MAC2311 Test 4 Name Key

You may use your calculator to **check** the definite integrals. **However, all work must be shown to find the definite integrals without using the calculator.**

1. $\int_0^2 y^2\sqrt{1+y^3}\,dy$

$u = 1+y^3$

$du = 3y^2\,dy$

$\frac{du}{3} = y^2\,dy$

$= \int \sqrt{u}\,\frac{du}{3}$

$= \frac{1}{3}\int u^{1/2}\,du$

$= \frac{1}{3}\frac{u^{3/2}}{3/2} = \frac{2}{9}u^{3/2} = \frac{2}{9}(1+y^3)^{3/2}\Big|_0^2$

$= \frac{2}{9}(1+2^3)^{3/2} - \frac{2}{9}(1+0^3)^{3/2}$

$= \frac{2}{9}(1+8)^{3/2} - \frac{2}{9}(1)^{3/2}$

$= \frac{2}{9}(9)^{3/2} - \frac{2}{9}(1)^{3/2} = \frac{2}{9}(27) - \frac{2}{9} = 2(3) - \frac{2}{9} = 6 - \frac{2}{9}$

$= \frac{54}{9} - \frac{2}{9} = \boxed{\frac{52}{9}} \approx 5.78$

2. $\int (1+\tan x)^3 \sec^2 x\,dx$

$u = 1+\tan x$

$du = \sec^2 x\,dx$

$= \int u^3\,du$

$= \frac{u^4}{4} = \frac{1}{4}u^4 = \boxed{\frac{1}{4}(1+\tan x)^4 + C}$

3. $\int \left(x+\frac{1}{x}\right)^2 dx = \int \left(x+\frac{1}{x}\right)\left(x+\frac{1}{x}\right)dx = \int \left(x^2+1+1+\frac{1}{x^2}\right)dx$

$= \int \left(x^2+2+x^{-2}\right)dx$

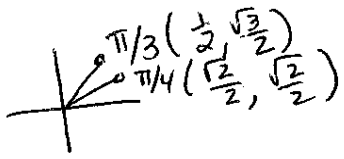
$= \frac{x^3}{3} + 2x + \frac{x^{-1}}{-1}$

$= \boxed{\frac{1}{3}x^3 + 2x - \frac{1}{x} + C}$

4. $\int x\left(\sqrt[3]{x}+\sqrt[4]{x}\right)dx = \int x\left(x^{1/3}+x^{1/4}\right)dx$

$= \int \left(x^{4/3}+x^{5/4}\right)dx$

$= \frac{x^{7/3}}{7/3} + \frac{x^{9/4}}{9/4} = \boxed{\frac{3}{7}x^{7/3} + \frac{4}{9}x^{9/4} + C}$



5. $\int_{\pi/4}^{\pi/3} secxtanx\,dx$ $= Sec\,x\Big|_{\pi/4}^{\pi/3} = Sec\left(\frac{\pi}{3}\right) - Sec\left(\frac{\pi}{4}\right)$

$= \boxed{2 - \frac{2}{\sqrt{2}} = \frac{2\sqrt{2}-2}{\sqrt{2}}} = \frac{(2\sqrt{2}-2)\sqrt{2}}{2} = (\sqrt{2}-1)\sqrt{2}$

$= \boxed{2-\sqrt{2}}$

6. $\int(1+\tan^2 x)\,dx$ $= \int Sec^2 x\,dx = \boxed{\tan x + C}$

7. Suppose that a volcano is erupting and readings of the rate at which solid materials are spewed into the atmosphere are given in the table. The time is measured in seconds and the units for the rate are metric tons per second. Use **three midpoint rectangles** to approximate the total quantity of erupted material.

time	0	1	2	3	4	5	6
rate	2	10	24	36	46	54	60

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width = 2

$A = 2(10) + 2(36) + 2(54) = 20 + 72 + 108$

$\boxed{A = 200}$

8. $\frac{d}{dx}\int_0^x \cos(\pi t^2)dt$ $= \boxed{\cos(\pi x^2)}$

9. $\frac{d}{dx}\int_0^\pi \cos(\pi t^2)dt$ $= \boxed{0}$

derivative of constant is zero

10. $\int_{-100}^{100} \sqrt[3]{x}\,dx$ $= \boxed{0}$

integral of odd function around zero is zero.