MAC2312 Test 2 Name Key
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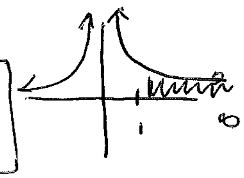
Show all work for credit.

4

1. a. Determine if the integral converges or diverges. If it converges, determine to what value it converges. Sketch an appropriate graph which indicates its meaning. $\int_1^\infty \frac{1}{x^4}dx$

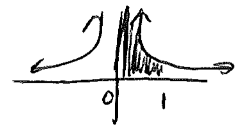
See next page for work

$\boxed{\frac{1}{3} \text{ Converges}}$



4

b. Determine if the integral converges or diverges. If it converges, determine to what value it converges. Sketch an appropriate graph which indicates its meaning. $\int_0^1 \frac{1}{x^4}dx$



See next page for work

$\boxed{\text{Diverges}}$

B 2. Select the appropriate method to integrate $\int \frac{e^x}{\sqrt{5+e^x}}dx$.

6

A. Partial fractions
B. U substitution
C. Trig substitution
D. Trig identity
E. Integration by parts
F. Long division

$u = e^x$
$du = e^x dx$
$\int \frac{1}{\sqrt{1+u}} du$

C 3. Select the appropriate method to integrate $\int \sqrt{25-e^{2x}}\,dx$.

6

A. Partial fractions
B. U substitution
C. Trig substitution
D. Trig identity
E. Integration by parts
F. Long division

$5^2-(e^x)^2 \Rightarrow$ $e^x = 5\sin\theta$
$e^x dx = 5\cos\theta\, d\theta$
$dx = \frac{\cancel{5}\cos\theta\, d\theta}{\cancel{5}\sin\theta}$

$\int \sqrt{25-25\sin^2\theta}\cdot\cot\theta\, d\theta$

6 A 4. Select the appropriate method to integrate $\int \frac{5}{(x+3)(x-4)}dx$.

A. Partial fractions
B. U substitution
C. Trig substitution
D. Trig identity
E. Integration by parts
F. Long division

$\int \left(\frac{A}{x+3} + \frac{B}{x-4}\right) dx$

F 5. Select the appropriate method to integrate $\int \frac{2x^3-1}{x-4}dx$. $= \int \left(2x^2+8x+32+\frac{127}{x-4}\right)dx$

(6)

A. Partial fractions
B. U substitution
C. Trig substitution
D. Trig identity
E. Integration by parts
F. Long division

$$2x^2+8x+32+\frac{127}{x-4}$$

$$
\begin{array}{r}
x-4\,\overline{)\,2x^3+0x^2+0x-1} \\
-(2x^3-8x^2) \\
\hline
8x^2+0x \\
-(8x^2-32x) \\
\hline
32x-1 \\
-(32x-128) \\
\hline
127
\end{array}
$$

E 6. Select the appropriate method to integrate $\int x^2 e^{-x}dx$.

(6)

A. Partial fractions
B. U substitution
C. Trig substitution
D. Trig identity
E. Integration by parts
F. Long division

$u = x^2$ $v = -e^{-x}$
$du = 2x\,dx$ $dv = e^{-x}dx$
$uv - \int v\,du$
$-x^2e^{-x} - \int -e^{-x}\cdot 2x\,dx$ By Parts again!

D 7. Select the appropriate method to integrate $\int \cos^2 x \sin^7 x\,dx$.

(6)

A. Partial fractions
B. U substitution
C. Trig substitution
D. Trig identity
E. Integration by parts
F. Long division

$u = \cos x$
$du = -\sin x\,dx$
$\int \cos^2 x \sin^6 x \sin x\,dx$
$\int \cos^2 x (\sin^2 x)^3 \sin x\,dx$
$\int \cos^2 x (1-\cos^2 x)^3 \sin x\,dx$
$\int -u^2(1-u^2)^3\,du$

$7\times 8 = 56$

Choose **8** of the following 10 problems to integrate. Clearly mark the **8** problems chosen to be graded.

✓ $\int \frac{1-\tan^2 x}{\sec^2 x}\,dx$ ✓ $\int \sin^2\left(\frac{x}{3}\right)dx$ ✓ $\int \frac{1}{(x^2+1)^2}\,dx$ ✓ $\int \frac{1}{x^2\sqrt{x^2-16}}\,dx$

7 each

✓ $\int x\sqrt{1-x^4}\,dx$ ✓ $\int x\sinh(2x)dx$ ✓ $\int \cos x \ln(\sin x)\,dx$ ✓ $\int \frac{x^2+2x-1}{x^3-x}\,dx$

✓ $\int \frac{3x}{x-4}\,dx$ ✓ $\int \frac{1}{x\sqrt{1+(\ln x)^2}}\,dx$

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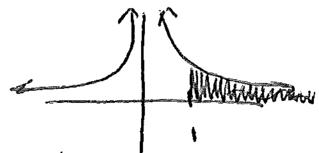
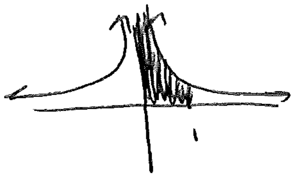
① a) $\int_1^\infty \frac{1}{x^4}\,dx = \int_1^\infty x^{-4}\,dx = \frac{x^{-3}}{-3} = \frac{-1}{3x^3}\Big|_1^\infty = \lim_{x\to\infty}\left(-\frac{1}{3x^3}\right) - \left(-\frac{1}{3}\right)$

$= 0 + \frac{1}{3} = \boxed{\frac{1}{3} \text{ Converges}}$



① b.) $\int_0^1 \frac{1}{x^4}\,dx = \frac{-1}{3x^3}\Big|_0^1 = -\frac{1}{3} - \lim_{x\to 0^+}\left(-\frac{1}{3x^3}\right) = -\frac{1}{3} + \infty$ $\boxed{\text{Diverges}}$



$\int \frac{1-\tan^2 x}{\sec^2 x}\,dx = \int \frac{1-(\sec^2 x - 1)}{\sec^2 x}\,dx = \int\left(\frac{2-\sec^2 x}{\sec^2 x}\right)dx = \int\left(\frac{2}{\sec^2 x} - 1\right)dx$

$= \int 2\cos^2 x\,dx - \int dx = \int \cancel{2}\left(\frac{1+\cos 2x}{\cancel{2}}\right)dx - x = \int (1+\cos 2x)\,dx - x$

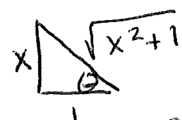
$= \cancel{x} + \frac{\sin 2x}{2} - \cancel{x} = \boxed{\frac{1}{2}\sin 2x + C}$

$\int \sin^2\left(\frac{x}{3}\right)dx = \int\left(\frac{1-\cos\left(\frac{2x}{3}\right)}{2}\right)dx = \int \frac{1}{2}\,dx - \int \frac{1}{2}\cos\left(\frac{2x}{3}\right)dx$

$= \frac{1}{2}x - \frac{1}{2}\sin\left(\frac{2x}{3}\right)\cdot\frac{3}{2} = \boxed{\frac{1}{2}x - \frac{3}{4}\sin\left(\frac{2x}{3}\right) + C}$

$\int \frac{1}{(x^2+1)^2}\,dx$ $\quad x^2+1^2 \Rightarrow x = \tan\theta$

$dx = \sec^2\theta\,d\theta$



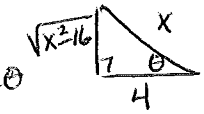
$= \int \frac{1}{(\tan^2\theta+1)^2}\sec^2\theta\,d\theta = \int \frac{\sec^2\theta}{(\sec^2\theta)^2}\,d\theta = \int \frac{1}{\sec^2\theta}\,d\theta = \int \cos^2\theta\,d\theta$

$= \int\left(\frac{1+\cos 2\theta}{2}\right)d\theta = \int \frac{1}{2}\,d\theta + \int \frac{1}{2}\cos 2\theta\,d\theta = \frac{1}{2}\theta + \frac{1}{2}\frac{\sin 2\theta}{2}$

$= \frac{1}{2}\theta + \frac{1}{4}\sin 2\theta = \frac{1}{2}\theta + \frac{1}{4}(2\sin\theta\cos\theta) = \frac{1}{2}\theta + \frac{1}{2}\sin\theta\cos\theta$

$= \frac{1}{2}\tan^{-1}x + \frac{1}{2}\left(\frac{x}{\sqrt{x^2+1}}\right)\left(\frac{1}{\sqrt{x^2+1}}\right) = \boxed{\frac{1}{2}\tan^{-1}x + \frac{1}{2}\cdot\frac{x}{x^2+1} + C}$

$$\int \frac{1}{x^2\sqrt{x^2-16}}\,dx \qquad x^2-4^2 \Rightarrow x = 4\sec\theta$$
$$dx = 4\sec\theta\tan\theta\,d\theta$$



$$\int \frac{1}{16\sec^2\theta\sqrt{16\sec^2\theta-16}}\cdot 4\sec\theta\tan\theta\,d\theta = \int \frac{\cancel{4}\sec\cancel{\theta}\tan\theta\,d\theta}{\underset{4}{\cancel{16}}\sec^2\theta\sqrt{16\underbrace{(\sec^2\theta-1)}_{\tan^2\theta}}}$$

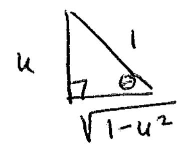
$$= \frac{1}{16}\int \frac{\tan\theta\,d\theta}{\sec\theta\,\cancel{\sec\theta}\,\cancel{\tan\theta}} = \frac{1}{16}\int \frac{1}{\sec\theta}\,d\theta = \frac{1}{16}\int \cos\theta\,d\theta$$

$$= \frac{1}{16}\sin\theta = \boxed{\frac{1}{16}\cdot\frac{\sqrt{x^2-16}}{x} + c}$$

$$\int x\sqrt{1-x^4}\,dx \qquad u = x^2, \quad du = 2x\,dx, \quad \frac{du}{2} = x\,dx$$

$$\int \sqrt{1-u^2}\,\frac{du}{2} = \frac{1}{2}\int\sqrt{1-u^2}\,du$$

$$1^2-u^2 \Rightarrow u = \sin\theta, \quad du = \cos\theta\,d\theta$$



$$= \frac{1}{2}\int\sqrt{\underbrace{1-\sin^2\theta}_{\cos^2\theta}}\cdot\cos\theta\,d\theta = \frac{1}{2}\int\cos\theta\cdot\cos\theta\,d\theta$$

$$= \frac{1}{2}\int\cos^2\theta\,d\theta = \frac{1}{2}\int\left(\frac{1+\cos 2\theta}{2}\right)d\theta = \frac{1}{4}\int(1+\cos 2\theta)\,d\theta = \frac{1}{4}\theta + \frac{1}{4}\,\frac{\sin 2\theta}{2}$$

$$= \frac{1}{4}\theta + \frac{1}{8}\sin 2\theta = \frac{1}{4}\theta + \frac{1}{8}(2\sin\theta\cos\theta) = \frac{1}{4}\theta + \frac{1}{4}\sin\theta\cos\theta$$

$$= \frac{1}{4}\sin^{-1}u + \frac{1}{4}u\cdot\sqrt{1-u^2} = \boxed{\frac{1}{4}\sin^{-1}(x^2) + \frac{1}{4}x^2\sqrt{1-x^4} + c}$$

$$\int x\sinh(2x)\,dx \qquad u = x, \quad du = dx \qquad v = \frac{\cosh(2x)}{2}, \quad dv = \sinh(2x)\,dx$$

$$uv - \int v\,du$$

$$= \frac{1}{2}x\cosh(2x) - \frac{1}{2}\int\cosh(2x)\,dx = \frac{1}{2}x\cosh(2x) - \frac{1}{2}\,\frac{\sinh(2x)}{2}$$

$$= \boxed{\frac{1}{2}x\cosh(2x) - \frac{1}{4}\sinh(2x) + c}$$

$$\int \cos x\cdot\ln(\sin x)\,dx \qquad u = \sin x, \quad du = \cos x\,dx \qquad = \int \ln u\,du$$

$$u = \ln u, \quad du = \frac{1}{u}\,du \qquad v = u, \quad dv = du$$

$$uv - \int v\,du$$

$$= u\ln u - \int u\cdot\frac{1}{u}\,du = u\ln u - \int du = u\ln u - u$$

$$= \boxed{\sin x\,\ln(\sin x) - \sin x + c}$$

$$\int \frac{x^2+2x-1}{x^3-x}\,dx = \int \frac{x^2+2x-1}{x(x^2-1)}\,dx = \int \frac{x^2+2x-1}{x(x+1)(x-1)}\,dx$$

$$= \int \left(\frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1}\right)dx = A\ln|x| + B\ln|x+1| + C\ln|x-1|$$

$$\begin{aligned} A(x+1)(x-1) &= Ax^2 - A \\ B(x)(x-1) &= Bx^2 - Bx \\ C(x)(x+1) &= Cx^2 + Cx \\ \hline & \ 1x^2 + 2x - 1 \end{aligned} \qquad \left\{\begin{aligned} -A &= -1 \\ -B + C &= +2 \\ A + B + C &= 1 \end{aligned}\right. \qquad A = 1$$

$$\begin{aligned} -B + C &= +2 \\ + \quad B + C &= 0 \\ \hline 2C &= +2 \\ C &= +1 \end{aligned} \qquad B = 1$$

$$= \ln|x| - \ln|x+1| + \ln|x-1| + C = \boxed{\ln\left|\frac{x(x-1)}{x+1}\right| + C}$$

$$\int \frac{3x}{x-4}\,dx \qquad \begin{array}{r} 3 + \frac{12}{x-4} \\ x-4\,\overline{)\,3x} \\ -3x+12 \\ \hline 12 \end{array} \qquad = \int \left(3 + \frac{12}{x-4}\right)dx = 3x + 12\ln|x-4|$$

$$= \boxed{3x + 12\ln|x-4| + C}$$

$$\int \frac{1}{x\sqrt{1+(\ln x)^2}}\,dx \qquad \begin{aligned} u &= \ln x \\ du &= \tfrac{1}{x}\,dx \end{aligned} \qquad = \int \frac{1}{\sqrt{1+u^2}}\,du = \sinh^{-1} u$$

$$= \boxed{\sinh^{-1}(\ln x) + C}$$