③ $y = 2 - 15x + 9x^2 - x^3$

a) $0 = 2 - 15x + 9x^2 - x^3$

$\frac{P}{Q} = \frac{\pm 1, \pm 2}{\pm 1} = \pm 1, \pm 2$

$$\cancel{1}\ \begin{array}{r|rrrr} & -1 & 9 & -15 & 2 \\ & \downarrow & -1 & 8 & -7 \\ \hline & -1 & 8 & -7 & -5 \end{array} \qquad \cancel{-1}\ \begin{array}{r|rrrr} & -1 & 9 & -15 & 2 \\ & \downarrow & 1 & -10 & 25 \\ \hline & -1 & 10 & -25 & 27 \end{array} \qquad \begin{array}{r|rrrr} 2 & -1 & 9 & -15 & 2 \\ & \downarrow & -2 & 14 & -2 \\ \hline & -1 & 7 & -1 & 0 \end{array}$$

$y = (x-2)(-x^2 + 7x - 1)$ $\qquad a = -1 \quad b = 7 \quad c = -1$

$x = \frac{-7 \pm \sqrt{7^2 - 4(-1)(-1)}}{2(-1)} = \frac{-7 \pm \sqrt{49-4}}{-2} = \frac{-7 \pm \sqrt{45}}{-2}$

$x = 2, \ \frac{-7 + \sqrt{45}}{-2}, \ \frac{-7 - \sqrt{45}}{-2}$

$x = 2, 0.15, 6.85$ $\qquad \boxed{(2,0)\ (0.15,0)\ (6.85,0)}$

b) $y = 2 - 15(0) + 9(0)^2 - (0)^3 = 2$

$\boxed{(0,2)}$

c.) y is a polynomial $\Rightarrow$ domain $\boxed{(-\infty, \infty)}$

d.) Polynomials do not have VA, HA, slant asym. $\Rightarrow$ $\boxed{\text{none}}$

e.) $\boxed{y' = -15 + 18x - 3x^2}$

f.) $0 = -15 + 18x - 3x^2$

$0 = -5 + 6x - x^2$

$0 = 5 - 6x + x^2$

$0 = (5 - x)(1 - x)$

$5 - x = 0 \qquad 1 - x = 0$

$x = 5 \qquad x = 1$

x	y
1	-5
5	27

crit. points

$$\begin{array}{ccccc} \searrow & & \nearrow & & \searrow \\ - - - & | & + + + & | & - - - \\ & 1 & & 5 & \end{array} \quad f'(x)$$

g.) $\boxed{\text{inc } (1,5)}$

h.) $\boxed{\text{dec } (-\infty,1) \cup (5,\infty)}$

i.) $\boxed{\text{local min } (1,-5)}$

j.) $\boxed{\text{local max } (5,27)}$

3 cont:

k.) $\boxed{y'' = 18 - 6x}$

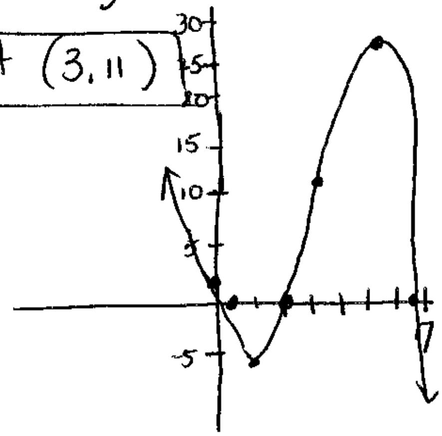
l.) $0 = 18 - 6x$

$6x = 18$

$x = 3$

$$\begin{array}{ccc} \text{Up} & & \text{Down} \\ +++ & | & --- \\ & 3 & \end{array} \quad f''(x)$$

$\boxed{\text{inf point } (3, 11)}$



m.) $\boxed{\text{Up } (-\infty, 3)}$

n.) $\boxed{\text{down } (3, \infty)}$

o.)

---

7. $y = 2x^5 - 5x^2 + 1$

a.) $0 = 2x^5 - 5x^2 + 1$

$\frac{P}{Q} = \frac{\pm 1}{\pm 1, \pm 2} = \pm 1, \pm \frac{1}{2}$

$$\cancel{-\tfrac{1}{2}} \;\begin{array}{|cccccc} 2 & 0 & 0 & -5 & 0 & 1 \\ \downarrow & -1 & \frac{1}{2} & -\frac{1}{4} & \frac{21}{8} & -\frac{21}{16} \\ \hline 2 & -1 & \frac{1}{2} & -\frac{21}{4} & \frac{21}{8} & -\frac{5}{16} \end{array}$$

$$\cancel{1} \;\begin{array}{|cccccc} 2 & 0 & 0 & -5 & 0 & 1 \\ \downarrow & 2 & 2 & 2 & -3 & -3 \\ \hline 2 & 2 & 2 & -3 & -3 & -2 \end{array}$$

$$\cancel{-1} \;\begin{array}{|cccccc} 2 & 0 & 0 & -5 & 0 & 1 \\ \downarrow & -2 & 2 & -2 & 7 & -7 \\ \hline 2 & -2 & 2 & -7 & 7 & -6 \end{array}$$

$$\cancel{\tfrac{1}{2}} \;\begin{array}{|cccccc} 2 & 0 & 0 & -5 & 0 & 1 \\ \downarrow & 1 & \frac{1}{2} & \frac{1}{4} & -\frac{19}{8} & -\frac{19}{16} \\ \hline 2 & 1 & \frac{1}{2} & -\frac{19}{4} & -\frac{19}{8} & -\frac{3}{16} \end{array}$$

no rational x int. $\Rightarrow$ use graphing calculator.

$x = 0.46 \quad x = -0.44 \quad x = 1.3$

$\boxed{(0.46, 0)\ (-0.44, 0)\ (1.3, 0)}$

b.) $y = 2(0)^5 - 5(0)^2 + 1 = 1 \quad \boxed{(0, 1)}$

c.) polynomial $\Rightarrow (-\infty, \infty)$

d.) $\boxed{\text{none}}$

e.) $y' = 10x^4 - 10x = 10x(x^3 - 1) = \boxed{10x(x-1)(x^2+x+1) = y'}$

f.) $0 = 10x(x-1)(x^2+x+1)$

$10x = 0 \qquad x - 1 = 0 \qquad x^2 + x + 1 = 0$

$x = 0 \qquad x = 1 \qquad$ imaginary

x	f(x)
0	1
1	-2

crit pts

7cont:

g.) $\boxed{\text{inc } (-\infty,0)\cup(1,\infty)}$

$\nearrow \quad \searrow \quad \nearrow$

$+++ \;|\; --- \;|\; ++++ \quad f'(x)$

$\quad 0 \qquad 1$

h.) $\boxed{\text{dec } (0,1)}$

i.) $\boxed{\text{local min } (1,-2)}$

j.) $\boxed{\text{local max } (0,1)}$

k.) $y'' = 40x^3 - 10 = 10(4x^3 - 1) = \boxed{10(\sqrt[3]{4}\,x - 1)(\sqrt[3]{16}\,x^2 + \sqrt[3]{4}\,x + 1) = y''}$

l.) $0 = 10(\sqrt[3]{4}\,x - 1)(\sqrt[3]{16}\,x^2 + \sqrt[3]{4}\,x + 1)$

$10 \neq 0$ none

$\sqrt[3]{4}\,x - 1 = 0$

$\sqrt[3]{4}\,x = 1$

$x = \frac{1}{\sqrt[3]{4}} \approx 0.63$

none

Down / Up

$--- \;|\; +++ \quad f''(x)$

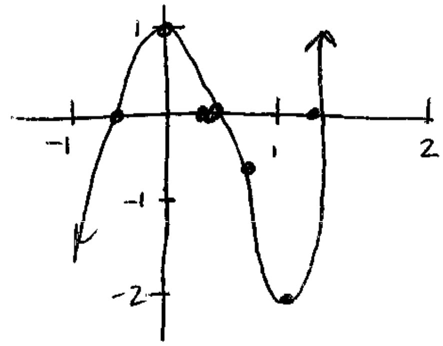
$\frac{1}{\sqrt[3]{4}}$

inflection point $\left(\frac{1}{\sqrt[3]{4}}, -0.79\right) \approx \boxed{(0.63, -0.79)}$

m.) $\boxed{\text{up } (0.63,\infty)}$

n.) $\boxed{\text{down } (-\infty, 0.63)}$

o.)



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11. $y = \frac{1}{x^2-9} = \frac{1}{(x+3)(x-3)}$

a.) $0 = \frac{1}{x^2-9}$

$0 \neq 1$ $\boxed{\text{none}}$

b.) $y = \frac{1}{0^2-9} = \frac{-1}{9}$ $\boxed{\left(0, -\frac{1}{9}\right)}$

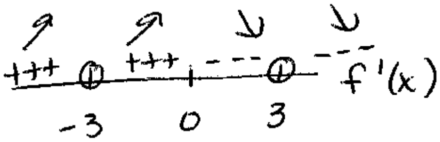
c.) $\boxed{x \neq \pm 3}$

d.) $\boxed{\text{VA: } x \neq \pm 3 \quad \text{HA: } y \neq 0}$ no slant

11 cont:

e.) $y' = \frac{(x^2-9)(0)-1(2x)}{(x^2-9)^2} = \boxed{\frac{-2x}{(x^2-9)^2} = y'}$

f.) $0 = \frac{-2x}{(x^2-9)^2}$



$0 = -2x$

$x = 0$

$\boxed{\text{crit point } (0, -\tfrac{1}{9})}$

g.) $\boxed{\text{inc } (-\infty, -3) \cup (-3, 0)}$

h.) $\boxed{\text{dec } (0, 3) \cup (3, \infty)}$

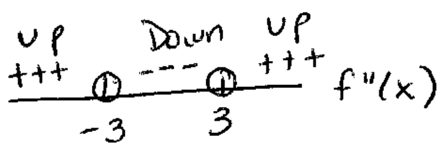
i.) $\boxed{\text{local min: none}}$

j.) $\boxed{\text{local max: } (0, -\tfrac{1}{9})}$

k.) $y'' = \frac{(x^2-9)^2(-2) - (-2x)(2)(x^2-9)(2x)}{(x^2-9)^4} = \frac{-2\cancel{(x^2-9)}[(x^2-9)-4x^2]}{(x^2-9)^{\cancel{4}3}}$

$y'' = \frac{-2(x^2-9-4x^2)}{(x^2-9)^3} = \frac{-2(-3x^2-9)}{(x^2-9)^3} = \boxed{\frac{+6(x^2+3)}{(x^2-9)^3} = y''}$

l.) $0 = \frac{6(x^2+3)}{(x^2-9)^3}$



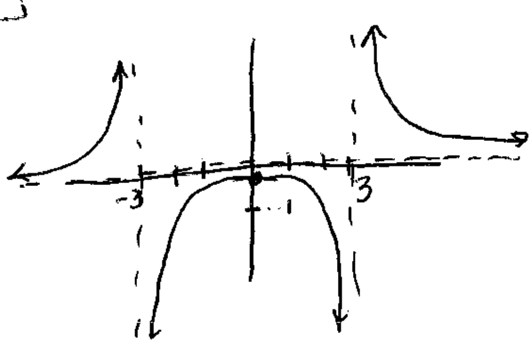
$0 = 6(x^2+3)$

$\boxed{\text{none real}}$

m.) $\boxed{\text{up } (-\infty, -3) \cup (3, \infty)}$

n.) $\boxed{\text{down } (-3, 3)}$

o.)



17. $y = \dfrac{x^2}{x^2+3}$

a.) $0 = \dfrac{x^2}{x^2+3}$

$0 = x^2$

$x = 0$ $\quad \boxed{(0,0)}$

b.) $y = \dfrac{0^2}{0^2+3} = 0 \quad \boxed{(0,0)}$

d.) $x^2+3 \neq 0$ $\quad \boxed{\text{HA: } y \neq 1}$ $\quad$ no slant

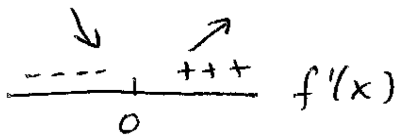
none VA.

c.) $\boxed{(-\infty,\infty)}$

e.) $y' = \dfrac{(x^2+3)(2x) - (x^2)(2x)}{(x^2+3)^2} = \dfrac{2x(x^2+3-x^2)}{(x^2+3)^2} = \dfrac{2x(3)}{(x^2+3)^2}$

$\boxed{y' = \dfrac{6x}{(x^2+3)^2}}$

f.) $0 = \dfrac{6x}{(x^2+3)^2}$



$0 = 6x$

$x = 0$ $\quad \boxed{\begin{matrix}\text{crit pt} \\ (0,0)\end{matrix}}$

g.) $\boxed{\text{inc } (0,\infty)}$

h.) $\boxed{\text{dec } (-\infty,0)}$

i.) $\boxed{\text{local min } (0,0)}$

j.) $\boxed{\text{local max: none}}$

k.) $y'' = \dfrac{(x^2+3)^2(6) - (6x)(2)(x^2+3)(2x)}{(x^2+3)^4} = \dfrac{6\cancel{(x^2+3)}[x^2+3-4x^2]}{(x^2+3)^{\cancel{4}\,3}}$

$y'' = \dfrac{6(-3x^2+3)}{(x^2+3)^3} = \dfrac{-18(x^2-1)}{(x^2+3)^3} = \boxed{\dfrac{-18(x+1)(x-1)}{(x^2+3)^3} = y''}$

17 cont:

l.) $0 = \dfrac{-18(x+1)(x-1)}{(x^2+3)^3}$

$0 = -18(x+1)(x-1)$

$-18 \neq 0$ none $x+1=0$, $x=-1$ $x-1=0$, $x=1$

$\boxed{\text{inf pts: } (-1, 0.25), (1, 0.25)}$

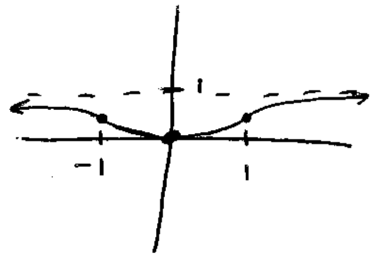
Down Up Down

--- +++ --- $f''(x)$

-1 1

m.) $\boxed{\text{up } (-1, 1)}$

n.) $\boxed{\text{down } (-\infty, -1) \cup (1, \infty)}$



o.)

---

23) $y = \dfrac{x}{\sqrt{x^2+1}} = \dfrac{x}{(x^2+1)^{1/2}}$

a) $0 = \dfrac{x}{\sqrt{x^2+1}}$

$0 = x$ $\boxed{(0,0)}$

b.) $y = \dfrac{0}{\sqrt{0^2+1}} = 0$ $\boxed{(0,0)}$

c.) $x^2+1 \geq 0$ always true $\Rightarrow$ $\boxed{(-\infty, \infty)}$

d.) $\boxed{\text{VA: none} \quad \text{HA: } y \neq 1 \quad \text{no slant}}$

e.) $y' = \dfrac{(x^2+1)^{1/2}(1) - x\left(\frac{1}{2}\right)(x^2+1)^{-1/2}(2x)}{x^2+1} = \dfrac{(x^2+1)^{-1/2}\left[(x^2+1)^1 - x^2\right]}{x^2+1}$

$y' = \dfrac{(x^2+1)^{-1/2}(1)}{x^2+1} = \boxed{\dfrac{1}{(x^2+1)^{3/2}} = y'}$

f.) $0 = \dfrac{1}{(x^2+1)^{3/2}}$

$0 \neq 1$

$\boxed{\text{no crit point}}$

+ + + + + $f'(x)$

g.) $\boxed{\text{inc } (-\infty, \infty)}$

23 cont.

h.) $\boxed{\text{dec none}}$

i.) $\boxed{\text{local min : none}}$

j.) $\boxed{\text{local max : none}}$

k.) $y'' = \dfrac{(x^2+1)^{3/2}(0) - 1\left(\frac{3}{2}\right)(x^2+1)^{1/2}(2x)}{(x^2+1)^3} = \dfrac{-3x(x^2+1)^{1/2}}{(x^2+1)^3}$

$$\boxed{y'' = \frac{-3x}{(x^2+1)^{5/2}}}$$

l.) $0 = \dfrac{-3x}{(x^2+1)^{5/2}}$

Up +++ | Down - - - - $f''(x)$, 0

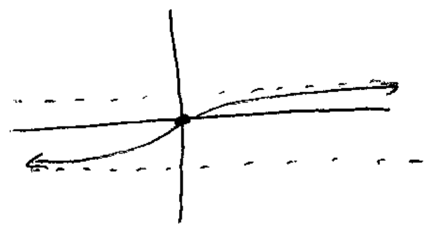
$0 = -3x$

$x = 0$ $\boxed{\text{inf point } (0,0)}$

m.) $\boxed{\text{Up } (-\infty, 0)}$

n.) $\boxed{\text{Down } (0, \infty)}$

o.)



(31.) $y = 3\sin x - \sin^3 x = \sin x(3 - \sin^2 x) = \sin x(\sqrt{3} + \sin x)(\sqrt{3} - \sin x)$

a.) $0 = \sin x(\sqrt{3} + \sin x)(\sqrt{3} - \sin x)$

$\sin x = 0$ $\qquad \sqrt{3} + \sin x = 0$ $\qquad \sin x = \sqrt{3} \approx 1.73$

$x = 0 + \pi k$ $\qquad \sin x = -\sqrt{3} \approx -1.73$ $\qquad$ DNE

$\sin^{-1}(-\sqrt{3}) = \text{DNE}$

$\boxed{(0,0)\ (\pi,0)\ (2\pi,0),\ (-\pi,0),\ (-2\pi,0)\ \ldots}$

b.) $y = 3\sin(0) - \sin^3(0) = 0$ $\quad \boxed{(0,0)}$

c.) $\boxed{(-\infty, \infty)}$

d.) $\boxed{\text{no VA, no HA, no SA}}$

e.) $y' = 3\cos x - 3\sin^2 x(\cos x) = 3\cos x(1 - \sin^2 x) = 3\cos x \cos^2 x$

$\boxed{y' = 3\cos^3 x}$

f.) $0 = 3\cos^3 x$

$0 = \cos x$

$x = \frac{\pi}{2} + \pi k$

$f'(x)$: $+++$ | $-\frac{7\pi}{2}$ | $---$ | $-\frac{5\pi}{2}$ | $+++$ | $-\frac{3\pi}{2}$ | $---$ | $-\frac{\pi}{2}$ | $+++$ | $\frac{\pi}{2}$ | $---$ | $\frac{3\pi}{2}$ | $+++$ | $\frac{5\pi}{2}$ | $---$ | $\frac{7\pi}{2}$ | $+++$

$\boxed{\left(\frac{\pi}{2}, 3\right)\left(\frac{3\pi}{2}, -2\right)\left(\frac{5\pi}{2}, 3\right)\left(\frac{7\pi}{2}, -2\right) \ldots}$

g.) $\boxed{\text{inc } \left(-\frac{5\pi}{2}, -\frac{3\pi}{2}\right) \cup \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{2}, \frac{5\pi}{2}\right) \cup \left(\frac{7\pi}{2}, \frac{9\pi}{2}\right) \ldots}$

h.) $\boxed{\text{dec } \left(-\frac{7\pi}{2}, -\frac{5\pi}{2}\right) \cup \left(-\frac{3\pi}{2}, -\frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)\left(\frac{5\pi}{2}, \frac{7\pi}{2}\right) \ldots}$

i.) $\boxed{\text{local min } \left(-\frac{5\pi}{2}, -2\right), \left(-\frac{\pi}{2}, -2\right), \left(\frac{3\pi}{2}, -2\right), \left(\frac{7\pi}{2}, -2\right) \ldots}$

j.) $\boxed{\text{local max } \left(-\frac{7\pi}{2}, 3\right), \left(-\frac{3\pi}{2}, 3\right), \left(\frac{\pi}{2}, 3\right), \left(\frac{5\pi}{2}, 3\right)}$

k.) $\boxed{y'' = 9\cos^2 x(-\sin x)}$

l.) $0 = 9\cos^2 x(-\sin x)$

$9\cos^2 x = 0 \qquad -\sin x = 0$

$\cos^2 x = 0 \qquad \sin x = 0$

$\cos x = 0 \qquad x = 0 + \pi k$

$x = \frac{\pi}{2} + \pi k$

Down Up Down Up Down

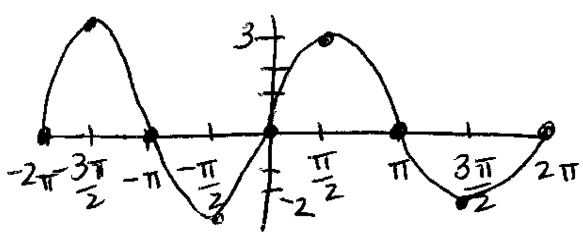
$f''(x)$: $---$ | $-\pi$ | $+++$ | $-\frac{\pi}{2}$ | $+++$ | 0 | $---$ | $\frac{\pi}{2}$ | $---$ | π | $+++$ | $\frac{3\pi}{2}$ | $+++$ | 2π | $---$

$\boxed{\text{inflection pts } (\pi, 0), (2\pi, 0), (0, 0) \ldots}$

m.) $\boxed{\text{up } (\cancel{-\frac{\pi}{2}, 0), (\pi, 0), (2\pi, 0)}}$ $\boxed{(-\pi, 0) \cup (\pi, 2\pi) \ldots}$

n.) $\boxed{\text{down } (0, \pi)(2\pi, 3\pi)} \ldots$

o.)



43. $y = \dfrac{x^2+1}{x+1}$

a.) $0 = \dfrac{x^2+1}{x+1} \Rightarrow 0 = x^2+1$ $\boxed{\text{none}}$

b.) $y = \dfrac{0^2+1}{0+1} = 1$ $\boxed{(0,1)}$

c.) $\boxed{x \neq -1}$

d.) $\boxed{VA: x \neq -1}$ $\boxed{HA: \text{none}}$ slant: $\boxed{y \neq x-1}$

$$\begin{array}{r} x - 1 \\ x+1 \overline{\smash{)}\, x^2 + 0x + 1} \\ -x^2 + {}^{-}x \\ \hline -x + 1 \\ +x + 1 \\ \hline 2 \end{array}$$

e.) $y' = \dfrac{(x+1)(2x) - (x^2+1)(1)}{(x+1)^2} = \dfrac{2x^2 + 2x - x^2 - 1}{(x+1)^2}$

$$\boxed{y' = \frac{x^2+2x-1}{(x+1)^2}} = \boxed{\cancel{\frac{(x-1)(x-1)}{(x+1)^2} = y'}}$$

f.) $\cancel{0 = \dfrac{(x-1)(x-1)}{(x+1)^2}}$

$\cancel{0 = (x-1)(x-1)}$

$x = \cancel{1}$ $\boxed{\cancel{\text{crit pt } (1,1)}}$

$+++ \quad --- \quad \ominus \quad --- \quad +++ \qquad f'(x)$

$-2.41 \quad -1 \quad 0.41$

$0 = \dfrac{x^2+2x-1}{(x+1)^2}$

$0 = x^2+2x-1$

DNF

$a = 1 \quad b = 2 \quad c = -1$

$x = \dfrac{-2 \pm \sqrt{2^2 - 4(1)(-1)}}{2(1)} = \dfrac{-2 \pm \sqrt{4+4}}{2}$

$x = \dfrac{-2 \pm \sqrt{8}}{2} = \dfrac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}$

$x = -1 + \sqrt{2} \approx 0.41$

$x = -1 - \sqrt{2} \approx -2.41$

$\boxed{\begin{array}{l} \text{crit point } (0.41, 0.83) \\ \qquad\qquad (-2.41, -4.83) \end{array}}$

g.) $\boxed{\text{inc } (-\infty, -2.41) \cup (0.41, \infty)}$

h.) $\boxed{\text{dec } (-2.41, -1) \cup (-1, 0.41)}$

i.) $\boxed{\text{local min } (0.41, 0.83)}$

j.) $\boxed{\text{local max } (-2.41, -4.83)}$

43 cont:

k.) $y'' = \dfrac{(x+1)^2(2x+2) - (x^2+2x-1)(2)(x+1)(1)}{(x+1)^4}$

$y'' = \dfrac{2\cancel{(x+1)}\left[(x+1)^2 - (x^2+2x-1)\right]}{(x+1)^{\cancel{4}\,3}}$

$y'' = \dfrac{2(\cancel{x^2}+\cancel{2x}+1-\cancel{x^2}-\cancel{2x}+1)}{(x+1)^3} = \dfrac{2(2)}{(x+1)^3} = \boxed{\dfrac{4}{(x+1)^3} = y''}$

l.) $0 = \dfrac{4}{(x+1)^3} \Rightarrow 0 = 4$ $\boxed{\text{none inf pt}}$

Down Up

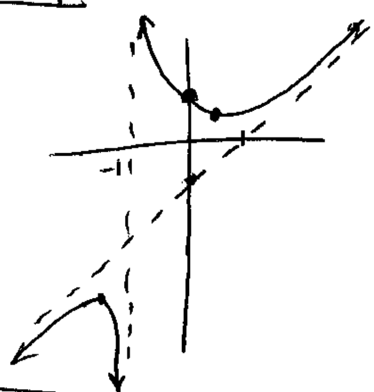
$- - -$ ① $+ + +$ $f''(x)$

-1

m.) $\boxed{\text{Up } (-1, \infty)}$

n.) $\boxed{\text{Down } (-\infty, -1)}$

o.)



(47.) $y = \dfrac{-2x^2+5x-1}{2x-1}$

a) $0 = \dfrac{-2x^2+5x-1}{2x-1}$

$0 = -2x^2+5x-1$

$a = -2$ $b = 5$ $c = -1$

$x = \dfrac{-5 \pm \sqrt{5^2-4(-2)(-1)}}{2(-2)}$

$x = \dfrac{-5 \pm \sqrt{25-8}}{-4} = \dfrac{-5 \pm \sqrt{17}}{-4}$

~~(-0.44, 0) (-4.56, 0)~~

$\boxed{(0.22, 0) \quad (2.28, 0)}$

b.) $y = \dfrac{-2(0)^2+5(0)-1}{2(0)-1} = \dfrac{-1}{-1} = 1$

$\boxed{(0, 1)}$

c.) $2x-1 \neq 0 \Rightarrow \boxed{x \neq 1/2}$

d.) $\boxed{\text{VA: } x \neq \tfrac{1}{2}}$ $\boxed{\text{HA: none}}$

slant asym: $\boxed{y \neq -1x+2}$

$$\begin{array}{r} -1x + 2 \\ 2x-1 \,\big)\, -2x^2+5x-1 \\ +2x^2 - x \quad \\ \hline 4x-1 \\ -4x+2 \\ \hline 1 \end{array}$$

47 cont.

e.) $y' = \dfrac{(2x-1)(-4x+5) - (-2x^2+5x-1)(2)}{(2x-1)^2}$

$y' = \dfrac{-8x^2 + 10x + 4x - 5 + 4x^2 - 10x + 2}{(2x-1)^2} = \boxed{\dfrac{-4x^2+4x-3}{(2x-1)^2} = y'}$

f.) $0 = \dfrac{-4x^2+4x-3}{(2x-1)^2} \Rightarrow 0 = -4x^2+4x-3$

$a=-4 \quad b=4 \quad c=-3$

$x = \dfrac{-4 \pm \sqrt{4^2-4(-4)(-3)}}{2(-4)} = \dfrac{-4 \pm \sqrt{16-48}}{-8} = \text{imag} \Rightarrow \boxed{\text{no critpt}}$

$- - - \quad \underset{1/2}{\oplus} \quad - - - \quad f'(x)$

g.) $\boxed{\text{inc : none}}$

h.) $\boxed{\text{dec : } (-\infty, \tfrac{1}{2}) \cup (\tfrac{1}{2}, \infty)}$

i.) $\boxed{\text{local min: none}}$

j.) $\boxed{\text{local max : none}}$

k.) $y'' = \dfrac{(2x-1)^2(-8x+4) - (-4x^2+4x-3)(2)(2x-1)(2)}{(2x-1)^4}$

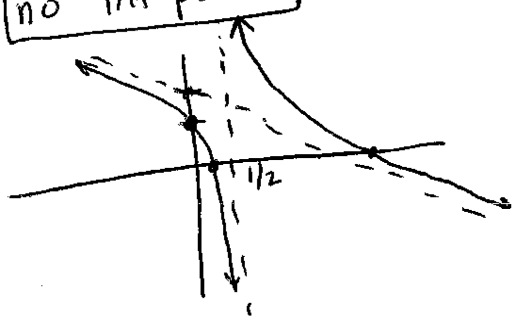
$y'' = \dfrac{4(2x-1)[(2x-1)(-2x+1) - (-4x^2+4x-3)]}{(2x-1)^{\cancel{4}\,3}}$

$y'' = \dfrac{4(-4x^2 + 2x + 2x - 1 + 4x^2 - 4x + 3)}{(2x-1)^3} = \dfrac{4(2)}{(2x-1)^3}$

$\boxed{y'' = \dfrac{8}{(2x-1)^3}}$

$- - - \quad \underset{1/2}{\oplus} \quad +++ \quad f''(x)$

l.) $0 = \dfrac{8}{(2x-1)^3} \Rightarrow 0 \neq 8$ $\boxed{\text{no inf points}}$



m.) $\boxed{\text{up } (1/2, \infty)}$

n.) $\boxed{\text{down } (-\infty, 1/2)}$

o.)

51. $y = \dfrac{2x^3+x^2+1}{x^2+1}$

$\dfrac{P}{Q} = \dfrac{\pm 1}{\pm 2, \pm 1} = \pm 1, \pm \frac{1}{2}$

a.) $0 = \dfrac{2x^3+x^2+1}{x^2+1} \Rightarrow 0 = 2x^3+x^2+1$

$$\begin{array}{r|rrrr} \cancel{1} & 2 & 1 & 0 & 1 \\ & \downarrow & 2 & 3 & 3 \\ \hline & 2 & 3 & 3 & \end{array} \qquad \begin{array}{r|rrrr} \cancel{\tfrac{1}{2}} & 2 & 1 & 0 & 1 \\ & \downarrow & 1 & 1 & \frac{1}{2} \\ \hline & 2 & 2 & 1 & \end{array} \qquad \begin{array}{r|rrrr} -1 & 2 & 1 & 0 & 1 \\ & \downarrow & -2 & 1 & -1 \\ \hline & 2 & -1 & 1 & 0 \end{array}$$

$x = -1$

$(x+1)(2x^2-x+1) = 0$

$a = 2 \quad b = -1 \quad c = 1$

$x = \dfrac{-1 \pm \sqrt{(-1)^2-4(2)(1)}}{2(2)} = \dfrac{-1 \pm \sqrt{1-8}}{4}$ = imag $\Rightarrow$ none.

$\boxed{(-1, 0)}$

b.) $y = \dfrac{2(0)^3+(0)^2+1}{0^2+1} = \dfrac{1}{1}$ $\boxed{(0, 1)}$

c.) $x^2+1 \neq 0$ none $\Rightarrow$ $\boxed{(-\infty, \infty)}$

d.) $\boxed{\text{VA: none}}$ $\boxed{\text{HA: none}}$ slant asym. $\boxed{y \neq 2x+1}$

$$\begin{array}{r|l} & y \neq 2x + 1 \\ x^2+0x+1 & 2x^3+x^2+0x+1 \\ & -\cancel{2x^3} \mp 0x^2 \mp 2x \;\downarrow \\ \hline & \cancel{x^2} \mp 2x + 1 \\ & -\cancel{x^2} \mp 0x \mp 1 \\ \hline & -2x + 0 \end{array}$$

e.) $y' = \dfrac{(x^2+1)(6x^2+2x) - (2x^3+x^2+1)(2x)}{(x^2+1)^2} = \dfrac{6x^4 + \cancel{2x^3} + 6x^2 + \cancel{2x} - 4x^4 - \cancel{2x^3} - \cancel{2x}}{(x^2+1)^2}$

$y' = \dfrac{2x^4+6x^2}{(x^2+1)^2} = \boxed{\dfrac{2x^2(x^2+3)}{(x^2+1)^2} = y'}$

f.) $0 = \dfrac{2x^2(x^2+3)}{(x^2+1)^2} \Rightarrow 0 = 2x^2(x^2+3)$

$2x^2 = 0 \qquad x^2+3 = 0$

$x = 0 \qquad$ none

$+++ \quad \underset{0}{|} \quad +++ \quad f'(x)$

$\boxed{\text{cp } (0, 1)}$

g.) $\boxed{\text{inc } (-\infty, \infty)}$

h.) $\boxed{\text{dec : none}}$

i.) $\boxed{\text{local min : none}}$

j.) $\boxed{\text{local max: none}}$

k.) $$y'' = \frac{(x^2+1)^2(8x^3+12x) - (2x^2)(x^2+3)(2)(x^2+1)^1(2x)}{(x^2+1)^4}$$

$$y'' = \frac{(x^2+1)^2(4x)(2x^2+3) - (2x^2)(x^2+3)(2)(x^2+1)(2x)}{(x^2+1)^4}$$

$$y'' = \frac{4x(\cancel{x^2+1})[(x^2+1)(2x^2+3) - (2x^2)(x^2+3)]}{(x^2+1)^{\cancel{4}3}}$$

$$y'' = \frac{4x(\cancel{2x^4} + 3x^2 + 2x^2 + 3 - \cancel{2x^4} - 6x^2)}{(x^2+1)^3}$$

$$y'' = \frac{4x(-x^2+3)}{(x^2+1)^3} = \frac{-4x(x^2-3)}{(x^2+1)^3} = \boxed{\frac{-4x(x+\sqrt{3})(x-\sqrt{3})}{(x^2+1)^3} = y''}$$

l.) $$0 = \frac{-4x(x+\sqrt{3})(x-\sqrt{3})}{(x^2+1)^3} \Rightarrow$$

$0 = -4x(x+\sqrt{3})(x-\sqrt{3})$

$-4x = 0$ $\quad x+\sqrt{3} = 0$ $\quad x - \sqrt{3} = 0$

$x = 0$ $\quad x = -\sqrt{3}$ $\quad x = \sqrt{3}$

infpts : $(-\sqrt{3}, -1.6)$ or $(-1.7, -1.6)$
$(0, 1)$
$(\sqrt{3}, 3.6)$ or $(1.7, 3.6)$

Up	Down	Up	Down
+++	---	+++	---

$-\sqrt{3}$ $\quad 0$ $\quad \sqrt{3}$ $\quad f''(x)$

m.) $\boxed{\text{up } (-\infty, -\sqrt{3}) \cup (0, \sqrt{3})}$

n.) $\boxed{\text{down } (-\sqrt{3}, 0) \cup (\sqrt{3}, \infty)}$

o.)