1. $\int \cos x\,(1+\sin^2 x)\,dx = \int (1+u^2)\,du = u + \frac{u^3}{3} = \boxed{\sin x + \frac{1}{3}\sin^3 x + C}$

$\boxed{\begin{array}{l} u = \sin x \\ du = \cos x\,dx \end{array}}$

3. $\int \frac{\sin x + \sec x}{\tan x}\,dx = \int \left(\frac{\sin x}{\tan x} + \frac{\sec x}{\tan x}\right)dx = \int (\cos x + \csc x)\,dx$

$= \boxed{\sin x + -\ln|\csc x + \cot x| + c}$

5. $\int \frac{t}{t^4+2}\,dt = \int \frac{1}{u^2+2}\,\frac{du}{2} = \frac{1}{2}\int \frac{1}{2\left(\frac{u^2}{2}+1\right)}\,du = \frac{1}{4}\int \frac{1}{\left(\frac{u}{\sqrt{2}}\right)^2+1}\,du$

$\boxed{\begin{array}{l} u = t^2 \\ du = 2t\,dt \end{array}}$ $\boxed{\begin{array}{l} v = \frac{u}{\sqrt{2}} \\ dv = \frac{1}{\sqrt{2}}\,du \end{array}}$

$= \frac{1}{4}\int \frac{1}{v^2+1}\cdot\sqrt{2}\,dv = \frac{\sqrt{2}}{4}\tan^{-1} v$

$= \frac{\sqrt{2}}{4}\tan^{-1}\left(\frac{u}{\sqrt{2}}\right) = \boxed{\frac{\sqrt{2}}{4}\tan^{-1}\left(\frac{t^2}{\sqrt{2}}\right) + c}$

7. $\int_{-1}^{1} \frac{e^{\tan^{-1} y}}{1+y^2}\,dy = \int e^u\,du = e^u = e^{\tan^{-1} y}\Big|_{-1}^{1} = e^{\tan^{-1}(1)} - e^{\tan^{-1}(-1)}$

$\boxed{\begin{array}{l} u = \tan^{-1} y \\ du = \frac{1}{1+y^2}\,dy \end{array}}$

$= \boxed{e^{\pi/4} - e^{-\pi/4}}$

9. $\int_1^3 r^4 \ln r\,dr = \frac{1}{5}r^5 \ln r - \int \frac{1}{5} r^4\,dr = \frac{1}{5}r^5 \ln r - \frac{1}{5}\frac{r^5}{5}$

$\boxed{\begin{array}{ll} u = \ln r & v = \frac{r^5}{5} \\ du = \frac{1}{r}\,dr & dv = r^4\,dr \end{array}}$

$= \left(\frac{1}{5}r^5 \ln r - \frac{1}{25}r^5\right)\Big|_1^3 = \frac{243}{5}\ln 3 - \frac{243}{25} - \left(0 - \frac{1}{25}\right)$

$= \boxed{\frac{243}{5}\ln 3 - \frac{242}{25}}$

11. $\int \frac{x-1}{x^2-4x+5}\,dx = \int \frac{\cancel{x-1}}{(\cancel{x-1})(x-5)} = \int \cancel{\frac{x-1}{u}}\cdot\frac{du}{2x-4} = \int \frac{x-1}{x^2-4x+4+1}\,dx$

$\cancel{\boxed{\begin{array}{l} u = x^2-4x+5 \\ du = (2x-4)\,dx \end{array}}}$

$= \int \frac{x-1}{(x-2)^2+1}\,dx = \int \cancel{\frac{x}{x-2}} = \int \frac{u+2-1}{u^2+1}\,du$

$\boxed{\begin{array}{l} u = x-2 \\ du = dx \end{array}} \Rightarrow x = u+2$

$= \int \frac{u+1}{u^2+1}\,du = \int \frac{u}{u^2+1}\,du + \int \frac{1}{u^2+1}\,du$

$= \int \frac{1}{v}\,\frac{dv}{2} + \tan^{-1} u = \frac{1}{2}\ln|v| + \tan^{-1} u$

$\boxed{\begin{array}{l} v = u^2+1 \\ dv = 2u\,du \end{array}}$

$= \boxed{\frac{1}{2}\ln|u^2+1| + \tan^{-1} u} = \boxed{\frac{1}{2}\ln|(x-2)^2+1| + \tan^{-1}(x-2) + c}$

(13) $\int \sin^5 t \cos^4 t \, dt$

$$\int \sin^4 t \cos^4 t \sin t \, dt = \int (1-\cos^2 t)^2 \cos^4 t \sin t \, dt$$

$\boxed{\begin{array}{l} u = \cos t \\ du = -\sin t \, dt \end{array}}$

$$= -\int (1-u^2)^2 u^4 \, du = -\int (1-2u^2+u^4) u^4 \, du$$

$$= -\int (u^4 - 2u^6 + u^8) \, du = -\frac{u^5}{5} + \frac{2u^7}{7} - \frac{u^9}{9}$$

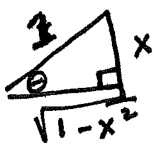
$$= \boxed{-\frac{1}{5}\cos^5 t + \frac{2}{7}\cos^7 t - \frac{1}{9}\cos^9 t + C}$$

(15.) $\int \frac{dx}{(1-x^2)^{3/2}} = \int \frac{\cos\theta \, d\theta}{(1-\sin^2\theta)^{3/2}} = \int \frac{\cos\theta \, d\theta}{(\cos^2\theta)^{3/2}} = \int \frac{\cos\theta \, d\theta}{\cos^3\theta}$

$x = 1\sin\theta$

$dx = \cos\theta \, d\theta$

$$= \int \frac{1}{\cos^2\theta} d\theta = \int \sec^2\theta \, d\theta = \tan\theta = \boxed{\frac{x}{\sqrt{1-x^2}} + C}$$



(17.) $\int_0^{\pi} t \cos^2 t \, dt = t\cos t \sin t + \cancel{\int (t\sin^2 t + \sin t \cos t) \, dt}$

$\boxed{\begin{array}{ll} u = t\cos t & v = \sin t \\ & dv = \cancel{\cos t \, dt} \\ du = (-t\sin t + \cos t)\, dt & \end{array}}$

$\boxed{\begin{array}{ll} u = t\sin t & v = -\cos t \\ du = (t\cos t + \cancel{\sin t})\, dt & dv = \sin t \, dt \end{array}}$

$= t\cos t \sin t \; \cancel{-\tfrac{?}{?}} \; t\cos t \sin t + \int (t\cos^2 t + \cos t \sin t)\, dt$

$+ \int \sin t \cos t \, dt$

$$\int t\left(\frac{1+\cos 2t}{2}\right) dt = \int \left(\frac{1}{2}t + \frac{1}{2}t\cos 2t\right) dt = \frac{1}{2}\frac{t^2}{2} + \frac{1}{2}\int t\cos 2t \, dt$$

$= \frac{1}{4}t^2 + \frac{1}{2}\int t\cos 2t \, dt$

$\boxed{\begin{array}{ll} u = t & v = \frac{\sin 2t}{2} \\ du = dt & dv = \cos 2t \, dt \end{array}}$

$$= \frac{1}{4}t^2 + \frac{1}{2}\left[\frac{1}{2}t\sin 2t - \int \frac{1}{2}\sin 2t \, dt\right]$$

$$= \frac{1}{4}t^2 + \frac{1}{4}t\sin 2t + \frac{1}{4}\frac{\cos 2t}{2}$$

$$= \left(\frac{1}{4}t^2 + \frac{1}{4}t\sin 2t + \frac{1}{8}\cos 2t\right)\Big|_0^{\pi}$$

$$= \frac{\pi^2}{4} + 0 + \cancel{\frac{1}{8}} - \left(0 + 0 + \cancel{\frac{1}{8}}\right) = \boxed{\frac{\pi^2}{4}}$$

(19.) $\int e^{(x+e^x)} dx = \int e^x \cdot e^{e^x} dx = \int e^u \, du = e^u = \boxed{e^{e^x} + C}$

$\boxed{\begin{array}{l} u = e^x \\ du = e^x \, dx \end{array}}$

(21.) $\int \tan^{-1}\sqrt{x}\,dx = x\tan^{-1}\sqrt{x} - \int \frac{x}{1+x}\cdot\frac{1}{2\sqrt{x}}\,dx$

$\boxed{\begin{array}{ll} u = \tan^{-1}\sqrt{x} & v = x \\ du = \frac{1}{1+x}\cdot\frac{1}{2\sqrt{x}}dx & dv = dx \end{array}}$

$= x\tan^{-1}\sqrt{x} - \int \frac{(\sqrt{x})^2}{1+(\sqrt{x})^2}\cdot\frac{1}{2\sqrt{x}}\,dx$

$\boxed{\begin{array}{l} u = \sqrt{x} \\ du = \frac{1}{2\sqrt{x}}dx \end{array}}$ $= x\tan^{-1}\sqrt{x} - \cancel{\tfrac{2}{2}} \int \frac{u^2}{1+u^2}\,du$

$u^2+1 \overline{)\,u^2}$, $-u^2 \mp 1$, -1; $1 - \frac{1}{u^2+1}$

$= x\tan^{-1}\sqrt{x} - \int\left(1 - \frac{1}{u^2+1}\right)du$

$= x\tan^{-1}\sqrt{x} - u + \tan^{-1}u = \boxed{x\tan^{-1}\sqrt{x} - \sqrt{x} + \tan^{-1}\sqrt{x} + C}$ ☺

(23.) $\int_0^1 (1+\sqrt{x})^8\,dx = \int \cancel{(1+u)^8}\cdot 2u\,du$

$\cancel{u = 1+\sqrt{x}}$, $\cancel{du = \frac{1}{2\sqrt{x}}dx}$ $\quad u = \cancel{\sqrt{x}}$, $du = \cancel{\frac{1}{2\sqrt{x}}}dx = \frac{1}{2u}dx$

$\boxed{\begin{array}{ll} u = 1+\sqrt{x} & \\ du = \frac{1}{2\sqrt{x}}dx \Rightarrow & 2\sqrt{x}\,du = dx \\ & 2(u-1)\,du = dx \end{array}}$

$= \int u^8\cdot 2(u-1)\,du$

$= \int 2(u^9 - u^8)\,du = 2\left(\frac{u^{10}}{10} - \frac{u^9}{9}\right)$

$= \frac{1}{5}u^{10} - \frac{2}{9}u^9 = \left[\frac{1}{5}(1+\sqrt{x})^{10} - \frac{2}{9}(1+\sqrt{x})^9\right]_0^1$

$= \frac{1}{5}\cdot 2^{10} - \frac{2}{9}\cdot 2^9 - \left(\frac{1}{5} - \frac{2}{9}\right)$

$= \boxed{\frac{1}{5}(2^{10}-1) - \frac{2}{9}(2^9-1)} = \frac{4097}{45}$ ☺

(25.) $\int \frac{3x^2-2}{x^2-2x-8}\,dx$

$x^2-2x-8 \overline{)\,3x^2+0x-2}$ with quotient 3; $-3x^2+6x+24$; $6x+22$

$\int\left(3 + \frac{6x+22}{(x+2)(x-4)}\right)dx = 3x + \int\frac{A}{x+2}\,dx + \int\frac{B}{x-4}\,dx = 3x + A\ln|x+2| + B\ln|x-4| + C$

$A(x-4) + B(x+2) = 6x+22$

$Ax - 4A$
$Bx + 2B$
$6x - 26$

$\begin{cases} A+B=6 \\ -4A+2B=+22 \end{cases}$

$4A+4B=24$
$-4A+2B=+22$
$6B=46$
$B=\frac{23}{3}$

$A+\frac{23}{3}=6$
$\cancel{A = 6+\frac{1}{3} = \frac{19}{3}}$
$A=-\frac{5}{3}$

$= \boxed{3x \mp \frac{5}{3}\ln|x+2| - \frac{23}{3}\ln|x-4| + C}$ ☺

(27.) $\int \frac{dx}{1+e^x} = \int \frac{1}{u}\cdot\frac{du}{(u-1)} = \int\frac{A}{u}\,du + \int\frac{B}{u-1}\,du = A\ln|u| + B\ln|u-1|$

$= A\ln|1+e^x| + B\ln|e^x| = A\ln|1+e^x| + Bx$

$= \boxed{-\ln|1+e^x| + x + C}$ ☺

$\boxed{\begin{array}{l} u = 1+e^x\,dx \\ du = e^x dx \Rightarrow du = (u-1)dx \end{array}}$

$A(u-1) + Bu = 1$

$Au - A$
Bu
$0u + 1$

$\begin{cases} A+B=0 \\ -A=1 \end{cases}$

$A=-1$
$B=+1$

(29.) $$\int \ln\left(x+\sqrt{x^2-1}\right)dx = x\ln\left(x+\sqrt{x^2+1}\right) - \int \frac{x}{x+\sqrt{x^2+1}}\left(\frac{1}{1}+\frac{x}{\sqrt{x^2+1}}\right)dx$$

$$\cancel{\frac{x+\sqrt{x^2-1}}{1}\cdot\frac{x-\sqrt{x^2-1}}{x-\sqrt{x^2-1}} = \frac{x^2-(x^2-1)}{x-\sqrt{x^2-1}} = \frac{1}{x-\sqrt{x^2-1}}}$$

$$= x\ln\left(x+\sqrt{x^2+1}\right) - \int \frac{x}{\cancel{x+\sqrt{x^2+1}}}\cdot\left(\frac{\cancel{\sqrt{x^2+1}+x}}{\sqrt{x^2+1}}\right)dx$$

$$\boxed{\begin{array}{ll} u=\ln\left(x+\sqrt{x^2+1}\right) & v=x \\ du=\left(\dfrac{1}{x+\sqrt{x^2+1}}\cdot\left(1+\dfrac{1\cdot 2x}{2\sqrt{x^2+1}}\right)\right)dx & dv=dx \end{array}}$$

$$= x\ln\left(x+\sqrt{x^2+1}\right) - \int \frac{x}{\sqrt{x^2+1}}\,dx$$

$$\boxed{\begin{array}{l} u=x^2+1 \\ du=2x\,dx \end{array}}$$

$$= x\ln\left(\sqrt{x^2+1}+x\right) - \int \frac{1}{\sqrt{u}}\,\frac{du}{2} = x\ln\left(x+\sqrt{x^2+1}\right) - \frac{1}{2}\int u^{-1/2}\,du$$

$$= x\ln\left(x+\sqrt{x^2+1}\right) - \frac{1}{\cancel{2}}\,u^{1/2}\cdot\frac{\cancel{2}}{1} = \boxed{x\ln\left(x+\sqrt{x^2+1}\right) - \sqrt{x^2+1}+C}$$

(31.) $$\int\sqrt{\frac{1+x}{1-x}}\,dx = \cancel{\int\sqrt{\frac{1+x}{1+x}\cdot\frac{1-x}{1-x}}\,dx = \int\sqrt{\frac{1-x^2}{(1-x)^2}}\,dx} = \int\frac{\sqrt{1-x^2}}{1-x}\,dx$$

$$a^2-x^2 \Rightarrow x=a\sin\theta$$

$$1^2-(\sqrt{x})^2 \Rightarrow \boxed{\begin{array}{l} \sqrt{x}=1\sin\theta \\ \dfrac{1}{2\sqrt{x}}\,dx=\cos\theta\,d\theta \\ dx=2\sin\theta\cos\theta\,d\theta \end{array}}$$

$$\cancel{= \int\sqrt{\frac{1+\sin^2\theta}{1-\sin^2\theta}}\cdot 2\sin\theta\cos\theta\,d\theta = \int\frac{\sqrt{1+\sin^2\theta}}{\cos\theta}\cdot 2\sin\theta\cos\theta\,d\theta}$$

$$= 2\int\sqrt{1+\sin^2\theta}\cdot\sin\theta\,d\theta$$

$$x^2-a^2 \Rightarrow x=a\sec\theta$$

$$(\sqrt{x})^2-1^2 \Rightarrow \boxed{\begin{array}{l} \sqrt{x}=1\sec\theta \\ \dfrac{1}{2\sqrt{x}}\,dx=\sec\theta\tan\theta\,d\theta \\ dx=2\sec^2\theta\tan\theta\,d\theta \end{array}}$$

$$\cancel{= \int\sqrt{\frac{1+\sec^2\theta}{1-\sec^2\theta}}\cdot 2\sec^2\theta\tan\theta\,d\theta = \int\sqrt{1+\sec^2\theta}}$$

$$\int\sqrt{\frac{1+x}{1-x}\cdot\frac{1+x}{1+x}}\,dx = \int\sqrt{\frac{(1+x)^2}{1-x^2}}\,dx = \int\frac{1+x}{\sqrt{1-x^2}}\,dx =$$

$$\int\frac{1}{\sqrt{1-x^2}}\,dx + \int\frac{x}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + \int\frac{1}{\sqrt{u}}\,\frac{du}{-2} = \sin^{-1}x - \frac{1}{2}\int u^{-1/2}\,du$$

$$\boxed{\begin{array}{l} u=1-x^2 \\ du=-2x\,dx \end{array}}$$

$$= \sin^{-1}x - \frac{1}{2}\,u^{1/2}\cdot\frac{2}{1} = \sin^{-1}x - u^{1/2} = \boxed{\sin^{-1}x - \sqrt{1-x^2}+C}$$

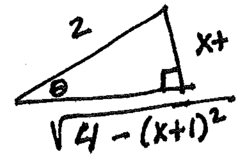
(33) $\int \sqrt{3-2x-x^2}\,dx$

$a^2-x^2 \Rightarrow x = a\sin\theta$

$2^2-(x+1)^2 \Rightarrow \boxed{\begin{array}{l} x+1 = 2\sin\theta \\ dx = 2\cos\theta\, d\theta \end{array}}$

$\int \sqrt{-x^2-2x+3}\,dx$

$\int \sqrt{-(x^2+2x+\quad 1\quad)+3+1}\,dx$



$\int \sqrt{4-(x+1)^2}\,dx$

$\int \sqrt{4-4\sin^2\theta}\cdot 2\cos\theta\, d\theta = \int \sqrt{4(1-\sin^2\theta)}\cdot 2\cos\theta\, d\theta = 4\int \cos^2\theta\, d\theta$

$= 4\int \frac{1+\cos 2\theta}{2}\,d\theta = 2\int (1+\cos 2\theta)\,d\theta = 2\theta + \cancel{2}\frac{\sin 2\theta}{\cancel{2}} = 2\theta + \sin 2\theta$

$= 2\theta + 2\sin\theta\cos\theta = 2\sin^{-1}\left(\frac{x+1}{2}\right) + \cancel{2}\left(\frac{x+1}{2}\right)\left(\frac{\sqrt{4-(x+1)^2}}{\cancel{2}}\right)$

$= \boxed{2\sin^{-1}\left(\frac{x+1}{2}\right) + \left(\frac{x+1}{2}\right)\sqrt{3-2x-x^2} + c}$

(35) $\int \cos 2x \cos 6x\,dx = \int \cos 2x\left(\cancel{2}\cos^2 3x - 1\right)dx$

$\cos(u+v) \cancel{= \cos x \cos y} - \sin x \sin y$

$\cos(8x) = \cancel{\cos 2x \cos 6x} - \sin 2x \sin 6x$

$\cos u \cos v = \frac{1}{2}[\cos(u+v) + \cos(u-v)]$

$\cos 2x \cos 6x = \frac{1}{2}[\cos(8x) + \cos(-4x)]$

$\frac{1}{2}\int \cos(8x)\,dx + \frac{1}{2}\int \cos(-4x)\,dx = \frac{1}{2}\frac{\sin(8x)}{8} + \frac{1}{2}\frac{\sin(-4x)}{-4}$

$= \frac{1}{16}\sin(8x) - \frac{1}{8}\sin(-4x) = \boxed{\frac{1}{16}\sin(8x) + \frac{1}{8}\sin(4x) + C}$

(37) $\int_0^{\pi/4} \tan^3\theta \sec^2\theta\, d\theta = \int u^3\,du = \frac{u^4}{4} = \frac{1}{4}\tan^4\theta \Big|_0^{\pi/4} = \frac{1}{4}(1-0) = \boxed{\frac{1}{4}}$

$\boxed{\begin{array}{l} u = \tan\theta \\ du = \sec^2\theta\, d\theta \end{array}}$

(39) $\int \frac{\sec\theta \tan\theta}{\sec^2\theta - \sec\theta}\,d\theta = \int \frac{\cancel{\sec\theta}\tan\theta}{\cancel{\sec\theta}(\sec\theta - 1)}\,d\theta = \int \frac{\tan\theta}{\sec\theta - 1}\,d\theta = \int \frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta} - 1}\,d\theta$

$= \int \frac{\frac{\sin\theta}{\cancel{\cos\theta}}}{\frac{1-\cos\theta}{\cancel{\cos\theta}}}\,d\theta = \int \frac{\sin\theta}{1-\cos\theta}\,d\theta = \int \frac{1}{u}\,du = \ln|u| = \boxed{\ln|1-\cos\theta| + C}$

$\boxed{\begin{array}{l} u = 1-\cos\theta \\ du = \sin\theta\, d\theta \end{array}}$

(41.) $\int \theta \tan^2\theta \, d\theta = \int \theta(\sec^2\theta - 1)\, d\theta = \int \theta \sec^2\theta \, d\theta - \int \theta \, d\theta$

$\cancel{u = \theta \quad v = \quad du = d\theta \quad dv = \tan^2\theta \, d\theta}$

$\boxed{u = \theta \quad v = \tan\theta \quad du = d\theta \quad dv = \sec^2\theta \, d\theta}$

$= \theta\tan\theta - \int \tan\theta \, d\theta - \frac{\theta^2}{2}$

$= \theta\tan\theta - \frac{1}{2}\theta^2 - \int \frac{\sin\theta}{\cos\theta} d\theta = \theta\tan\theta - \frac{1}{2}\theta^2 + \int \frac{1}{u} du$

$\boxed{u = \cos\theta \quad du = -\sin\theta \, d\theta}$

$= \theta\tan\theta - \frac{1}{2}\theta^2 + \ln|u| = \boxed{\theta\tan\theta - \frac{1}{2}\theta^2 + \ln|\cos\theta| + c}$

(43.) * $\int \frac{\sqrt{x}}{1+x^3} dx = \int \frac{u}{1+u^6} \cdot 2u\,du = \int \frac{2u^2}{1+u^6} du = \int \frac{2}{1+v^2} \frac{dv}{3}$

$\cancel{u = x^2 \Rightarrow x = \sqrt{u} \quad du = 2x\,dx}$

$\boxed{u = \sqrt{x} \Rightarrow x = u^2 \quad du = \frac{1}{2\sqrt{x}} dx \Rightarrow dx = 2u\,du}$

$\boxed{v = u^3 \quad dv = 3u^2 du}$

$= \frac{2}{3}\tan^{-1} v = \frac{2}{3}\tan^{-1}(u^3)$

$= \boxed{\frac{2}{3}\tan^{-1}(x^{3/2}) + c}$

(45.) $\int x^5 e^{-x^3} dx = \int -u\, e^u \frac{du}{-3} = \frac{1}{3}\int u e^u du = \frac{1}{3}\left[u e^u - \int e^u du\right]$

$\boxed{u = -x^3 \quad du = -3x^2 dx}$

$\boxed{u = u \quad v = e^u \quad du = du \quad dv = e^u du}$

$= \frac{1}{3}\left[u e^u - e^u\right] = \frac{1}{3}(-x^3)e^{-x^3} - \frac{1}{3}e^{-x^3}$

$= \boxed{-\frac{1}{3}x^3 e^{-x^3} - \frac{1}{3}e^{-x^3} + c}$

(47) $\int x^3(x-1)^{-4} dx = \int \frac{x^3}{(x-1)^4} dx = \int \frac{(u+1)^3}{u^4} du = \int \frac{u^3 + 3u^2 + 3u + 1}{u^4} du$

$\cancel{u = (x-1)^4 \Rightarrow \sqrt[4]{u} + 1 = x \quad du = 4(x-1)^3 dx}$

$\boxed{u = x-1 \quad du = dx} \Rightarrow x = u+1$

$= \int \left(\frac{1}{u} + 3u^{-2} + 3u^{-3} + u^{-4}\right) du = \ln|u| - 3u^{-1} + \frac{3u^{-2}}{-2} + \frac{u^{-3}}{-3} = \ln|u| - \frac{3}{u} - \frac{3}{2u^2} - \frac{1}{3u^3}$

$= \boxed{\ln|x-1| - \frac{3}{x-1} - \frac{3}{2}(x-1)^{-2} - \frac{1}{3}(x-1)^{-3} + c}$

(49) $\int \frac{1}{x\sqrt{4x+1}} dx = \int \frac{1}{\left(\frac{u^2-1}{4}\right) u} \cdot \frac{2u}{4} du = \int \frac{2}{u^2-1} du = -2\tanh^{-1} u$

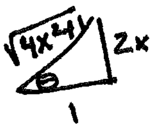
$\boxed{u = \sqrt{4x+1} \quad du = \frac{4}{2\sqrt{4x+1}} dx} \Rightarrow x = \frac{u^2-1}{4}$

$= \boxed{-2\tanh^{-1}(\sqrt{4x+1}) + c}$

(51) $$\int \frac{1}{x\sqrt{4x^2+1}}\,dx = \int \frac{1}{\cancel{\frac{1}{2}}\tan\theta\sqrt{\tan^2\theta+1}}\,\frac{\sec^2\theta\,d\theta}{\cancel{2}} = \int \frac{\sec^2\theta\,d\theta}{\tan\theta\sqrt{\sec^2\theta}}$$

$x^2+a^2 \Rightarrow x = a\tan\theta$

$(2x)^2+1^2$ $\boxed{\begin{array}{l}2x = 1\tan\theta \\ 2dx = \sec^2\theta\,d\theta\end{array}}$



$$= \int \frac{\sec\theta}{\tan\theta}\,d\theta = \int \frac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}} = \int \csc\theta$$

$$= -\ln|\cancel{\sec}^{\csc}\theta + \cot\theta|$$

$$= -\ln\left|\frac{\sqrt{4x^2+1}}{2x} + \frac{1}{2x}\right| = \boxed{-\ln\left|\frac{\sqrt{4x^2+1}+1}{2x}\right| + c}$$

(53) $$\int x^2\sinh(mx)\,dx = \frac{x^2}{m}\cosh(mx) - \int \frac{2x}{m}\cosh(mx)\,dx$$

$\boxed{\begin{array}{ll}u = x^2 & V = \frac{\cosh(mx)}{m} \\ du = 2x\,dx & dv = \sinh(mx)\,dx\end{array}}$ $\boxed{\begin{array}{ll}u = x & V = \frac{\sinh(mx)}{m} \\ du = dx & dv = \cosh(mx)\,dx\end{array}}$

$$= \frac{x^2}{m}\cosh(mx) - \frac{2}{m}\left[\frac{x\sinh(mx)}{m} - \int \frac{\sinh(mx)}{m}\,dx\right]$$

$$= \boxed{\frac{1}{m}x^2\cosh(mx) - \frac{2}{m^2}x\sinh(mx) + \frac{2}{m^2}\frac{\cosh(mx)}{m} + C}$$

(55) $$\int \frac{dx}{x+x\sqrt{x}} = \int \frac{dx}{x(1+\sqrt{x})} = \int \frac{1}{(u-1)^2 u}\cdot 2(u-1)\,du = \int \frac{2u-2}{u(u-1)^2}\,du$$

$\boxed{\begin{array}{l}u = 1+\sqrt{x} \\ du = \frac{1}{2\sqrt{x}}\,dx\end{array}}$ $\Rightarrow x = (u-1)^2$

$\Rightarrow dx = 2\sqrt{x}\,du$

$dx = 2(u-1)\,du$

$$= \int \frac{A}{u}\,du + \int \frac{B}{u-1}\,du + \int \frac{C}{(u-1)^2}\,du$$

$$= A\ln|u| + B\ln|u-1| + C\,\frac{(u-1)^{-1}}{-1}$$

$A(u-1)^2 + B(u-1)u + C(u) = 2u-2$

$$\left\{\begin{array}{lll}Au^2 & -2Au & +A \\ Bu^2 & -Bu & \\ & +Cu & \end{array}\right.$$

$0u^2 + 2u - 2$

$A = -2$

$A + B = 0 \Rightarrow B = +2$

$-2A - B + C = 2$

$4 - 2 + C = 2$

$C = 0$

$$= -2\ln|u| + 2\ln|u-1|$$

$$= -2\ln|1+\sqrt{x}| + 2\ln|\sqrt{x}| + c$$

$$= \boxed{2\ln\left|\frac{\sqrt{x}}{1+\sqrt{x}}\right| + C}$$

(57) $$\int x\sqrt[3]{x+c}\,dx = \int (u-c)\sqrt[3]{u}\,du = \int \left(u^{4/3} - cu^{1/3}\right)du$$

$\boxed{\begin{array}{l}u = x+c \\ du = dx\end{array}}$ $\Rightarrow x = u-c$

$$= \frac{3u^{7/3}}{7} - \frac{3c}{4}u^{4/3} = \boxed{\frac{3}{7}(x+c)^{7/3} - \frac{3}{4}c(x+c)^{4/3} + C_1}$$

(59) $\int \cos x \cos^3(\sin x)\,dx = \int \cos^3 u \; du = \int \cos^2 u \cos u\, du$

$u = \sin x$
$du = \cos x\, dx$

$= \int (1-\sin^2 u)\cos u \; du = \int (1 - v^2)\,dv = v - \frac{v^3}{3}$

$v = \sin u$
$dv = \cos u\, du$

$= \sin u - \frac{1}{3}\sin^3 u = \boxed{\sin(\sin x) - \frac{1}{3}\sin^3(\sin x) + C}$

(61.) $\int \frac{d\theta}{1+\cos\theta} = \int \frac{1}{1+\cos\theta}\cdot\frac{1-\cos\theta}{1-\cos\theta}\,d\theta = \int \frac{1-\cos\theta}{1-\cos^2\theta}\,d\theta = \int \frac{1-\cos\theta}{\sin^2\theta}\,d\theta$

$= \int\left(\frac{1}{\sin^2\theta} - \frac{\cos\theta}{\sin^2\theta}\right)d\theta = \int \csc^2\theta\, d\theta - \int\frac{\cos\theta}{\sin^2\theta}\,d\theta = -\cot\theta - \int\frac{\cos\theta}{\sin^2\theta}\,d\theta$

$u = \sin\theta$
$du = \cos\theta\, d\theta$

$= -\cot\theta - \int \frac{du}{u^2} = -\cot\theta - \frac{u^{-1}}{-1} = -\cot\theta + \frac{1}{\sin\theta}$

$= \boxed{-\cot\theta + \csc\theta + C}$

(63.) $\int \sqrt{x}\, e^{\sqrt{x}}\,dx = \int u e^u \cdot 2u\,du = \int 2u^2 e^u\, du$

$u = \sqrt{x}$
$du = \frac{1}{2\sqrt{x}}dx \Rightarrow dx = 2u\,du$

$u = 2u^2 \qquad v = e^u$
$du = 4u\,du \qquad dv = e^u du$

$u = 4 \qquad v = e^u$
$du = du \qquad dv = e^u du$

$= 2u^2 e^u - \int 4u e^u du$

$= 2u^2e^u - 4\left[ue^u - \int e^u du\right] = 2u^2e^u - 4ue^u + 4e^u = \boxed{2xe^{\sqrt{x}} - 4\sqrt{x}e^{\sqrt{x}} + 4e^{\sqrt{x}} + C}$

(65.) $\int \frac{\sin 2x\, dx}{1+\cos^4 x} = \int \frac{2\sin x\cos x}{1+\cos^4 x}\,dx = \int\frac{-2u}{1+u^4}\,du$

$u = \cos x$
$du = -\sin x\,dx$

$v = u^2$
$dv = 2u\,du$

$= \int -\frac{dv}{1+v^2} = -\tan^{-1} v = -\tan^{-1}(u^2)$

$= \boxed{-\tan^{-1}(\cos^2 x) + C}$

(67.) $\int \frac{1}{\sqrt{x+1}+\sqrt{x}}\,dx = \int \frac{1}{\sqrt{x+1}+\sqrt{x}}\cdot\frac{\sqrt{x+1}-\sqrt{x}}{\sqrt{x+1}-\sqrt{x}}\,dx = \int\frac{\sqrt{x+1}-\sqrt{x}}{x+1-x}\,dx$

$= \int(\sqrt{x+1}-\sqrt{x})\,dx = \int\left((x+1)^{1/2} - x^{1/2}\right)dx = \boxed{\frac{2(x+1)^{3/2}}{3} - \frac{2x^{3/2}}{3} + C}$

(69) $\displaystyle\int_1^{\sqrt{3}} \frac{\sqrt{1+x^2}}{x^2}\,dx = \int_{\pi/4}^{\pi/3} \frac{\sqrt{1+\tan^2\theta}}{\tan^2\theta}\sec^2\theta\,d\theta = \int_{\pi/4}^{\pi/3} \frac{\sec^3\theta}{\tan^2\theta}\,d\theta$

$a^2+x^2 \Rightarrow x = a\tan\theta$

$1^2+x^2 \Rightarrow \boxed{\begin{array}{l} x = \tan\theta \\ dx = \sec^2\theta\,d\theta \end{array}}$

$\sqrt{3} = \tan\theta \Rightarrow \theta = \pi/3$

$1 = \tan\theta \Rightarrow \theta = \pi/4$

$\displaystyle = \int_{\pi/4}^{\pi/3} \frac{1}{\cos^3\theta}\cdot\frac{\cos^2\theta}{\sin^2\theta}\,d\theta = \int_{\pi/4}^{\pi/3} \frac{1}{\cos\theta\sin^2\theta}\,d\theta$

$\boxed{\begin{array}{l} u = \sin\theta \\ du = \cos\theta\,d\theta \end{array}}$

$\displaystyle = \int \frac{1}{\cos\theta\, u^2}\,\frac{d\theta}{\cos\theta} = \int \frac{d\theta}{\cos^2\theta\, u^2}$

$\displaystyle = \int \frac{du}{(1-\sin^2\theta)u^2} = \int \frac{du}{(1-u^2)u^2} = \int_{\sqrt{2}/2}^{\sqrt{3}/2} \frac{du}{(1+u)(1-u)u^2} =$

$\displaystyle = \int_{\sqrt{2}/2}^{\sqrt{3}/2} \left(\frac{A}{1+u} + \frac{B}{1-u} + \frac{C}{u} + \frac{D}{u^2}\right)du$

$A(1-u)u^2 = Au^2 - Au^3$

$B(1+u)u^2 = Bu^2 + Bu^3$

$C(1+u)(1-u)u = Cu - Cu^3$

$D(1+u)(1-u) = D - Du^2$

$0u^3 + 0u^2 + 0u + 1$

$-A+B-C=0$

$A+B-D=0$

$C=0$

$D=1$

$-A+B=0$

$A+B=1$

$2B=1$

$B=\frac{1}{2}$

$A=\frac{1}{2}$

$\displaystyle = \int_{\sqrt{2}/2}^{\sqrt{3}/2} \left(\frac{1}{2(1+u)} + \frac{1}{2(1-u)} + \frac{1}{u^2}\right)du$

$\displaystyle = \frac{1}{2}\ln|1+u| + \frac{1}{2}\frac{\ln|1-u|}{-1} + \frac{u^{-1}}{-1}$

$\displaystyle = \left(\frac{1}{2}\ln|1+u| - \frac{1}{2}\ln|1-u| - \frac{1}{u}\right)\Big|_{\sqrt{2}/2}^{\sqrt{3}/2}$

$\displaystyle = \frac{1}{2}\ln\left|1+\frac{\sqrt{3}}{2}\right| - \frac{1}{2}\ln\left|1-\frac{\sqrt{3}}{2}\right| - \frac{2}{\sqrt{3}}$

$\displaystyle - \frac{1}{2}\ln\left|1+\frac{\sqrt{2}}{2}\right| + \frac{1}{2}\ln\left|1-\frac{\sqrt{2}}{2}\right| + \frac{2}{\sqrt{2}}$

$\displaystyle = \frac{1}{2}\ln\left|\frac{2+\sqrt{3}}{2}\right| - \frac{1}{2}\ln\left|\frac{2-\sqrt{3}}{2}\right| - \frac{2}{\sqrt{3}} - \frac{1}{2}\ln\left|\frac{2+\sqrt{2}}{2}\right| + \frac{1}{2}\ln\left|\frac{2-\sqrt{2}}{2}\right| + \frac{2}{\sqrt{2}}$

$\displaystyle = \frac{1}{2}\ln|2+\sqrt{3}| - \cancel{\frac{1}{2}\ln|2|} - \frac{1}{2}\ln|2-\sqrt{3}| + \cancel{\frac{1}{2}\ln|2|} - \frac{2}{\sqrt{3}} - \frac{1}{2}\ln|2+\sqrt{2}| + \cancel{\frac{1}{2}\ln|2|}$

$\displaystyle + \frac{1}{2}\ln|2-\sqrt{2}| - \cancel{\frac{1}{2}\ln|2|} + \frac{2}{\sqrt{2}}$

$\displaystyle = \frac{1}{2}\ln|2+\sqrt{3}| - \frac{1}{2}\ln|2-\sqrt{3}| - \frac{2}{\sqrt{3}} - \frac{1}{2}\ln|2+\sqrt{2}| + \frac{1}{2}\ln|2-\sqrt{2}| + \frac{2}{\sqrt{2}}$

$\displaystyle = \frac{1}{2}\ln\left|\frac{2+\sqrt{3}}{2-\sqrt{3}}\right| - \frac{2}{\sqrt{3}} + \frac{1}{2}\ln\left|\frac{2-\sqrt{2}}{2+\sqrt{2}}\right| + \frac{2}{\sqrt{2}}$

$\displaystyle = \frac{1}{2}\ln\left|\frac{2+\sqrt{3}}{2-\sqrt{3}}\cdot\frac{2+\sqrt{3}}{2+\sqrt{3}}\right| - \frac{2}{\sqrt{3}} + \frac{1}{2}\ln\left|\frac{2-\sqrt{2}}{2+\sqrt{2}}\cdot\frac{2-\sqrt{2}}{2-\sqrt{2}}\right| + \frac{2}{\sqrt{2}}$

$\displaystyle = \frac{1}{2}\ln\left|\frac{(2+\sqrt{3})^2}{1}\right| - \frac{2}{\sqrt{3}} + \frac{1}{2}\ln\left|\frac{(2-\sqrt{2})^2}{2}\right| + \frac{2}{\sqrt{2}}$

$\displaystyle = \boxed{\ln|2+\sqrt{3}| - \frac{2}{\sqrt{3}} + \ln|2-\sqrt{2}| - \frac{1}{2}\ln 2 + \frac{2}{\sqrt{2}}}$

69. $\int_1^{\sqrt{3}} \frac{\sqrt{1+x^2}}{x^2}\,dx = \int \frac{\sqrt{1+\tan^2\theta}}{\tan^2\theta}\,\sec^2\theta\, d\theta = \int \frac{\sqrt{\sec^2\theta}}{\tan^2\theta}\,d\theta = \int \frac{\sec\theta}{\tan^2\theta}\,d\theta$

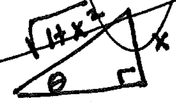
$a^2+x^2 \Rightarrow x = a\tan\theta$

$1^2+x^2 \Rightarrow \boxed{x = 1\tan\theta}$

$\boxed{dx = \sec^2\theta\, d\theta}$

$= \int \frac{\frac{1}{\cos\theta}}{\frac{\sin^2\theta}{\cos^2\theta}}\,d\theta = \int \frac{1}{\cos\theta}\cdot\frac{\cos^2\theta}{\sin^2\theta}\,d\theta = \int \frac{\cos\theta}{\sin^2\theta}\,d\theta$

$\boxed{\begin{array}{l} u = \sin\theta \\ du = \cos\theta\, d\theta \end{array}}$



$= \int \frac{1}{u^2}\,du = \frac{u^{-1}}{-1} = \frac{-1}{u} = \frac{-1}{\sin\theta} = -\csc\theta$

$= -\frac{\sqrt{1+x^2}}{x}\Big|_1^{\sqrt{3}} = -\frac{2}{\sqrt{3}} + \frac{\sqrt{2}}{1} = \frac{-2+\sqrt{6}}{\sqrt{3}}$

71. $\int \frac{e^{2x}}{1+e^x}\,dx = \int \frac{u-1}{u}\,du = \int\left(1-\frac{1}{u}\right)du = u - \ln|u|$

$\boxed{\begin{array}{l} u = 1+e^x \\ du = e^x\,dx \end{array}} \Rightarrow e^x = u-1$

$= 1+e^x - \ln|1+e^x| + C = \boxed{e^x - \ln|1+e^x| + C}$

73. $\int \frac{x+\arcsin x}{\sqrt{1-x^2}}\,dx = \int \frac{x}{\sqrt{1-x^2}}\,dx + \int \frac{\arcsin x}{\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{u}}\,\frac{du}{-2} + \int u\,du$

$\boxed{\begin{array}{l} u = \arcsin x \\ du = \frac{1}{\sqrt{1-x^2}}\,dx \end{array}}$ $\boxed{\begin{array}{l} u = 1-x^2 \\ du = -2x\,dx \end{array}}$

$= -\frac{1}{2}\,u^{1/2}\cdot\frac{2}{1} + \frac{u^2}{2}$

$= \boxed{-\left(1-x^2\right)^{1/2} + \frac{1}{2}\left(\arcsin x\right)^2 + C}$

75. $\int \frac{1}{(x-2)(x^2+4)}\,dx = \int \frac{A}{x-2}\,dx + \int \frac{Bx+C}{x^2+4}\,dx = \int \frac{A}{x-2}\,dx + \int \frac{Bx}{x^2+4}\,dx + \int \frac{C\,dx}{x^2+4}$

$u = x^2+4$, $du = 2x\,dx$; $u = \frac{x}{2}$, $du = \frac{1}{2}dx$

$\int \frac{B}{u}\frac{du}{2} + \int \frac{C\,dx}{4\left(\left(\frac{x}{2}\right)^2+1\right)}$

$= \boxed{A\ln|x-2| + \frac{B}{2}\ln|x^2+4| + \frac{C}{2}\tan^{-1}\left(\frac{x}{2}\right) + C}$

$= \boxed{\frac{1}{8}\ln|x-2| - \frac{1}{16}\ln|x^2+4| - \frac{1}{8}\tan^{-1}\left(\frac{x}{2}\right) + C}$

$A(x^2+4) + Bx(x-2) + C(x-2) = 1$

$Ax^2 \qquad +4A$
$Bx^2 - 2Bx$
$\qquad +Cx \quad -2C$
$+$
$0x^2 + 0x + 1$

$\begin{cases} A+B=0 \\ -2B+C=0 \\ 4A-2C=1 \end{cases}$

$5A - 3B = 1$
$8A = 1$
$A = 1/8$

$B = -\frac{1}{8}$
$C = -\frac{1}{4}$

77. $\int \frac{xe^x}{\sqrt{1+e^x}}\,dx = \int \frac{\ln(u-1)}{\sqrt{u}}\,du$

$\boxed{\begin{array}{l} u = 1+e^x \\ du = e^x\,dx \end{array}} \Rightarrow u-1 = e^x$, $\ln(u-1) = x$

$\boxed{\begin{array}{ll} u = \ln(u-1) & v = 2u^{1/2} \\ du = \frac{1}{u-1}\,du & dv = \frac{1}{\sqrt{u}}\,du \end{array}}$

$v = u-1 \Rightarrow u = v+1$
$dv = du$

$= 2\sqrt{u}\,\ln(u-1) - \int \frac{2\sqrt{u}}{u-1}\,du$

$= 2\sqrt{u}\,\ln(u-1) - 2\int \frac{\sqrt{v+1}}{v}\,dv$

$u = \sqrt{1+e^x} \Rightarrow x = \ln(u-1)^2$

$du = \frac{e^x}{2\sqrt{1+e^x}}\,dx$

$\int \frac{2u\,x\,du}{u} = \int 2\ln(u-1)^2\,du$

(73.) $\displaystyle\int \frac{x+\arcsin x}{\sqrt{1-x^2}}\,dx = \int \frac{x}{\sqrt{1-x^2}}\,dx + \int \frac{\arcsin x}{\sqrt{1-x^2}}\,dx$ $\boxed{\begin{array}{l} u=\arcsin x \\ du=\frac{1}{\sqrt{1-x^2}}\,dx \end{array}}$

$\boxed{\begin{array}{l} u=1-x^2 \\ du=-2x\,dx \end{array}}$ $\displaystyle = \int \frac{1}{\sqrt{u}}\,\frac{du}{-2} + \int u\,du$

$\displaystyle = -\frac{1}{2}u^{1/2}\cdot\frac{2}{1} + \frac{u^2}{2} = \boxed{-(1-x^2)^{1/2}+\frac{1}{2}(\arcsin x)^2+c}$

(77.) $\displaystyle\int \frac{xe^x}{\sqrt{1+e^x}}\,dx = \int \frac{\ln(u-1)\,du}{\sqrt{u}}$

$\boxed{\begin{array}{l} u=1+e^x \\ du=e^x\,dx \end{array}} \Rightarrow u-1=e^x$, $\ln(u-1)=x$

$\boxed{\begin{array}{ll} u=\ln(u-1) & v=u^{1/2}\cdot\frac{2}{1} \\ du=\frac{1}{u-1}\,du & dv=\sqrt{u}\,du \end{array}}$

$$\begin{array}{r|l} & 1+\frac{1}{v^2-1} \\ v^2-1 & v^2 \\ & -v^2+1 \\ \hline & 1 \end{array}$$

$A(v-1)=Av-A$
$B(v+1)=Bv+B$
$0v+1$

$\displaystyle = 2\sqrt{u}\ln(u-1) - \int \frac{2\sqrt{u}}{u-1}\,du$ $\boxed{\begin{array}{l} v=\sqrt{u} \\ dv=\frac{1}{2\sqrt{u}}\,du \end{array}}$

$\displaystyle = 2\sqrt{u}\ln(u-1) - 2\int \frac{v}{v^2-1}\,2v\,dv$

$\displaystyle = 2\sqrt{u}\ln(u-1) - 4\int \frac{v^2}{v^2-1}\,dv = 2\sqrt{u}\ln(u-1) - 4\int\left(1+\frac{1}{v^2-1}\right)dv$

$A+B=0$
$-A+B=1$
$2B=1$
$B=1/2$
$A=-1/2$

$\displaystyle = 2\sqrt{u}\ln(u-1) - 4v - 4\int\left(\frac{A}{v+1}+\frac{B}{v-1}\right)dv$

$\displaystyle = 2\sqrt{u}\ln(u-1) - 4v - 4\left(-\frac{1}{2}\ln|v+1| + \frac{1}{2}\ln|v-1|\right)$

$= 2\sqrt{u}\ln(u-1) - 4\sqrt{u} + 2\ln|\sqrt{u}+1| - 2\ln|\sqrt{u}-1|$

$= 2\sqrt{1+e^x}\ln(e^x) - 4\sqrt{1+e^x} + 2\ln|\sqrt{1+e^x}+1| - 2\ln|\sqrt{1+e^x}-1|$

$= \cancel{2\sqrt{1+e^x}}\ 2x\sqrt{1+e^x} - 4\sqrt{1+e^x} + 2\ln|\sqrt{1+e^x}+1| - 2\ln|\sqrt{1+e^x}-1| + c$

$\displaystyle = \boxed{2x\sqrt{1+e^x} - 4\sqrt{1+e^x} + 2\ln\left|\frac{\sqrt{1+e^x}+1}{\sqrt{1+e^x}-1}\right| + c}$

(79.) $\displaystyle\int x\sin^2 x\cos x\,dx = \int \sin^{-1}(u)\cdot u^2\,du$

$\boxed{\begin{array}{l} u=\sin x \\ du=\cos x\,dx \end{array}} \Rightarrow x=\sin^{-1}u$

$\boxed{\begin{array}{ll} u=\sin^{-1}(u) & v=\frac{u^3}{3} \\ du=\frac{1}{\sqrt{1-u^2}}\,du & dv=u^2\,du \end{array}}$

$a^2-x^2 \Rightarrow x=a\sin\theta$
$1^2-u^2 \Rightarrow \boxed{\begin{array}{l} u=\sin\theta \\ du=\cos\theta\,d\theta \end{array}}$

$\boxed{\begin{array}{l} V=\cos\theta \\ dV=-\sin\theta\,d\theta \end{array}}$

$\displaystyle = \frac{1}{3}u^3\sin^{-1}(u) - \int \frac{\frac{1}{3}u^3}{\sqrt{1-u^2}}\,du = \frac{1}{3}u^3\sin^{-1}(u) - \frac{1}{3}\int \frac{\sin^3\theta}{\sqrt{1-\sin^2\theta}}\,\cos\theta\,d\theta$

$\displaystyle = \frac{1}{3}u^3\sin^{-1}(u) - \frac{1}{3}\int \frac{\sin^3\theta}{\cancel{\cos\theta}}\,\cancel{\cos\theta}\,d\theta = \frac{1}{3}u^3\sin^{-1}(u) - \frac{1}{3}\int \sin^2\theta\sin\theta\,d\theta$

$\displaystyle = \frac{1}{3}u^3\sin^{-1}(u) - \frac{1}{3}\int(1-\cos^2\theta)\sin\theta\,d\theta = \frac{1}{3}u^3\sin^{-1}(u) + \frac{1}{3}\int(1-v^2)\,dv$

$\displaystyle = \frac{1}{3}u^3\sin^{-1}(u) + \frac{1}{3}v - \frac{1}{3}\frac{v^3}{3} = \frac{1}{3}u^3\sin^{-1}(u) + \frac{1}{3}\cos\theta - \frac{1}{9}\cos^3\theta = \frac{1}{3}u^3\sin^{-1}(u) + \frac{1}{3}\frac{\sqrt{1-u^2}}{1} - \frac{1}{9}\left(\frac{\sqrt{1-u^2}}{1}\right)^3$

$\displaystyle = \boxed{\frac{x}{3}\sin x + \frac{1}{3}\sqrt{1-\sin^2 x} - \frac{1}{9}\left(\sqrt{1-\sin^2 x}\right)^3 + c}$

*

(77) $\int \frac{xe^x}{\sqrt{1+e^x}}\,dx = \int \frac{xe^x}{\sqrt{1+e^x}} \cdot \frac{\sqrt{1+e^x}}{\sqrt{1+e^x}}\,dx = \int \frac{xe^x\sqrt{1-e^x}}{\sqrt{1-e^{2x}}}\,dx$

$a^2 + x^2 \Rightarrow x = a\tan\theta$

$1^2 + (e^{x/2})^2 \Rightarrow e^{x/2} = \tan\theta$

$x = 2\ln(\tan\theta) \quad \frac{1}{2}e^{x/2}\,dx = \sec^2\theta\,d\theta$

$\int \frac{2\ln(\tan\theta)\tan^2\theta}{\sqrt{1+\tan^2\theta}} \cdot \sec^2\theta \cdot 2\tan\theta\,d\theta$

$= 4\int \frac{\ln(\tan\theta)\tan^3\theta\sec^2\theta}{\sec\theta}\,d\theta$

$u = \ln(\tan\theta)$

$du = \frac{1}{\tan\theta} \cdot \sec^2\theta\,d\theta$

(81) $\int \sqrt{1-\sin x}\,dx = \int \frac{\sqrt{1-\sin x}}{1} \cdot \frac{\sqrt{1+\sin x}}{\sqrt{1+\sin x}}\,dx = \int \frac{\sqrt{1-\sin^2 x}}{\sqrt{1+\sin x}}\,dx$

$= \int \frac{\sqrt{\cos^2 x}}{\sqrt{1+\sin x}}\,dx = \int \frac{\cos x}{\sqrt{1+\sin x}}\,dx = \int \frac{1}{\sqrt{u}}\,du = u^{1/2} \cdot \frac{2}{1} = \boxed{2\sqrt{1+\sin x} + c}$

$u = 1+\sin x$

$du = \cos x\,dx$

(79) $\int x\sin^2 x\cos x\,dx = \int x\left(\frac{1-\cos x}{2}\right)\cos x\,dx = \int \frac{1}{2}x\cos x\,dx - \int \frac{1}{2}x\cos^2 x\,dx$

$u = x \quad v = \sin x$

$du = dx \quad dv = \cos x\,dx$

$u = x \quad v = \int \cos^2 x\,dx$

$du = dx \quad dv = \cos^2 x\,dx$

$= \frac{1}{2}x\sin x - \frac{1}{2}\int \sin x\,dx - \frac{1}{2}\left(\frac{1}{2}x^2 + \frac{1}{2}x\sin x - \int\left(\frac{1}{2}x + \frac{1}{2}\sin x\right)dx\right)$

$= \frac{1}{2}x\sin x + \frac{1}{2}\cos x - \frac{1}{4}x^2 + \frac{1}{4}x\sin x + \frac{1}{4}\frac{x^2}{2} + \frac{1}{4}\cos x$

$= \boxed{\frac{3}{4}x\sin x + \frac{3}{4}\cos x - \frac{1}{8}x^2 + c}$

$v = \int \cos^2 x\,dx = \int \frac{1+\cos x}{2}\,dx = \frac{1}{2}x + \frac{1}{2}\sin x$