MAC2311 Test 3 Name Key

Show all work for credit.

1. Find two positive integers such that the sum of the first number and four times the second number is 1000 and the product of the numbers is as large as possible.

$x + 4y = 1000 \implies x = 1000 - 4y$

max $= xy$

$P = (1000 - 4y)y$

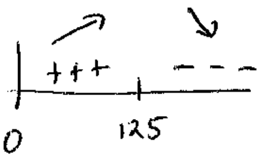
$P = 1000y - 4y^2$

$P' = 1000 - 8y$

$0 = 1000 - 8y$

$8y = 1000$

$y = 125$



local max $y = 125$

$y = 125$

$x = 1000 - 4(125)$

$x = 500$

$\boxed{\text{125 and 500}}$

2. $\lim_{x\to\infty} \frac{3x^4+x-5}{6x^4-2x^2+1} = \frac{1}{2}$

HA: $y \neq \frac{3}{6}$

$y \neq \frac{1}{2}$

3. The velocity of a wave of length L in deep water is given by $v(L) = K\sqrt{\frac{L}{C} + \frac{C}{L}}$ where $K = 5$ and $C = 2$. What is the length of the wave that gives the minimum velocity?

$$V = 5\sqrt{\frac{L}{2} + \frac{2}{L}} = 5\left(\frac{1}{2}L + 2L^{-1}\right)^{1/2}$$

$$V' = \frac{5}{2}\left(\frac{1}{2}L + 2L^{-1}\right)^{-1/2}\left(\frac{1}{2} - 2L^{-2}\right)$$

$$V' = \frac{5}{2\sqrt{\frac{1}{2}L + 2L^{-1}}}\left(\frac{1}{2} - 2L^{-2}\right)$$

$$0 = \frac{5}{2\sqrt{\frac{1}{2}L + 2L^{-1}}}\left(\frac{1}{2} - 2L^{-2}\right)$$

$$0 = 5\left(\frac{1}{2} - 2L^{-2}\right)$$

$$0 = \frac{1}{2} - 2L^{-2}$$

$$2L^{-2} = \frac{1}{2}$$

$$\frac{2}{L^2} = \frac{1}{2}$$

$$4 = L^2$$

$$\pm 2 = L$$

$$2 = L$$

$$\boxed{f(x) = \frac{1}{10}x^5 + \frac{1}{4}x^4 - \frac{2}{3}x^3 + \frac{5}{2}x^2 - \frac{251}{60}x + 2}$$

4. Given $f''(x) = 2x^3 + 3x^2 - 4x + 5$ and $f(0) = 2$ and $f(1) = 0$, find $f(x)$.

$$\int f''(x)\,dx = f'(x) = \int (2x^3 + 3x^2 - 4x + 5)\,dx$$

$$f'(x) = \frac{2x^4}{4} + \frac{3x^3}{3} - \frac{4x^2}{2} + 5x + C_1$$

$$f'(x) = \frac{1}{2}x^4 + x^3 - 2x^2 + 5x + C_1$$

$$\int f'(x)\,dx = f(x) = \int\left(\frac{1}{2}x^4 + x^3 - 2x^2 + 5x + C_1\right)dx$$

$$f(x) = \frac{\frac{1}{2}x^5}{5} + \frac{x^4}{4} - \frac{2x^3}{3} + \frac{5x^2}{2} + C_1x + C_2$$

$$f(x) = \frac{1}{10}x^5 + \frac{1}{4}x^4 - \frac{2}{3}x^3 + \frac{5}{2}x^2 + C_1x + C_2$$

$$C_2 = 2$$

$$0 = \frac{1}{10} + \frac{1}{4} - \frac{2}{3} + \frac{5}{2} + C_1 + 2 \implies C_1 = -\left(\frac{6}{60} + \frac{15}{60} - \frac{40}{60} + \frac{150}{60} + \frac{120}{60}\right)$$

$$C_1 = -251/60$$

5. Given a particle with acceleration $a(t) = sint + 3cost$ and $s(0) = 0$ and $v(0) = 2$, find the particle's position equation.

$$\int a(t)\,dt = v(t) = \int (\sin t + 3\cos t)\,dt = -\cos t + 3\sin t + C_1$$

$$-\cos 0 + 3\sin 0 + C_1 = 2$$

$$-1 + 0 + C_1 = 2$$

$$C_1 = 3$$

$$v(t) = -\cos t + 3\sin t + 3$$

$$\int v(t)\,dt = s(t) = \int(-\cos t + 3\sin t + 3)\,dt$$

$$s(t) = -\sin t - 3\cos t + 3t + C_2$$

$$-\sin 0 - 3\cos 0 + 3(0) + C_2 = 0$$

$$0 - 3 + 0 + C_2 = 0$$

$$C_2 = 3$$

$$\boxed{s(t) = -\sin t - 3\cos t + 3t + 3}$$

6. Sketch the graph of $f(x)$ that satisfies the given conditions:

$f(0) = 0,$

$f'(-2) = f'(1) = f'(9) = 0,$ cp

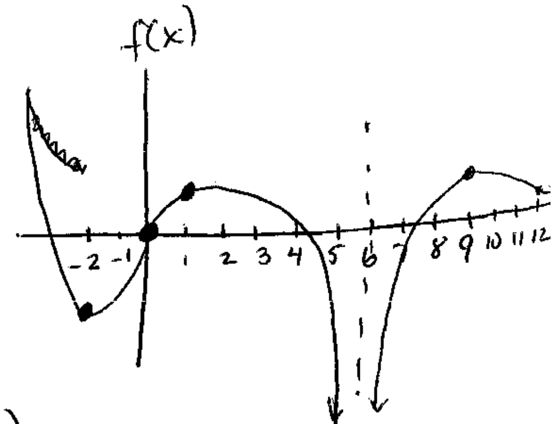
$\lim_{x\to\infty} f(x) = 0,$ HA $\quad \lim_{x\to 6} f(x) = -\infty,$ VA

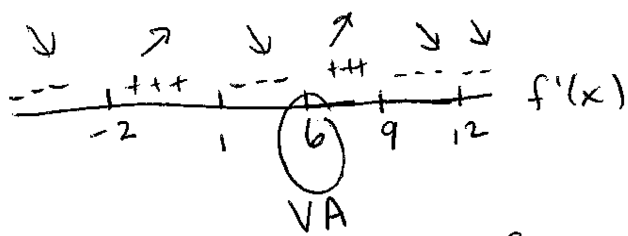
$f'(x) < 0$ for $(-\infty,-2), (1,6), (9,\infty)$

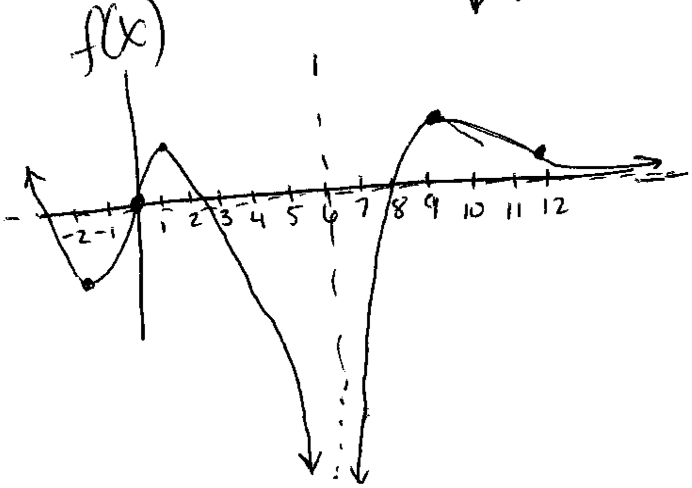
$f'(x) > 0$ for $(-2,1), (6,9)$

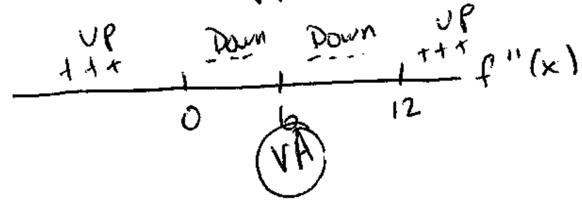
$f''(x) > 0$ for $(-\infty,0), (12,\infty)$

$f''(x) < 0$ for $(0,6), (6,12)$

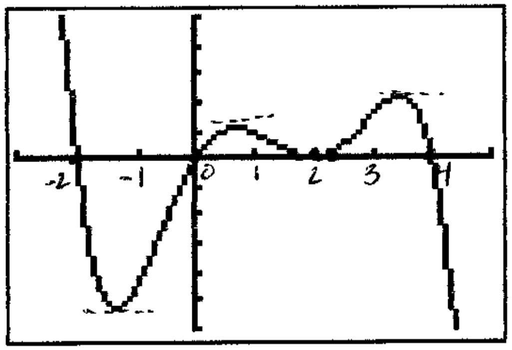








7. The following is the graph of $f'(x)$. Each tick mark represents one unit.
(Please note that it is the graph of $f'(x)$ and NOT the graph of $f(x)$.)



max   min   max

+++   ---   ++   +++   --- $f'(x)$

-2   0   2   4

Down   Up   Down   Up   Down

---   +++   ---   +++   --- $f''(x)$

-1.2   .5   2   3.5

a.) On what interval(s) is $f(x)$ increasing? $(-\infty, -2) \cup (0, 4)$

b.) On what interval(s) is $f(x)$ decreasing? $(-2, 0) \cup (4, \infty)$

c.) For what value(s) does $f(x)$ have a local maximum? $x = -2, 4$

d.) For what value(s) does $f(x)$ have a local minimum? $x = 0$

e.) Approximate the interval(s) where $f(x)$ will be concave up.

$(-1.2, .5) \cup (2, 3.5)$

f.) Approximate the interval(s) where $f(x)$ will be concave down.

$(-\infty, -1.2) \cup (.5, 2) \cup (3.5, \infty)$