Theorem 7: If *f* is a one-to-one differentiable function with inverse function $g = f^{-1}$ and $f'(g(a)) \neq 0$, then the inverse function is differentiable at *a* and $g'(a) = \dfrac{1}{f'(g(a))}$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

$$\sinh 2x = 2\sinh x \cosh x$$

$$\cosh^2 x = \frac{\cosh 2x + 1}{2}$$

$$\sinh^2 x = \frac{\cosh 2x - 1}{2}$$

$$a^2 + x^2 \Rightarrow x = a\tan\theta$$

$$a^2 - x^2 \Rightarrow x = a\sin\theta$$

$$x^2 - a^2 \Rightarrow x = a\sec\theta$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$$

$$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

$$\int \sinh x dx = \cosh x + C$$

$$\int \cosh x dx = \sinh x + C$$

$$\int \sec h^2 x dx = \tanh x + C$$

$$\int - \csc h^2 x dx = \coth x + C$$

$$\int - \sec hx \tanh x dx = \sec hx + C$$

$$\int - \csc hx \coth x dx = \csc hx + C$$

$$\int \sec x dx = \ln|\sec x + \tan x| + C$$

$$\int \csc x dx = - \ln|\csc x + \cot x| + C$$

$$\int \sec^3 x dx = \frac{1}{2} \sec x \tan x +$$

$$\frac{1}{2} \ln|\sec x + \tan x| + C$$

$$\int u dv = uv - \int v du$$

$$\int \frac{1}{\sqrt{1 - x^2}} dx = \sin^{-1} x + C$$

$$\int \frac{-1}{\sqrt{1 - x^2}} dx = \cos^{-1} x + C$$

$$\int \frac{1}{1 + x^2} dx = \tan^{-1} x + C$$

$$\int \frac{-1}{1 + x^2} dx = \cot^{-1} x + C$$

$$\int \frac{1}{|x|\sqrt{x^2 - 1}} dx = \sec^{-1} x + C$$

$$\int \frac{-1}{|x|\sqrt{x^2 - 1}} dx = \csc^{-1} x + C$$

$$\int \frac{1}{\sqrt{x^2 - 1}} dx = \cosh^{-1} x + C$$

$$\int \frac{1}{\sqrt{1 + x^2}} dx = \sinh^{-1} x + C$$

$$\int \frac{1}{1 - x^2} dx = \tanh^{-1} x + C$$

Arc length: $L = \int_a^b \sqrt{1 + [f'(x)]^2} dx$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

Surface Area: $\int_a^b 2\pi f(x)\sqrt{1+[f'(x)]^2}\,dx$ When around the x-axis

Surface Area: $\int_a^b 2\pi f(y)\sqrt{1+[f'(y)]^2}\,dy$ When around the y-axis

Surface Area: $\int_a^b 2\pi x\sqrt{1+[f'(x)]^2}\,dx$ When around the y-axis

Surface Area: $\int_a^b 2\pi y\sqrt{1+[f'(y)]^2}\,dy$ When around the x-axis

$$L=\int_a^b \sqrt{\left(\frac{dy}{dt}\right)^2+\left(\frac{dx}{dt}\right)^2}\,dt$$

$$SA=\int_a^b 2\pi y\sqrt{\left(\frac{dy}{dt}\right)^2+\left(\frac{dx}{dt}\right)^2}\,dt$$

$$A=\int_a^b \frac{1}{2}r^2\,d\theta$$

$$L=\int_a^b \sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}\,d\theta$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\Rightarrow c=\sqrt{|a^2-b^2|}$$

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1\Rightarrow c=\sqrt{|a^2+b^2|}$$

$$4py=x^2$$

$$F = \rho g d A$$

$$\bar{x} = \frac{1}{A}\int_a^b x f(x)\,dx$$

$$\bar{y} = \frac{1}{A}\int_a^b \frac{1}{2}[f(x)]^2\,dx$$

Geometric Series: $\sum_{n=1}^{\infty} ar^{n-1}$ converges to $\frac{a}{1-r}$ if $|r| < 1$.

Divergence Test: If $\lim_{n\to\infty} a_n$ does not exist or if $\lim_{n\to\infty} a_n \neq 0$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

Integral Test: Suppose *f* is a continuous, positive, decreasing function on $[1,\infty)$ and let $a_n = f(n)$. Then the series $\sum_{n=1}^{\infty} a_n$ is convergent if and only if the improper integral $\int_1^{\infty} f(x)\,dx$ is convergent.

P series Test: The *p*-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if $p > 1$ and divergent if $p \leq 1$.

Comparison Test: Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms.
If $\sum b_n$ is convergent and $a_n \leq b_n$ for all *n*, then $\sum a_n$ also converges.
If $\sum a_n$ is divergent and $a_n \leq b_n$ for all *n*, then $\sum b_n$ also diverges.

Limit Comparison Test: Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms. If $\lim_{n\to\infty} \frac{a_n}{b_n} = c$ where *c* is a finite number and $c > 0$, then either both series converge or both series diverge.

Alternating Series Test: If the alternating series $\sum_{n=1}^{\infty}(-1)^{n-1}b_n$ satisfies

1) $b_{n+1} \le b_n$ for all *n* and
2) $\lim_{n\to\infty} b_n = 0$ then the series converges.

Ratio Test: Suppose that $\sum a_n$ is a series with positive terms. If $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = L < 1$ then the series converges. If $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = L > 1$ then the series diverges.

Root Test: Suppose that $\sum a_n$ is a series with positive terms. If $\lim_{n\to\infty} \sqrt[n]{a_n} = L < 1$ then the series converges. If $\lim_{n\to\infty} \sqrt[n]{a_n} = L > 1$ then the series diverges.

If a series $\sum a_n$ converges and $\sum |a_n|$ converges, then $\sum a_n$ is **absolutely convergent.**
If a series $\sum a_n$ converges and $\sum |a_n|$ diverges, then $\sum a_n$ is **conditionally convergent.**

MacLaurin Series: $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$

Taylor Series: $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

$$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$$