MAC2312 Test 1 Summer 2006 Name Key

Prof Howard (3-16)

Show all work for credit.

For each of the following problems 1-7, find the derivative and **do not simplify**.

1. $g(t) = \dfrac{e^{2t}}{1+\sin t}$

$$\boxed{g'(t) = \frac{(1+\sin t)e^{2t}\cdot 2 - e^{2t}(\cos t)}{(1+\sin t)^2}}$$

2. $h(u) = 5^{\sqrt{u}}$

$$\boxed{h'(u) = 5^{\sqrt{u}} \cdot \frac{1}{2\sqrt{u}} \cdot \ln 5}$$

3. $y = \ln(\sec 5x + \tanh x)$

$$\boxed{y' = \frac{1}{\sec 5x + \tanh x} \cdot \left[\sec(5x)\tan(5x)\cdot 5 + \operatorname{sech}^2 x\right]}$$

4. $y = \tan^{-1}(\sin\sqrt{x})$

$$\boxed{y' = \frac{1}{1+\sin^2\sqrt{x}} \cdot \cos\sqrt{x} \cdot \frac{1}{2\sqrt{x}}}$$

5. $y = \cosh^{-1}(\sinh x)$

$$\boxed{y' = \frac{1}{\sqrt{\sinh^2 x - 1}} \cdot \cosh x}$$

6. $f(x) = (\sec x)^x$

$$\ln f(x) = x \ln(\sec x)$$

$$\frac{1}{f(x)} \cdot f'(x) = x \cdot \frac{1}{\cancel{\sec x}} \cdot (\cancel{\sec x}\tan x) + \ln(\sec x)\cdot 1$$

$$\boxed{f'(x) = [x\tan x + \ln(\sec x)](\sec x)^x}$$

7. $f(x) = (\tanh 3x)^{\ln x}$

$$\ln f(x) = \ln x \cdot \ln(\tanh 3x)$$

$$\frac{1}{f(x)} \cdot f'(x) = \ln x \cdot \frac{1}{\tanh 3x} \cdot 3 \cdot \operatorname{sech}^2(3x) + \ln(\tanh 3x) \cdot \frac{1}{x}$$

$$\boxed{f'(x) = \left[\frac{3\ln x\ \operatorname{sech}^2(3x)}{\tanh(3x)} + \frac{\ln(\tanh 3x)}{x}\right](\tanh 3x)^{\ln x}}$$

8. Given $f(x)=\sqrt{5x-1}+4$

a.) Find $g'(6)$ where $f^{-1}(x)=g(x)$ by finding the inverse and taking its derivative.

$$y = \sqrt{5x-1} + 4$$
$$x = \sqrt{5y-1} + 4$$
$$(x-4)^2 = 5y-1$$
$$\frac{(x-4)^2+1}{5} = y = g(x)$$

$$\cancel{g(x) = \tfrac{1}{5}[(x-4)^2+1] + \tfrac{1}{5}}$$
$$g(x) = \frac{1}{5}(x-4)^2 + \frac{1}{5}$$
$$g'(x) = \frac{2}{5}(x-4)(1)$$
$$g'(6) = \frac{2}{5}(6-4)(1) = \boxed{\frac{4}{5}}$$

b.) Find $g'(6)$ where $f^{-1}(x)=g(x)$ by using Theorem 7.

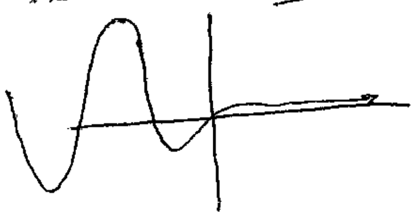
$$f'(x) = \frac{1}{2\sqrt{5x-1}} \cdot 5$$

$$g(6) = ?$$
$$6 = \sqrt{5x-1} + 4$$
$$2 = \sqrt{5x-1}$$
$$4 = 5x-1 \Rightarrow x = 1$$

$$g(6) = 1$$
$$g'(6) = \frac{1}{\frac{5}{2\sqrt{5(1)-1}}} = \frac{1}{\frac{5}{2\cdot 2}} = \boxed{\frac{4}{5}}$$

For each of the following problems, find the limit.

9. $\lim\limits_{x\to\infty} e^{-x}\sin x = \boxed{0}$



10. $\lim\limits_{x\to 0}\dfrac{\tan \pi x}{\ln(1+x)} = \dfrac{0}{0}$ ⓛ $\lim\limits_{x\to 0} \dfrac{\sec^2(\pi x)\cdot\pi}{\frac{1}{1+x}\cdot 1} = \dfrac{\pi}{1} = \boxed{\pi}$

11. $\lim_{x\to 0}\frac{e^{4x}-1-4x}{x^2} \overset{L}{=} \lim_{x\to 0}\frac{4e^{4x}-4}{2x} \overset{L}{=} \lim_{x\to 0}\frac{16e^{4x}}{2} = \frac{16}{2}$

$\frac{0}{0}$ $\quad$ $\frac{0}{0}$

$= \boxed{8}$

12. $\lim_{x\to 0}(1-8x)^{1/x} = L$ $\quad 1^{\infty}$

$\lim_{x\to 0} \ln(1-8x)^{1/x} = \ln L$

$\lim_{x\to 0} \frac{1}{x}\ln(1-8x) = \ln L$

$\lim_{x\to 0} \frac{\ln(1-8x)}{x} = \ln L$

$\overset{L}{} \lim_{x\to 0} \frac{\frac{1}{1-8x}\cdot -8}{1} = \ln L$

$\lim_{x\to 0} \frac{-8}{1-8x} = \ln L$

$e^{-8} = e^{\ln L}$

$\boxed{e^{-8} = L}$

13. $\lim_{x\to 1^+}\left(\frac{x}{1-x}-\frac{1}{x}\right) =$

$\frac{1}{0^-} - \frac{1}{1}$

$-\infty - 1 = \boxed{+\infty}$

14. $\lim_{x\to 0^+}\ln(\sinh x) =$

$\ln 0^+ = \boxed{-\infty}$

15. $\lim_{x\to 0}\frac{\sin x}{\cos x} = \frac{0}{1} = \boxed{0}$