Death

MAC2311 Calc I Name Key

D. Howard 3-16

Calculate y' for each of the following. **Do not simplify.**

1. $y = (x^4 - 3x^2 + 5)^3$

$y' = 3(x^4 - 3x^2 + 5)^2 (4x^3 - 6x)$

2. $y = 2x\sqrt{x^2 + 1}$

$y' = (2x)\frac{1}{2}(x^2+1)^{-1/2}(2x) + (x^2+1)^{1/2}(2)$

3. $y = \frac{sinx}{3x - tanx}$

$y' = \dfrac{(3x - \tan x)(\cos x) - \sin x\,(3 - \sec^2 x)}{(3x - \tan x)^2}$

4. $y = \sec(7 - 2x)$

$y' = \sec(7-2x)\tan(7-2x)\,(-2)$

Calculate y' for each of the following.

5. $4xy = cosy$

$$4xy' + y(4) = -\sin y \cdot y'$$
$$4y = -4xy' - \sin y \cdot y'$$
$$\frac{4y}{-4x - \sin y} = y'$$

6. $\sqrt{y} = \frac{y}{1+x^2}$

$$\frac{1}{2} y^{-1/2} y' = \frac{(1+x^2) y' - y(2x)}{(1+x^2)^2}$$
$$\frac{1}{2} y^{-1/2} (1+x^2)^2 y' - (1+x^2) y' = -2xy$$
$$y' = \frac{-2xy}{\frac{1}{2} y^{-1/2} (1+x^2)^2 - (1+x^2)}$$

7. The position function of a particle is given by $s = t^3 - 3.5t^2 + 2t$ for $t \geq 0$.

a.) Find the velocity at time *t*.

$$v = 3t^2 - 7t + 2$$

b.) Find the acceleration at time *t*.

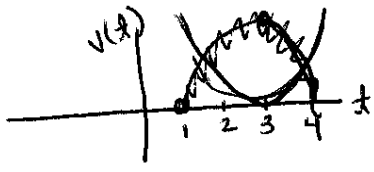
$$a = 6t - 7$$

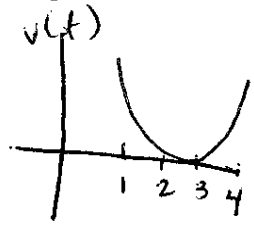
c.) When is the particle at rest?

$$v = 0$$
$$0 = 3t^2 - 7t + 2$$
$$0 = (3t - 1)(t - 2)$$
$$3t - 1 = 0 \qquad t - 2 = 0$$
$$t = 1/3 \qquad t = 2$$

8. Sketch a graph of a velocity function whose particle will be speeding up between $3 \le t \le 4$, but slowing down between $1 \le t \le 3$. Many answers are possible.

slowing down : acceleration and velocity are opposite signs
speeding up : acceleration and velocity are same sign.





9. Sodium chlorate crystals are easy to grow in the shape of cubes by allowing a solution of water and sodium chlorate to evaporate slowly. If V is the volume of such a cube with side length x, calculate $\frac{dV}{dx}$ when $x = 3$ mm and explain its meaning.

$V = x^3$

$\frac{dV}{dx} = 3x^2 \Big|_{x=3} = 3(3)^2 = 27 \frac{mm^3}{mm}$

As you increase the length of the side of a cube, the volume increases. When the length has been increased to 3mm, the change in the volume is $27\ mm^2$.

10. A paper cup has the shape of a cone where the height is always three times the radius of the cup. If water is poured into the cup at a rate of 2 cubic cm/sec, how fast is the water level rising when the water is 5 cm deep? The volume of a cone is $V = \frac{1}{3}\pi r^2 h$.

$\frac{dV}{dt} = 2\frac{cm^3}{sec}$ $\frac{dh}{dt} = ?$ $h = 5cm$ $h = 3r$

$\frac{dV}{dt} = \frac{1}{3}\pi r^2 \frac{dh}{dt} + h\left(\frac{1}{3}\right)\pi(2r)\frac{dr}{dt}$ OR

since $h = 3r \Rightarrow r = \frac{1}{3}h$

$V = \frac{1}{3}\pi\left(\frac{h}{3}\right)^2 h = \frac{1}{3}\pi\left(\frac{1}{9}\right)h^3$

$V = \frac{\pi}{27}h^3$

since $h = 3r$

$\frac{dh}{dt} = 3\frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{1}{3}\frac{dh}{dt}$

$\frac{dV}{dt} = 3\frac{\pi}{27}h^2\frac{dh}{dt} = \frac{\pi}{9}h^2\frac{dh}{dt}$

$2 = \frac{1}{3}\pi(r)^2\frac{dh}{dt} + (5)\left(\frac{1}{3}\pi\right)(2r)\left(\frac{1}{3}\frac{dh}{dt}\right)$

$(2) = \frac{\pi}{9}(5)^2\frac{dh}{dt}$

$2 = \frac{25}{9}\pi\frac{dh}{dt}$

$\frac{18}{25\pi} = \frac{dh}{dt}$

But have to find $r \Rightarrow 5 = 3r$

$r = \frac{5}{3}$

$2 = \frac{1}{3}\pi\left(\frac{5}{3}\right)^2\frac{dh}{dt} + (5)\left(\frac{1}{3}\pi\right)(2)\left(\frac{5}{3}\right)\left(\frac{1}{3}\right)\frac{dr}{dt}$

$2 = \frac{25}{27}\pi\frac{dh}{dt} + \frac{50\pi}{27}\frac{dr}{dt} = \frac{75}{27}\pi\frac{dr}{dt} = \frac{35}{9}\pi\frac{dr}{dt}$