6 points each (36 total points)

MAC1114 Test 3 Name Key

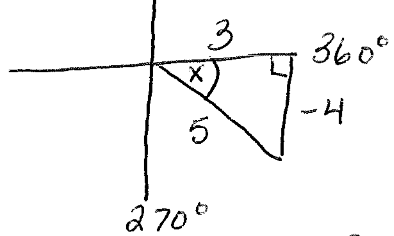
(Deborah Howard 3-16)

Show all work for credit. Use exact values only.

1. Given $sinx = \frac{-4}{5}$ where $270° < x < 360°$ and $siny = \frac{5}{13}$ where $90° < y < 180°$, find $\cos(x + y)$.

(6pt)

$\sin x = \frac{-4}{5} = \frac{O}{H}$



$A^2 + (-4)^2 = 5^2$

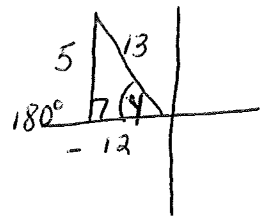
$A^2 + 16 = 25$

$A^2 = 9$

$A = \pm 3$

$\cos x = \frac{A}{H} = \frac{3}{5}$

90° $\sin y = \frac{5}{13} = \frac{O}{H}$



$A^2 + 5^2 = 13^2$

$A^2 + 25 = 169$

$A^2 = 144$

$A = \pm 12$

$\cos y = \frac{A}{H} = \frac{-12}{13}$

$\cos(x+y) =$

$\cos x \cos y - \sin x \sin y$

$\left(\frac{3}{5}\right)\left(\frac{-12}{13}\right) - \left(\frac{-4}{5}\right)\left(\frac{5}{13}\right)$

$\frac{-36}{65} + \frac{20}{65}$

$\boxed{\frac{-16}{65}}$

2. Verify the identity. Show all work for credit. Work on one side of the identity only. $cotx - tanx = \frac{\cos(2x)}{\text{sinxcosx}}$

(6)

$$= \frac{\cos^2 x - \sin^2 x}{\sin x \cos x}$$

$$= \frac{\cos^{\cancel{2}} x}{\sin x \cancel{\cos x}} - \frac{\sin^{\cancel{2}} x}{\cancel{\sin x} \cos x}$$

$$= \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}$$

$$= \boxed{\cot x - \tan x}$$

3. Verify the identity. Show all work for credit. Work on one side of the identity only. $\sin^2 x - \sin^2 y = \cos^2 y - \cos^2 x$

$$1 - \cos^2 x - (1 - \cos^2 y) =$$

$$\cancel{1} - \cos^2 x \cancel{-1} + \cos^2 y =$$

$$\boxed{\cos^2 y - \cos^2 x} =$$

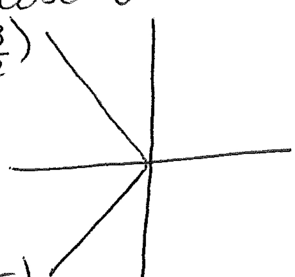
4. Solve algebraically for all values in radians. You must show work for credit.
$2\cos\theta + 1 = 0$

$$2\cos\theta = -1$$

$$\cos\theta = -\frac{1}{2}$$

$2\pi/3$ $\cos\theta < 0$

$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$



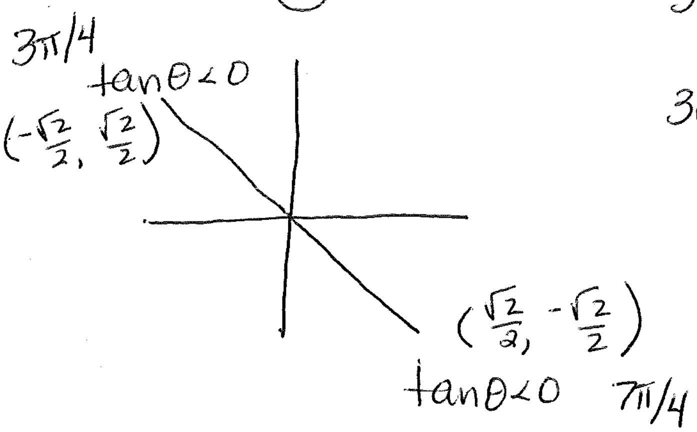
$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ $\cos\theta < 0$

$\frac{4\pi}{3}$

$$\boxed{\begin{aligned} \theta &= \frac{2\pi}{3} + 2\pi k \\ \theta &= \frac{4\pi}{3} + 2\pi k \end{aligned}}$$

5. Solve algebraically for all values in radians. You must show work for credit.
$\tan(3\theta) = -1$

$\tan(w) = -1$



$3\theta = w = \frac{3\pi}{4} + 2\pi k$

$3\theta = w = \frac{7\pi}{4} + 2\pi k$

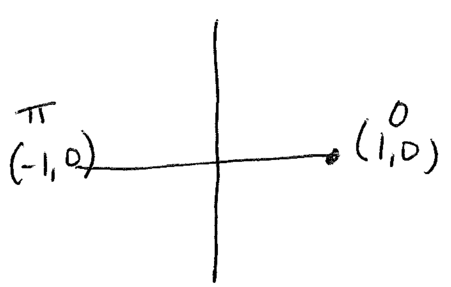
OR $3\theta = w = \frac{3\pi}{4} + \pi k$

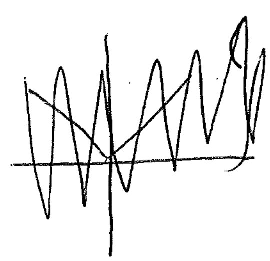
$\boxed{\theta = \frac{\pi}{4} + \frac{2}{3}\pi k \quad \theta = \frac{7\pi}{12} + \frac{2}{3}\pi k}$ OR $\boxed{\theta = \frac{\pi}{4} + \frac{\pi}{3}k}$

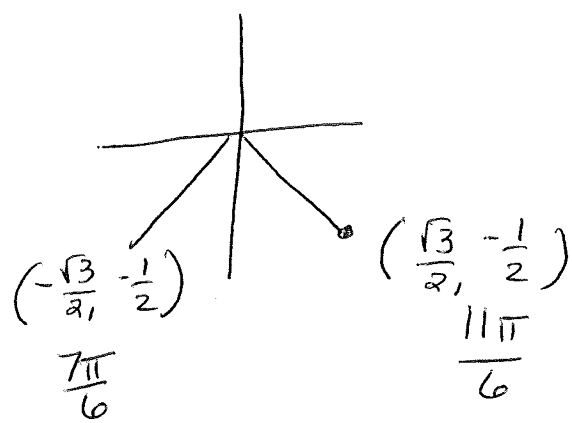
6. Solve algebraically for all values in radians. You must show work for credit.
$2\sin^2(x) + \sin(x) = 0$

$\sin x\,(2\sin x + 1) = 0$

$\sin x = 0 \qquad 2\sin x + 1 = 0 \implies \sin x = -\frac{1}{2}$







$\boxed{x = 0 + 2\pi k \quad x = \pi + 2\pi k \quad x = \frac{7\pi}{6} + 2\pi k \quad x = \frac{11\pi}{6} + 2\pi k}$

OR

$\boxed{x = 0 + \pi k \quad x = \frac{7\pi}{6} + 2\pi k \quad x = \frac{11\pi}{6} + 2\pi k}$