MAC1114 Test 2 Chapters 4-5   Name ___Key___

Show all work for credit. All answers should be using exact values (no rounding or decimals). Complete graphs show any x intercepts, y intercepts, asymptotes, and end behavior. Only show **one** period of graphs.
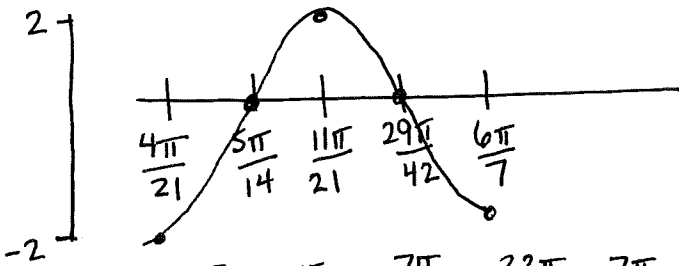
flip x axis

1. Graph $y = -2\cos(3x - \frac{4\pi}{7})$ A B C

Phase shift $= \phi = -\frac{C}{B} = -\frac{-\frac{4\pi}{7}}{\frac{3}{1}} = \frac{4\pi}{7} \cdot \frac{1}{3} = \frac{4\pi}{21}$ start $= \frac{8\pi}{42}$

Period $= T = \frac{2\pi}{B} = \frac{2\pi}{3}$

ticks $= \frac{T}{4} = \frac{2\pi}{3} \cdot \frac{1}{4} = \frac{\pi}{6}$ count $= \frac{7\pi}{42}$
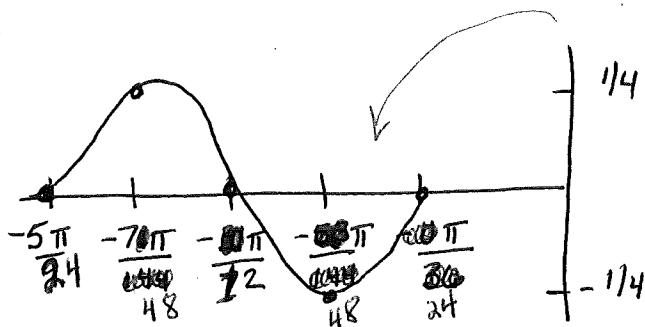

$\frac{4\pi}{21}$ $\frac{5\pi}{14}$ $\frac{11\pi}{21}$ $\frac{29\pi}{42}$ $\frac{6\pi}{7}$

$\frac{8\pi}{42} + \frac{7\pi}{42} = \frac{15\pi}{42} + \frac{7\pi}{42} = \frac{22\pi}{42} + \frac{7\pi}{42} = \frac{29\pi}{42} + \frac{7\pi}{42} = \frac{36\pi}{42}$

2. Graph $y = \frac{1}{4}\sin(8x + \frac{5\pi}{3})$

A B C

Phase shift $= \phi = -\frac{C}{B} = \frac{-\frac{5\pi}{3}}{\frac{8}{1}} = -\frac{5\pi}{3} \cdot \frac{1}{8} = -\frac{5\pi}{24}$ start $= -\frac{10\pi}{48}$

Period $= T = \frac{2\pi}{B} = \frac{2\pi}{8} = \frac{\pi}{4}$

ticks $= \frac{T}{4} = \frac{\pi}{4} \cdot \frac{1}{4} = \frac{\pi}{16}$ count $= \frac{3\pi}{48}$

$-\frac{5\pi}{24}$ $-\frac{7\pi}{48}$ $-\frac{\pi}{12}$ $-\frac{\pi}{48}$ $\frac{\pi}{24}$

$\frac{1}{4}$

$-\frac{1}{4}$

$-\frac{5\pi}{9} + \frac{9\pi}{144} = \frac{-7\pi}{144}$

$\frac{+9\pi}{144} = \frac{-62}{144}$

$\frac{+9\pi}{144} = \frac{-53}{144}$

$\frac{+9\pi}{144} = \frac{-44}{144}$

$$y = \frac{4}{\sin(5x - \pi)}$$

3. Graph $y = 4\csc(5x - \pi)$ start

phase shift $= \phi = -\frac{c}{B} = -\frac{-\pi}{5} = \frac{\pi}{5}$ $= \frac{2\pi}{10}$
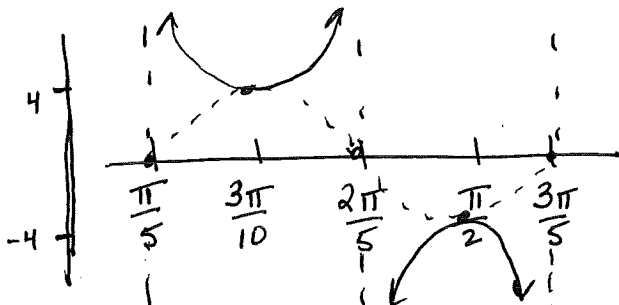
period $= T = \frac{2\pi}{B} = \frac{2\pi}{5}$

ticks $= \frac{T}{4} = \frac{2\pi}{5} \cdot \frac{1}{4} = \frac{\pi}{10}$ count

$\frac{2\pi}{10} + \frac{\pi}{10} = \frac{3\pi}{10}$

$+\frac{\pi}{10} = \frac{4\pi}{10}$

$+\frac{\pi}{10} = \frac{5\pi}{10}$

$+\frac{\pi}{10} = \frac{6\pi}{10}$

4. Graph $y = 2\sec(\frac{1}{2}x + \frac{2\pi}{5})$ $= \frac{2}{\cos(\frac{1}{2}x + \frac{2\pi}{5})}$

phase shift $= \phi = -\frac{c}{B} = -\frac{\frac{2\pi}{5}}{\frac{1}{2}} = -\frac{2\pi}{5} \cdot \frac{2}{1} = -\frac{4\pi}{5}$ start

period $= T = \frac{2\pi}{B} = \frac{2\pi}{\frac{1}{2}} = 2\pi \cdot \frac{2}{1} = 4\pi$

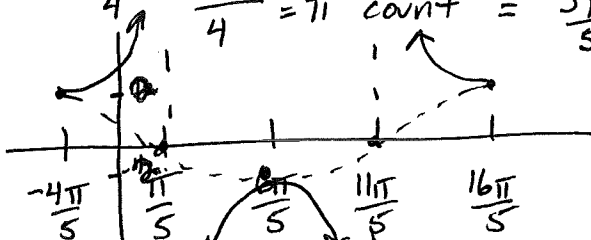ticks $= \frac{T}{4} = \frac{4\pi}{4} = \pi$ count $= \frac{5\pi}{5}$

$-\frac{4\pi}{5} + \frac{5\pi}{5} = \frac{\pi}{5}$

$+\frac{5\pi}{5} = \frac{6\pi}{5}$

$+\frac{5\pi}{5} = \frac{11\pi}{5}$

$+\frac{5\pi}{5} = \frac{16\pi}{5}$
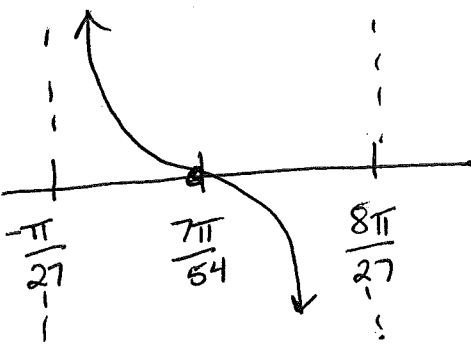

5. Graph $y = \cot(3x + \frac{\pi}{9})$

phase shift $= \phi = -\frac{c}{B} = -\frac{\frac{\pi}{9}}{\frac{3}{1}} = -\frac{\pi}{9} \cdot \frac{1}{3} = -\frac{\pi}{27}$ start $= \frac{-2\pi}{54}$

period $= \frac{\pi}{B} = \frac{\pi}{3}$

ticks $= \frac{T}{2} = \frac{\pi}{3} \cdot \frac{1}{2} = \frac{\pi}{6}$ count $= \frac{9\pi}{54}$

$-\frac{2\pi}{54} + \frac{9\pi}{54} = \frac{7\pi}{54}$

$+\frac{9\pi}{54} = \frac{16\pi}{54}$

6.

7. Verify the identity. Only work on one side.

$$\sin^2 x\,(1 + \cot x) + \cos^2 x(1 - \tan x) + \cot^2 x = \csc^2 x$$

$\sin^2 x + \sin^2 x \cot x + \cos^2 x - \cos^2 x \tan x + \cot^2 x$	Distribute
$\sin^2 x + \sin^2 x\,\dfrac{\cos x}{\sin x} + \cos^2 x - \cos^2 x\,\dfrac{\sin x}{\cos x} + \dfrac{\cos^2 x}{\sin^2 x}$	Quotient Id
$\sin^2 x + \sin x \cos x + \cos^2 x - \cos x \sin x + \dfrac{\cos^2 x}{\sin^2 x}$	Cancel like factors
$\left(\sin^2 x + \cos^2 x\right) + \dfrac{\cos^2 x}{\sin^2 x}$	Cancel like terms
	Pythag Id
$\dfrac{\sin^2 x}{\sin^2 x}\,\dfrac{1}{1} + \dfrac{\cos^2 x}{\sin^2 x}$	LCD
$\dfrac{\sin^2 x + \cos^2 x}{\sin^2 x}$	Pythag Id
	Reciprocal Id.
$\dfrac{1}{\sin^2 x}$	
$\boxed{\csc^2 x} \;=\; \boxed{\csc^2 x}$	