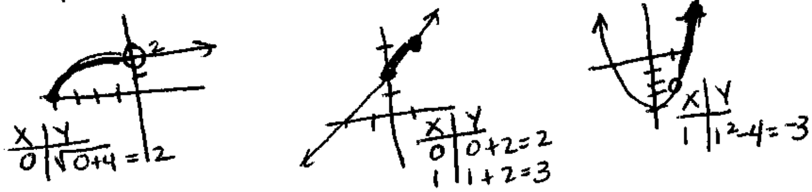
MAC2311 Test 1 Name Key

Show all work for credit.

1. Given $f(x) = \begin{cases} \sqrt{x+4}\,; x < 0 \\ x+2\,;\ 0 \le x \le 1 \\ x^2-4\,; x > 1 \end{cases}$

$y = \sqrt{x+4}$ $y = x+2$ $y = x^2-4$



a.) Sketch the graph of $f(x)$.

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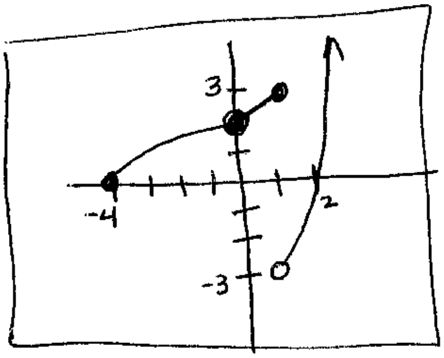

2 b.) $\lim_{x\to 0^+} f(x) =$ $0+2 = \boxed{2}$

2 c.) $\lim_{x\to 0^-} f(x) =$ $\sqrt{0+4} = \boxed{2}$

2 d.) $\lim_{x\to 0} f(x) =$ $\boxed{2}$

2 e.) $\lim_{x\to 1^+} f(x) =$ $1^2-4 = \boxed{-3}$

2 f.) $\lim_{x\to 1^-} f(x) =$ $1+2 = \boxed{3}$

2 g.) $\lim_{x\to 1} f(x) =$ $\boxed{\text{DNE}}$

2 h.) $f(\overset{0}{\cancel{1}}) =$ $0+2 = \boxed{2}$

2 i.) $f(1) =$ $1+2 = \boxed{3}$

j.) Is $f(x)$ continuous at $x = 0$? Explain.

6 Yes, $\lim_{x\to 0} f(x) = f(0) = 2$

k.) Is $f(x)$ continuous at $x = 1$? Explain.

6 No, $\lim_{x\to 1} f(x) = \text{DNE}$ while $f(1) = 3$

2. Evaluate $\frac{f(x+h)-f(x)}{h}$ when $f(x) = x^2 - 4$.

$$= \frac{(x+h)^2 - 4 - (x^2-4)}{h}$$

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$$= \frac{\cancel{x^2} + xh + xh + h^2 \cancel{-4} \cancel{-x^2} \cancel{+4}}{h}$$

$$= \frac{2xh + h^2}{h} = \boxed{2x+h}$$

3. If the recommended adult dosage for a drug is D (in mg), then to determine the appropriate dosage c for a child of age a, pharmacists use the equation $c = 0.0417D(a+1)$. Suppose the dosage for an adult is 200 mg. $D = 200$

a.) Find the slope of the graph of c. What does it represent in the context of the problem.

$$C = 0.0417(200)(a+1)$$

$$C = 8.34(a+1) = 8.34a + 8.34$$

3

$$\boxed{m = 8.34} = \frac{8.34}{1}\frac{\Delta C}{\Delta a}$$

6

For every 1 year of age the recommended dosage of a child increases by 8.34mg for an adult dosage of 200mg.

b.) What is the dosage for a 5-year old?

6

$$C = 8.34(5) + 8.34 = \boxed{50.04 \text{ mg}}$$

4. If $f(x)$ contains the point (3, 10), use your transformation rules to find a point on $\frac{-1}{2}f(x) - 3$.

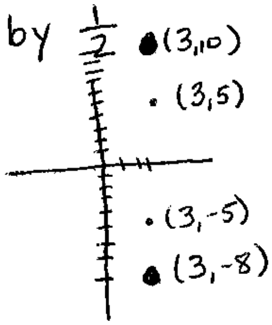
x	$y = f(x)$
3	10

$\Rightarrow$

x	$-\frac{1}{2}f(x) - 3$
3	$-\frac{1}{2}(10) - 3 = -8$

reflect @ x axis
Shrink vertically by $\frac{1}{2}$
shift down 2

9



$\boxed{(3, -8)}$

5. If $(x) = \sqrt{2x+1}$, find $f(x)$ and $g(x)$ such that $(f \circ g)(x) = F(x)$.

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$$(f \circ g)(x) = f(g(x)) = \sqrt{(2x+1)}$$

$$\boxed{\begin{aligned} f(x) &= \sqrt{x} \\ g(x) &= 2x+1 \end{aligned}}$$

6. $\lim_{h\to 0} \frac{\sqrt{1+h}-1}{h} = \frac{\sqrt{1+0}-1}{0} = \frac{0}{0} = ?$

multiply by conjugate

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$$\lim_{h\to 0} \frac{(\sqrt{1+h}-1)}{h} \cdot \frac{(\sqrt{1+h}+1)}{\sqrt{1+h}+1} = \lim_{h\to 0} \frac{\cancel{1}+h-\cancel{1}}{h(\sqrt{1+h}+1)}$$

$$= \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{1+h}+1)} = \lim_{h\to 0} \frac{1}{\sqrt{1+h}+1} = \frac{1}{\sqrt{1+0}+1} = \frac{1}{1+1}$$

$$= \boxed{\frac{1}{2}}$$

$\frac{0}{0} = ?$

7. $\lim_{x\to -1} \frac{(x^2-1)}{x+1} = \lim_{x\to -1} \frac{\cancel{(x+1)}(x-1)}{\cancel{(x+1)}} = \lim_{x\to -1} (x-1) = -1-1 = \boxed{-2}$

6

8. Prove by using the Intermediate Value Theorem that $f(x) = x^2 + 5x + 1$ has a root in the interval [-1, 1].

$N=0$

$$f(-1) = (-1)^2 + 5(-1) + 1 = -3$$

$$f(1) = (1)^2 + 5(1) + 1 = 7$$

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Since $f(-1) \le 0 \le f(1)$ and $f(x)$ is continuous $-1 \le x \le 1$, then there exists $x = c$ such that $f(c) = 0$.