5 points each (Total 55 points)

10 Test 3 **Student Name:** Key

1. Use partial fractions to **set up but do not solve** for the constants A, B, C, D, or E.

$$\frac{5x+13}{(x+5)(x-3)^2(x^2+7)}$$

$$\boxed{\frac{A}{x+5}+\frac{B}{x-3}+\frac{C}{(x-3)^2}+\frac{Dx+E}{x^2+7}}$$

2. Use partial fractions to decompose $\frac{2x-1}{(x+5)(x-3)}$. Solve for the constants.

$$\frac{A}{x+5}+\frac{B}{x-3}=\frac{2x-1}{(x+5)(x-3)}$$

$$A(x-3)+B(x+5)=2x-1$$

$$Ax-3A+Bx+5B=2x-1$$

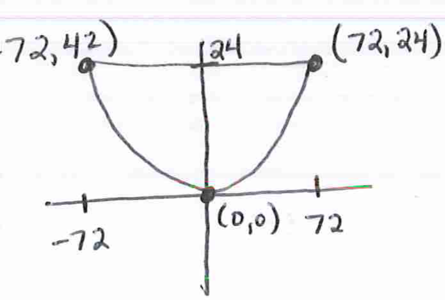
$$\begin{cases} A+B=2 \\ -3A+5B=-1 \end{cases}$$

$$3\cancel{A}+3B=6$$
$$+\cancel{-3}A+5B=-1$$
$$8B=5$$
$$B=\frac{5}{8}$$

$$A+B=2$$
$$A+\frac{5}{8}=2$$
$$A=2-\frac{5}{8}=\frac{16}{8}-\frac{5}{8}=\frac{11}{8}$$

$$\boxed{\frac{11}{8(x+5)}+\frac{5}{8(x-3)}}$$

3. **Satellite Dish.** A satellite dish measures 144 feet across its opening and 24 feet deep at its center. The receiver should be placed at the focus of the parabolic dish. Where is the focus?
The focus is [ ] feet away from the vertex of the satellite dish.



$$\frac{144}{2}=72$$

$$(x-h)^2=4p(y-k)$$
$$(x-0)^2=4p(y-0)$$
$$x^2=4py$$
$$72^2=4p(24)$$
$$5184=96p$$

$$p=\frac{5184 \div 4}{96 \div 4}=\frac{1296}{24}=54$$

$$p=54$$

$$\boxed{54 \text{ feet}}$$

4. Find the center, vertices, foci, asymptotes, and graph. $\frac{(x+3)^2}{4} - \frac{(y-2)^2}{25} = 1$

hyperbola open L/R

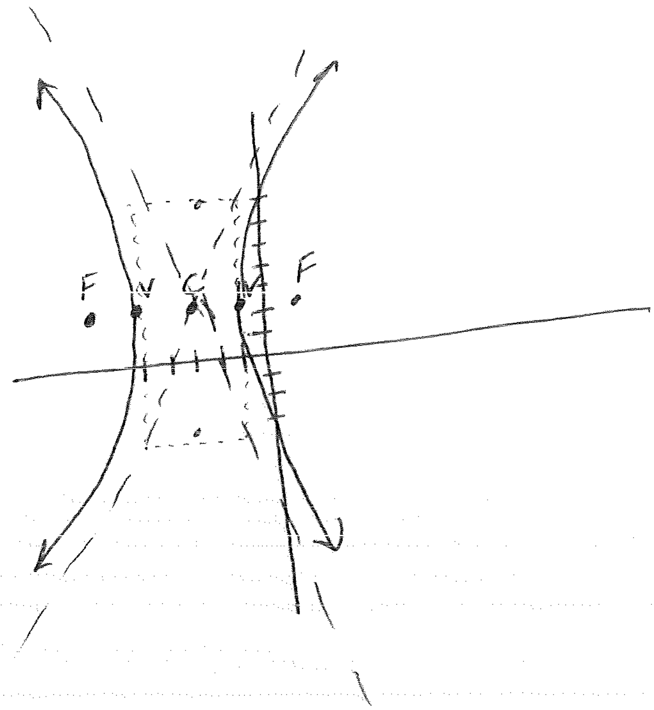
$C(-3, 2)$

$a = 2$ L/R

$b = 5$ U/D

$c = \sqrt{a^2+b^2} = \sqrt{4+25} = \sqrt{29}$ L/R

$$\boxed{\begin{array}{l} y-2 = \pm\frac{5}{2}(x+3) \\ C(-3,2) \\ F(-3 \pm \sqrt{29}, 2) \\ V(-1,2) \\ V(-5,2) \end{array}}$$



5. Find the center, vertices, foci, and graph. $25x^2 + 2y^2 - 8y - 42 = 0$

ellipse

$25x^2 + 2y^2 - 8y + \square = 42$

$25(x-0)^2 + 2(y^2 - 4y + \boxed{4}) = 42 + 8$

$\left(\frac{4}{2}\right)^2 = 4$

$\frac{25(x-0)^2}{50} + \frac{2(y-2)^2}{50} = \frac{50}{50}$

$\frac{(x-0)^2}{2} + \frac{(y-2)^2}{25} = 1$

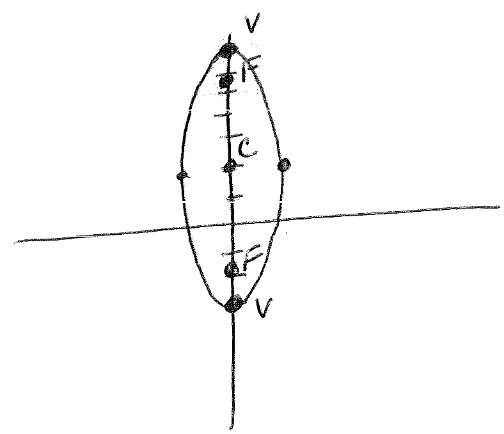
$C(0,2)$

$a = \sqrt{2}$ L/R

$b = 5$ U/D

$c = \sqrt{|a^2-b^2|} = \sqrt{|2-25|} = \sqrt{23}$ U/D

$$\boxed{\begin{array}{l} C(0,2) \\ F(0, 2 \pm \sqrt{23}) \\ V(0,7) \\ V(0,-3) \end{array}}$$



6. Write the first four terms of the sequence. Assume n starts at 1. $a_n = \dfrac{(n+5)!}{(n+6)!}$

$$a_n = \frac{(n+5)!}{(n+6)!} = \frac{6!}{7!}, \frac{7!}{8!}, \frac{8!}{9!}, \frac{9!}{10!}, \ldots$$

$$= \boxed{\frac{1}{7}, \frac{1}{8}, \frac{1}{9}, \frac{1}{10}}, \ldots$$

$a_0 = -2 \quad a_1 \quad a_2 \quad a_3 \qquad a_{15} = 3(15) - 2 = 43$

7. Find the sum of the first 15 terms of $\underbrace{1 + 4}_{+3} + \underbrace{7}_{+3} + 10 + \ldots =$

arithmetic ~~geometric~~ $d = 3$

$a_n = a_1 + d(n-1)$

$a_n = 3n - 2$

$$S_n = \frac{n(a_1 + a_n)}{2} = \frac{15(1 + 43)}{2} = \frac{15(44)}{2} = \boxed{330}$$

$1+4+7+10+13+16+19+22+25+28+31+34+37+40+43 = 330$

$\overbrace{2^0 + 2^1 + 2^2 + \ldots 2^9}^{10 \text{ terms}}$

8. Find the sum of $\underbrace{1 + 2}_{\times 2} + \underbrace{4}_{\times 2} + 8 + \ldots + 2^9 = \displaystyle\sum_{n=1}^{10} 1(2)^{n-1}$

geometric $r = 2$

$a_n = a_1(r)^{n-1}$

$a_n = 1(2)^{n-1}$

$$S_n = \frac{a_1(1-r^n)}{1-r} = \frac{1(1-2^{10})}{1-2} = \frac{1 - 1024}{-1} = \frac{-1023}{-1} = \boxed{1023}$$

$1+2+4+8+16+32+64+128+256+512 = 1023$

9. Find the sum and round your answer to three decimal places. $\displaystyle\sum_{n=0}^{11} 10(0.5)^n =$

$r = 0.5$

#9 $\sum_{n=0}^{11} 10(0.5)^n = S_n = \frac{a_1(1-r^n)}{1-r} = \frac{10(1-0.5^{12})}{1-0.5} = \boxed{19.995}$

$10 + 5 + \frac{5}{2} + \dots + 10(0.5^{11})$

$a_1 \quad a_2 \quad \dots \quad a_{12}$

12 terms since starting at 0

10. Find the sum and write as an exact answer. If it is not possible to find the sum, enter NS.

$$\sum_{n=1}^{\infty}\left(\frac{1}{11}\right)^n \qquad r = \frac{1}{11} \qquad a_1 = \frac{1}{11}$$

$S = \frac{a_1}{1-r}$ $\quad |r| < 1$ ☺

$$S = \frac{\frac{1}{11}}{1-\frac{1}{11}} = \frac{\frac{1}{11}}{\frac{10}{11}} = \frac{1}{11}\cdot\frac{11}{10} = \boxed{\frac{1}{10}}$$

11. Find the sum and write as an exact answer. If it is not possible to find the sum, enter NS.

$$\sum_{n=0}^{\infty}\left(\frac{17}{16}\right)^n \qquad r = \frac{17}{16} \qquad a_1 = 1$$

r is not < 1

$\boxed{NS}$