MAC2311 Test 3 Name Key

Show all work for credit.

1. Given $f(x) = x^4 + 8x^3 + 200$ domain $(-\infty, \infty)$

a.) Find the critical points. List both the x and y values.

7

$f'(x) = 4x^3 + 24x^2 = 4x^2(x+6)$

$0 = 4x^2(x+6)$

$x = 0 \quad x = -6$

$- - - \quad + + + \quad + + +$

$-6 \quad 0$

x	f(x)
0	200
-6	-232

$(0, 200)$

$(-6, -232)$

b.) List the intervals of increasing and decreasing.

7

inc $(-6, \infty)$

dec $(-\infty, -6)$

c.) Find the inflection points. List both the x and y values.

7 $f''(x) = 12x^2 + 48x = 12x(x+4)$

$0 = 12x(x+4)$

$x = 0 \quad x = -4$

up Down up

$+ + + \quad - - - \quad + + +$

$-4 \quad 0$

x	f(x)
0	200
-4	-56

$(0, 200)$

$(-4, -56)$

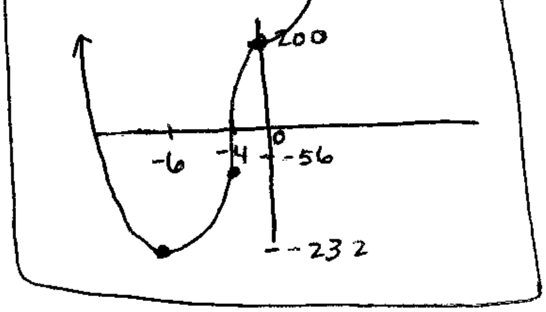
d.) List the intervals of concavity.

7

Up $(-\infty, -4) \cup (0, \infty)$

Down $(-4, 0)$

7 e. Sketch the graph and label your critical points and inflection points.



(7) 2. $\lim_{x\to\infty} \frac{x^2+x-5}{6x^4-2x^2+1} =$ $\boxed{0}$

HA : $y \neq 0$

3. If a resistor of R ohms is connected across a batter of E volts with internal resistance r ohms, then the power (in watts) in the external resistor is $P = \frac{E^2R}{(R+r)^2}$.

(12)

If E and r are fixed but the power varies with R, what is the value for R that will yield a maximum power?

$$P' = \frac{(R+r)^2(E^2) - (E^2R)(2)(R+r)(1)}{(R+r)^4} = \frac{E^2\cancel{(R+r)}[(R+r)(1) - 2R]}{(R+r)^3}$$

$$P' = \frac{E^2(R+r-2R)}{(R+r)^3} = \frac{E^2(-R+r)}{(R+r)^3}$$

$$0 = E^2(-R+r)$$

$$0 = -R+r$$

$$R = r$$

$$\boxed{R = r}$$

+++ | ---

R=r

max

(12) 4. Given $f''(x) = 2 + \cos x$ and $f(0) = -1$ and $f(\pi/2) = 0$, find $f(x)$.

$$f'(x) = \int (2+\cos x)\,dx = 2x + \sin x + C_1$$

$$f(x) = \int (2x + \sin x + C_1)\,dx = \frac{2x^2}{2} - \cos x + C_1 x + C_2$$

$$f(x) = x^2 - \cos x + C_1 x + C_2$$

$$f(0) = \cancel{0^2} - \cos 0 + C_1\cancel{(0)} + C_2 = -1$$

$$-1 + C_2 = -1$$

$$C_2 = 0$$

$$f(x) = x^2 - \cos x + C_1 x$$

$$f(\pi/2) = \left(\frac{\pi}{2}\right)^2 - \cancel{\cos\frac{\pi}{2}} + C_1(\pi/2) = 0$$

$$\left(\frac{\pi}{2}\right)^2 + \frac{\pi}{2} C_1 = 0$$

$$\left(\frac{\pi}{2}\right)^2 = -\frac{\pi}{2} C_1$$

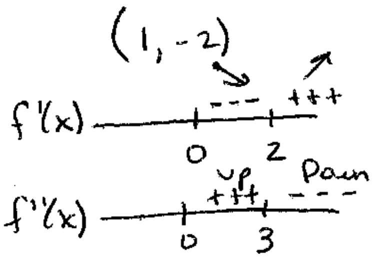
$$-\pi/2 = C_1$$

$$\boxed{f(x) = x^2 - \cos x - \frac{\pi}{2}x}$$

12

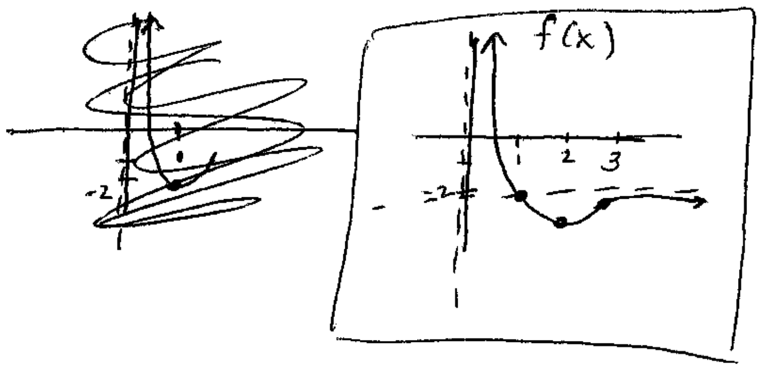
5. Sketch the graph of a function that satisfies the given conditions:

$f(1) = -2, f'(x) < 0$ for $0 < x < 2, f'(x) > 0$ for $x > 2, f''(x) > 0$ for $0 < x < 3, f''(x) < 0$ for $x > 3, \lim_{x\to\infty} f(x) = -2, \lim_{x\to 0^+} f(x) = \infty$



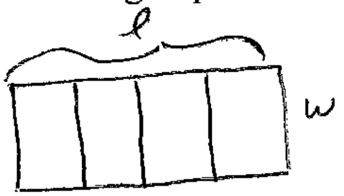
HA: $y \neq -2$

VA: $x \neq 0$



6. A farmer with 750 feet of fencing wants to enclose a rectangular area and then

12 divide it into four equal pens with fencing parallel to one side of the rectangle. What is the largest possible total area of the four pens?



$750 = 2l + 5w \Rightarrow l = \frac{750 - 5w}{2} = 375 - 2.5w = l$

$A = lw$

$A = (375 - 2.5w)w = 375w - 2.5w^2$

$A' = 375 - 5w$

$0 = 375 - 5w$

$5w = 375$

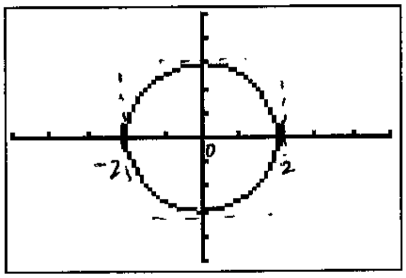
$w = 75$

$l = 375 - 2.5(75) = 187.5$

$A = (75)(187.5) = \boxed{14{,}062.5 \text{ sq ft} = \text{max Area}}$

9

7. For the graph of $f(x)$ below, list the critical values. Each tick mark is one unit.



$\boxed{x = 0, 2, -2}$

$f'(x) = 0$ at $x = 0$

$f'(x) = \text{DNE}$ at $x = \pm 2$