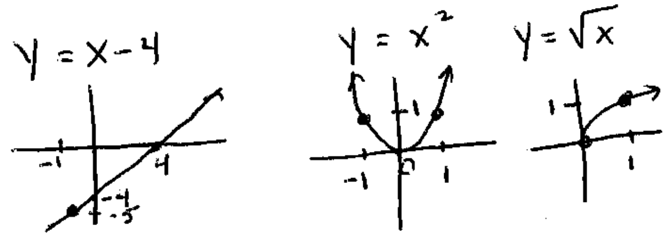
MAC2311 Test 1 Name Key

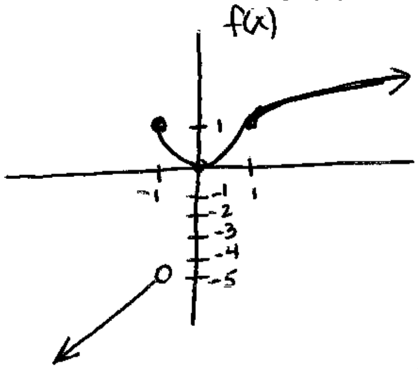
Show all work for credit.

Each part 4 points

1. Given $f(x) = \begin{cases} x-4\,; x < -1 \\ x^2\,;\ -1 \le x \le 1 \\ \sqrt{x}\,; x > 1 \end{cases}$



a.) Sketch the graph of $f(x)$.



b.) $\lim_{x\to -1^+} f(x) =$ $\lim_{x\to -1^+}(x^2) = (-1)^2 = 1$

c.) $\lim_{x\to -1^-} f(x) =$ $\lim_{x\to -1^-}(x-4) = -1-4 = -5$

d.) $\lim_{x\to -1} f(x) =$ $\lim_{x\to -1} f(x) =$ DNE

e.) $\lim_{x\to 1^+} f(x) =$ $\lim_{x\to 1^+}(\sqrt{x}) = \sqrt{1} = 1$

f.) $\lim_{x\to 1^-} f(x) =$ $\lim_{x\to 1^-}(x^2) = (1)^2 = 1$

g.) $\lim_{x\to 1} f(x) =$ $\lim_{x\to 1} f(x) = 1$

h.) $f(-1) =$ $(-1)^2 = 1$

i.) $f(1) =$ $(1)^2 = 1$

j.) Is $f(x)$ continuous at $x = -1$? Explain.

No, $f(-1) \neq \lim_{x\to -1^+} f(x) \neq \lim_{x\to -1^-} f(x)$

k.) Is $f(x)$ continuous at $x = 1$? Explain.

Yes, $f(1) = \lim_{x\to 1^+} f(x) = \lim_{x\to 1^-} f(x)$

2. Evaluate $\frac{f(x+h)-f(x)}{h}$ when $f(x) = 2x - 5$.

$$DQ = \frac{2(x+h)-5 - (2x-5)}{h}$$

(8)

$$DQ = \frac{\cancel{2x} + 2h - \cancel{5} - \cancel{2x} + \cancel{5}}{h}$$

$$DQ = \frac{2h}{h} = \boxed{2}$$

3. At the surface of the ocean, the water pressure is the same as the air pressure above water, 15 psi. Below the surface, the water pressure increases by 4.34 psi every 10 feet of descent.

a.) Find the equation that expression the water pressure as a function of the depth below the ocean surface. $P = f(d)$

0 feet ⇒ 15psi

Yintercept

$b = 15$

$$m = \frac{4.34 \text{ psi}}{10 \text{ ft}} = .434$$

(4)

$$\boxed{P = .434d + 15}$$

b.) At what depth is the pressure 100 psi?

(4)

$$100 = .434d + 15$$

$$85 = .434d$$

$$\frac{85}{.434} = d = \boxed{195.85 \text{ feet below the surface}}$$

4. If $f(x)$ contains the point (3, 10), use your transformation rules to find a point on $2f(x-3)$.

move right = Add 3 to xvalue

vertical stretch = multiply "y" value by 2

(8)

$(3+3, 10 * 2)$

$\boxed{(6, 20)}$

5. If $F(x) = \frac{cosx}{3+cosx}$, find $f(x)$ and $g(x)$ such that $(f \circ g)(x) = F(x)$.

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$(f \circ g)(x) = f(g(x))$

$\boxed{\text{let } f(x) = \frac{x}{3+x} \quad \text{and } g(x) = \cos x}$

6. $\lim_{h\to 0} \frac{(4+h)^2-16}{h} = \lim_{h\to 0} \frac{(4+h)(4+h)-16}{h}$

$= \lim_{h\to 0} \frac{\cancel{16}+4h+4h+h^2-\cancel{16}}{h}$

8

$= \lim_{h\to 0} \frac{8h+h^2}{h}$

$= \lim_{h\to 0} 8+h = 8$

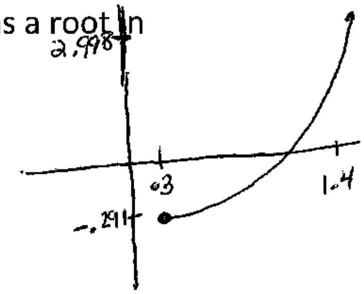
7. $\lim_{x\to -1}(x+1)(x-5) = (-1+1)(-1-5)$

4

$= 0 \cdot -6$

$= \boxed{0}$

8. Prove by using the Intermediate Value Theorem that $tanx = 2x$ has a root in the interval (0.3, 1.4).



Radians mode only

$\tan x - 2x = 0$

* $f(.3) = \tan(.3) - 2(.3) = -.291$

* $f(1.4) = \tan(1.4) - 2(1.4) = 2.998$

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* Since $\tan(x) - 2x$ is continuous on $[.3, 1.4]$

* and $\underset{f(a)}{-.291} \le \underset{N}{0} \le \underset{f(b)}{2.998}$, then there exists "c" such that $.3 \le c \le 1.4$ such that

$f(c) = 0$.

7. Solve $6x^2 - x < 1$ algebraically.

$6x^2 - x - 1 < 0$

$(3x+1)(2x-1) < 0$

$3x + 1 = 0 \quad 2x - 1 = 0$

$x = -\frac{1}{3} \quad x = \frac{1}{2}$

test $x = 0$

$-1 < 0$

True

$-\frac{1}{3} \quad \frac{1}{2}$

$\boxed{\text{OR } -\frac{1}{3} < x < \frac{1}{2} \text{ OR } \left(-\frac{1}{3}, \frac{1}{2}\right)}$

8. The points $(-12,6), (0,8)$, and $(8,-4)$ lie on the graph of $y = f(x)$. Determine three points that lie on the graph of $y = g(x)$.

a.) $g(x) = f(x+1) - 1$

subtract 1 to x

subtract 1 to y

$\boxed{(-13, 5)\ (-1, 7)\ (7, -5)}$

b.) $g(x) = -2f(x)$

multiply y by -2

$\boxed{(-12, -12)\ (0, -16)\ (8, 8)}$