MAC1105   Test #4   Name___*Key*___

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Show all work for credit. If the problem specifies you to use a particular method, you must show the work accordingly. If the problem does not specify you to use a particular method, you may use any appropriate method to solve. However, you still must show your work for credit.

1. a. Complete the table for the graphs of $f(x) = 4^x$ and $f(x) = 4^x - 2$.

x	$f(x) = 4^x$	$f(x) = 4^x - 2$
-2	$4^{-2} = \frac{1}{16} = .0625$	-1.9375
-1	$4^{-1} = \frac{1}{4} = .25$	-1.75
0	$4^0 = 1$	-1
1	$4^1 = 4$	2
2	$4^2 = 16$	14

b. How has the graph of $f(x) = 4^x - 2$ changed in comparison to the standard graph of
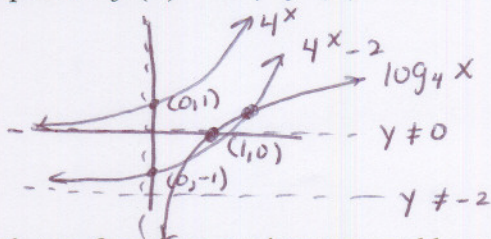
$f(x) = 4^x$?   Down 2

c. Complete the table for the graph of $f(x) = \log_4 x$.

$4^y = x$

x	$f(x) = \log_4 x$
$4^{-2} = \frac{1}{16}$	-2
$4^{-1} = \frac{1}{4}$	-1
$4^0 = 1$	0
$4^1 = 4$	1
$4^2 = 16$	2

d. Sketch the graphs of $f(x) = 4^x$, $f(x) = 4^x - 2$, and $f(x) = \log_4 x$ on the same axis.

e. In part d, which two functions are inverses and how do you know?

$y = 4^x$ and $y = \log_4 x$
are symmetric wrt the $y = x$ line.

2. a. Solve $5^{x^2-x-4} = 25$ algebraically.
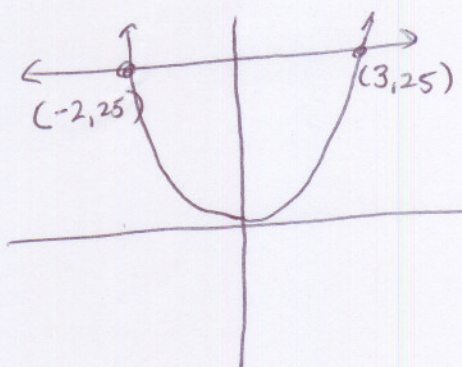
$$5^{x^2-x-4} = 5^2$$

$$x^2-x-4 = 2$$

$$x^2-x-6 = 0$$

$$(x+2)(x-3) = 0$$

$$\boxed{x=-2 \qquad x=3}$$

b. Solve $5^{x^2-x-4} = 25$ graphically.

$$Y_1 = 5\wedge(x^2-x-4)$$
$$Y_2 = 25$$

$(3,25)$

$(-2,25)$

$$\boxed{x=-2,3}$$

3. A color TV loses 30% of its value every 2 years. The value of a TV set t years after it was purchased for $700 is given by the function $V(t) = 700(0.7)^{t/2}$. Calculate the value of the TV in 5 years.

$ value

years after purchase

$$V(5) = 700(.7)^{5/2} = \boxed{\overset{\$}{2}\,86.97}$$

$$700(.7)\wedge(5/2)$$

4. Simplify each of the following.

a. $\log_3 81 = \boxed{4}$

$3^? = 81$

b. $\log_{10} \dfrac{1}{1000} = \boxed{-3}$

$10^? = \frac{1}{1000}$

c. $\log_8 2 = \boxed{1/3}$

$8^? = 2$

d. $\ln e^4 = \boxed{4}$

$e^? = e^4$

5. Suppose that \$6000 is invested at 7% interest compounded annually. Find the future value of the investment at the end of 10 years. $A = P(1+\dfrac{r}{n})^{nt}$

$P = 6000$

$r = .07$

$n = 1$

$t = 10$

$A = 6000\left(1 + \dfrac{.07}{1}\right)^{(1)(10)} = 6000(1.07)^{10}$

$= \boxed{11,802.91}$

6. a. Solve $\log_3(4-10x) = 1$ algebraically.

$3^1 = 4 - 10x$

$-1 = -10x$

$\boxed{\dfrac{1}{10} = x}$

7. Solve $\log_b 1024 = 5$ algebraically.

$b^5 = 1024$

$\boxed{b = 4}$

8. Chemists use a logarithmic scale called pH to measure acidity given by the function $pH = -\log_{10}[H^+]$ where H^+ is the hydrogen ion concentration of the solution. If spinach has a pH of 5.5, find its hydrogen ion concentration.

$$5.5 = -\log_{10}[H^+]$$
$$-5.5 = \log_{10}[H^+]$$
$$10^{-5.5} = H^+ = 3.16 \times 10^{-6}$$

9. Given $f(x) = \log_{10} x$.

a. $f(3) = \log_{10}(3) = 0.477$

b. $f(10) = \log_{10}(10) = 1$

10. a. Express as a sum and difference of logarithms. $\log_3 \dfrac{y^3 z^2}{k^4}$

$$= \log_3 y^3 + \log_3 z^2 - \log_3 k^4$$
$$= 3\log_3 y + 2\log_3 z - 4\log_3 k$$

b. Express as a single logarithm. $\ln\sqrt{x} + \ln y^2 - 4\ln(3z)$

$$= \ln\sqrt{x} + \ln y^2 - \ln(3z)^4$$
$$= \ln\left[\frac{\sqrt{x}\ y^2}{(3z)^4}\right] = \ln\left[\frac{x^{1/2}\ y^2}{81z^4}\right]$$