

Calculate  $y'$  for each of the following. Do not simplify.

1.  $y = \left(\frac{x}{1-x^2}\right)^3$

①

$$y' = 3 \left(\frac{x}{1-x^2}\right)^2 \left[ \frac{(1-x^2)(1) - (x)(-2x)}{(1-x^2)^2} \right]$$

2.  $y = \sec x \tan x$

①

$$y' = \sec x \sec^2 x + \tan x \sec x \tan x$$

3.  $y = \cos^2(5x) = [\cos(5x)]^2$

①

$$y' = 2 \cos(5x) (-\sin(5x)) (5)$$

4.  $y = \sqrt{x} + \frac{1}{\sqrt[3]{x^4}} = x^{1/2} + x^{-4/3}$

①

$$y' = \frac{1}{2} x^{-1/2} - \frac{4}{3} x^{-7/3}$$

5. If  $F(x) = f(g(x))$ , where  $f(-2) = 8$ ,  $f'(-2) = 4$ ,  $f'(5) = 3$ ,  $g(5) = -2$ ,

① and  $g'(5) = 6$ , find  $F'(5)$ .

$$F'(x) = f'(g(x)) g'(x)$$

$$F'(5) = f'(g(5)) g'(5)$$

$$F'(5) = f'(-2) \cdot 6$$

$$F'(5) = 4 \cdot 6 = \boxed{24}$$

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6.  $\cos x + \sqrt{y} = 5$        $-\sin x + \frac{1}{2} y^{-1/2} y' = 0$

$$\frac{1}{2} y^{-1/2} y' = \sin x$$

$$\frac{1}{2\sqrt{y}} y' = \sin x$$

$$y' = 2\sqrt{y} \sin x$$

①

7.  $\frac{3x}{y} + 2 = x^2$

①  $\frac{(y)(3) - (3x)(y')}{y^2} + 0 = 2x$

$$3y - 3xy' = 2xy^2$$

$$3y - 2xy^2 = 3xy'$$

$$\frac{3y - 2xy^2}{3x} = y'$$

8. Boyle's Law states that when a sample of gas is compressed at a constant temperature, the pressure  $P$  and volume  $V$  satisfy the equation  $PV = C$ , where  $C$  is a constant. Suppose that at a certain instant the volume is 600 cubic centimeters, the pressure is 150 kPa, and the pressure is increasing at a rate of 20 kPa/min. At what rate is the volume decreasing at this instant? Include your units in your answer.

⑩

$$PV = C$$

$$P \frac{dV}{dt} + V \frac{dP}{dt} = 0$$

$$(150) \frac{dV}{dt} + (600)(20) = 0$$

$$\frac{dV}{dt} = -\frac{12000}{150} = -80 \frac{\text{cm}^3}{\text{min}}$$

9. Evaluate  $\lim_{x \rightarrow 0} \frac{\sin(4x)}{x} = \lim_{x \rightarrow 0} 4 \frac{\sin(4x)}{4x} = 4(1) = \boxed{4}$

$$\begin{aligned} V &= 600 \text{ cm}^3 \\ P &= 150 \text{ kPa} \\ \frac{dP}{dt} &= 20 \frac{\text{kPa}}{\text{min}} \\ \frac{dV}{dt} &= ? < 0 \end{aligned}$$

⑤

10. The position function of a particle is given by  $s = 0.01t^4 - 0.04t^3$  for  $t \geq 0$ .

a.) Find the velocity at time  $t$ .

①  $v = 0.04t^3 - 0.12t^2$

b.) Find the acceleration at time  $t$ .

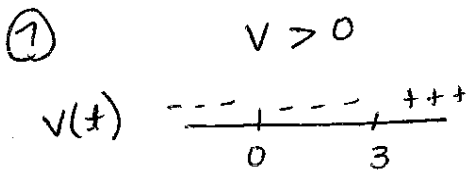
①  $a = 0.12t^2 - 0.24t$

c.) When is the particle at rest?  $v = 0$

①  $0.04t^3 - 0.12t^2 = 0$   
 $0.04t^2(t - 3) = 0$   
 $t = 0 \quad t = 3$

$t = 0 \text{ sec and } t = 3 \text{ sec}$

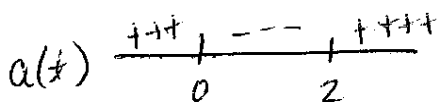
d.) When is the particle moving in a positive direction?



$(3, \infty)$   
after 3 seconds

e.) When is the particle speeding up?

①  $a(t) = 0$   
 $0.12t^2 - 0.24t = 0$   
 $0.12t(t - 2) = 0$   
 $t = 0 \quad t = 2$



speeding up when  $v > 0$  and  $a > 0$   
 $(3, \infty)$

and  
 speeding up when  $v < 0$  and  $a < 0$   
 $(0, 2)$

