MAC2311 Calc I Name Key

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Calculate y' for each of the following. **Do not simplify.**

1. $y = \left(\frac{x}{1-x^2}\right)^3$ (1)

$$y' = 3\left(\frac{x}{1-x^2}\right)^2\left[\frac{(1-x^2)(1) - (x)(-2x)}{(1-x^2)^2}\right]$$

2. $y = secxtanx$ (1)

$$y' = \sec x \sec^2 x + \tan x \sec x \tan x$$

3. $y = cos^2(5x) = [\cos(5x)]^2$ (1)

$$y' = 2\cos(5x)(-\sin(5x))(5)$$

4. $y = \sqrt{x} + \frac{1}{\sqrt[3]{x^4}} = x^{1/2} + x^{-4/3}$ (1)

$$y' = \frac{1}{2}x^{-1/2} - \frac{4}{3}x^{-7/3}$$

5. If $F(x) = f(g(x))$, where $f(-2) = 8, f'(-2) = 4, f'(5) = 3, g(5) = -2,$ and $g'(5) = 6$, find $F'(5)$. (1)

$$F'(x) = f'(g(x))\,g'(x)$$
$$F'(5) = f'(g(5))\,g'(5)$$
$$F'(5) = f'(-2)\cdot 6$$
$$F'(5) = 4 \cdot 6 = \boxed{24}$$

Calculate y' for each of the following.

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6. $\cos x + \sqrt{y} = 5$

$-\sin x + \frac{1}{2}y^{-1/2}y' = 0$

$\frac{1}{2}y^{-1/2}y' = \sin x$

$\frac{1}{2\sqrt{y}}y' = \sin x$

$y' = 2\sqrt{y}\sin x$

(1)

7. $\frac{3x}{y} + 2 = x^2$

$\frac{(y)(3) - (3x)(y')}{y^2} + 0 = 2x$

$3y - 3xy' = 2xy^2$

$3y - 2xy^2 = 3xy'$

$\frac{3y - 2xy^2}{3x} = y'$

(1)

8. Boyle's Law states that when a sample of gas is compressed at a constant temperature, the pressure P and volume V satisfy the equation $PV = C$, where C is a constant. Suppose that at a certain instant the volume is 600 cubic centimeters, the pressure is 150 kPa, and the pressure is increasing at a rate of 20 kPa/min. At what rate is the volume decreasing at this instant? Include your units in your answer.

$V = 600\ cm^3$

$P = 150\ kPa$

$\frac{dP}{dt} = +20\ \frac{kPa}{min}$

$\frac{dV}{dt} = ? < 0$

$PV = C$

$P\frac{dV}{dt} + V\frac{dP}{dt} = 0$

$(150)\frac{dV}{dt} + (600)(20) = 0$

$dV/dt = -\frac{12000}{150} = -80\ \frac{cm^3}{min}$

(1)

9. Evaluate $\lim_{x\to 0}\frac{\sin(4x)}{x}$

$\lim_{x\to 0} 4\cdot\frac{\sin(4x)}{4x} = 4(1) = 4$

(5)

10. The position function of a particle is given by $s = 0.01t^4 - 0.04t^3$ for $t \geq 0$.

a.) Find the velocity at time *t*.

(1) $v = 0.04t^3 - 0.12t^2$

b.) Find the acceleration at time *t*.

(1) $a = 0.12t^2 - 0.24t$

c.) When is the particle at rest? $v = 0$

(1) $0.04t^3 - 0.12t^2 = 0$

$0.04t^2(t-3) = 0$

$t = 0 \quad t = 3$

$t = 0$ sec and $t = 3$ sec

d.) When is the particle moving in a positive direction?

(1) $v > 0$

$v(t)$ — + + + | — — — | + + +

0 3

$(3, \infty)$

after 3 seconds

e.) When is the particle speeding up?

(1) $a(t) = 0$

$0.12t^2 - 0.24t = 0$

$0.12t(t-2) = 0$

$t = 0 \quad t = 2$

$a(t)$ + + + | — — — | + + + +

0 2

Speeding up when $v > 0$ and $a > 0$

$(3, \infty)$

and

Speeding up when $v < 0$ and $a < 0$

$(0, 2)$

$v(t)$ — — — | — — — | + + + +

0 1 2 3

$a(t)$ — — — | — — — | + + + | + + + +

0 1 2 3