MAC2312 Test #4 Summer 2014 Name Key
Prof Howard 3-16

Show all work for credit.

1. Determine whether the **sequence** converges or diverges. If it converges, find the limit.

$$a_n = \frac{5n^2+3}{2n^2-1}$$

$$\lim_{n\to\infty} \frac{5n^2+3}{2n^2-1} = \frac{5}{2}$$

$\boxed{\text{Sequence converges to } \frac{5}{2}}$

2. Determine whether the **series** is convergent or divergent. If it is convergent, find its sum.

$10 - 2 + 0.4 - 0.08 + \cdots$

$$S = \frac{a_1}{1-r} \text{ if } |r|<1$$

$$10 - 2 + \frac{2}{5} - \frac{2}{25} + \ldots$$

geometric series

$r = -\frac{1}{5}$ $a_1 = 10$

$$S = \frac{10}{1-\left(-\frac{1}{5}\right)} = \frac{10}{\frac{6}{5}} = \frac{10}{1}\cdot\frac{5}{6} = \boxed{\frac{25}{3}}$$

converges.

3. Derive the Maclaurin series for $f(x) = \cosh(3x)$

$$f(x) = \cosh(3x)\Big|_{x=0} = 1 = 3^0$$

$$f'(x) = 3\sinh(3x)\Big|_{x=0} = 3 = 3^1$$

$$f''(x) = 3\cdot 3\cosh(3x)\Big|_{x=0} = 3^2$$

$$f'''(x) = 3\cdot 3\cdot 3\sin(3x)\Big|_{x=0} = 3^3$$

$\vdots$

$$f(x) = \frac{3^0x^0}{0!} + \frac{3^1x^1}{1!} + \frac{3^2x^2}{2!} + \frac{3^3x^3}{3!} + \ldots$$

$$\boxed{\cosh(3x) = \sum_{n=0}^{\infty} \frac{3^n x^n}{n!} = \sum_{n=0}^{\infty} \frac{(3x)^n}{n!}}$$

Pick **7** of the 10 series and test for absolute convergence, conditional convergence, or divergence. Also, state which test you used.

$\sum_{n=1}^{\infty} \frac{(2n+1)^n}{n^{2n}}$ AC by $\sqrt{\ }$.T.

$\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ D by L.C.T.

$\sum_{n=1}^{\infty} \frac{2^n n!}{(n+2)!}$ D by R.T.

$\sum_{n=1}^{\infty} n^2 e^{-n^3}$ AC by I.T.

$\sum_{n=1}^{\infty} \frac{(-1)^n(n^2+1)}{(n^3-5)}$ CC by A.S.T and L.C.T

$\sum_{n=1}^{\infty} (-1)^n(4n+1)$ D by AST

$$\sum_{n=1}^{\infty} \frac{\sin(4n)}{4^n}$$ A.C. by C.T.

$$\sum_{n=1}^{\infty} \frac{(-3)^n}{(2n+1)!}$$ A.c by R.T.

$$\sum_{n=1}^{\infty} \frac{6^n}{5^n - 1}$$ Diverges by L.C.T.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n\sqrt{n^2-1}}$$ A.c. by L.C.T.

$$\sum_{n=1}^{\infty} \frac{(-3)^n}{(2n+1)!}$$

AC? $\sum_{n=1}^{\infty} \left| \frac{(-3)^n}{(2n+1)!} \right| = \sum_{n=1}^{\infty} \frac{3^n}{(2n+1)!}$

R.T. positive ☺

$$\lim_{n\to\infty} \frac{\frac{3^{n+1}}{(2n+3)!}}{\frac{3^n}{(2n+1)!}} = \lim_{n\to\infty} \frac{\cancel{3^n} \cdot 3^1 \ \cancel{(2n+1)!}}{(2n+3)(2n+2)\cancel{(2n+1)!}\ \cancel{3^n}} = \lim_{n\to\infty} \frac{3}{(2n+3)(2n+2)}$$

$= 0 < 1$ converges

$\therefore$ AC

$\sum_{n=1}^{\infty} \frac{6^n}{5^n - 1} \approx \sum_{n=1}^{\infty} \left(\frac{6}{5}\right)^n$ diverges geo series $\left|r = \frac{6}{5}\right| > 1$

LCT positive ☺

$$\lim_{n\to\infty} \frac{\frac{6^n}{5^n-1}}{\frac{6^n}{5^n}} = \lim_{n\to\infty} \frac{6^n \cdot 5^n}{(5^n-1)6^n} = 1 > 0$$ converge or diverge together

$\therefore$ Diverges

$\sum_{n=1}^{\infty} \frac{(-1)^n}{n\sqrt{n^2-1}}$ AST $\lim_{n\to\infty} \frac{1}{n\sqrt{n^2-1}} = 0$ and decreasing

$\therefore$ Converges.

AC? $\sum_{n=1}^{\infty} \left| \frac{(-1)^n}{n\sqrt{n^2-1}} \right| = \sum_{n=1}^{\infty} \frac{1}{n\sqrt{n^2-1}} \approx \sum_{n=1}^{\infty} \frac{1}{n\sqrt{n^2}} = \sum_{n=1}^{\infty} \frac{1}{n^2}$

Converges by PST

LCT positive ☺

$$\lim_{n\to\infty} \frac{\frac{1}{n\sqrt{n^2-1}}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{n^2}{n\sqrt{n^2-1}} = \lim_{n\to\infty} \frac{n}{\sqrt{n^2-1}} = 1$$ converge together

$\therefore$ AC

$$\sum_{n=1}^{\infty} \frac{(-1)^n (n^2+1)}{n^3-5}$$

AST: $\lim_{n\to\infty} \frac{n^2+1}{n^3-5} = 0$ and decreasing

$\therefore$ converges.

Ac? $\sum_{n=1}^{\infty} \left|\frac{(-1)^n(n^2+1)}{n^3-5}\right| = \sum_{n=1}^{\infty} \frac{n^2+1}{n^3-5} \approx \sum_{n=1}^{\infty} \frac{n^2}{n^3} = \sum_{n=1}^{\infty} \frac{1}{n}$ diverges by PST

LCT positive ☺

$\lim_{n\to\infty} \frac{\frac{n^2+1}{n^3-5}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n^3+1}{n^3-5} = 1 > 0$ diverge or converge together

$\therefore$ absolute series diverges

$\therefore$ CC

$$\sum_{n=1}^{\infty} (-1)^n(4n+1)$$

AST: $\lim_{n\to\infty} 4n+1 = 4 \neq 0$ Diverges

$\therefore$ Diverges

$$\sum_{n=1}^{\infty} \frac{\sin(4n)}{4^n} \leq \sum_{n=1}^{\infty} \frac{1}{4^n}$$ converges by geo series $|r = \frac{1}{4}| < 1$

C.T. positive ☺ if you take absolute value!

if larger series converges so does smaller series

$\therefore$ Ac

$$\sum_{n=1}^{\infty} \frac{(2n+1)^n}{(n^2)^n} = \sum_{n=1}^{\infty} \left(\frac{2n+1}{n^2}\right)^n$$

$\underline{\sqrt{\ }.T.}$ positive ☺

$$\lim_{n\to\infty} \sqrt[n]{\left(\frac{(2n+1)}{n^2}\right)^n} = \lim_{n\to\infty} \frac{2n+1}{n^2} = 0 < 1 \text{ converges}$$

$\therefore$ Since all positive AC

$$\sum_{n=1}^{\infty} \frac{n}{n^2+1} \approx \sum_{n=1}^{\infty} \frac{n}{n^2} = \sum_{n=1}^{\infty} \frac{1}{n}$$ Diverges by PST

$\underline{LCT}$ positive ☺

$$\lim_{n\to\infty} \frac{\frac{n}{n^2+1}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n^2}{n^2+1} = 1 > 0$$ converge or diverge together.

$\therefore$ Diverges

$$\sum_{n=1}^{\infty} \frac{2^n n!}{(n+2)!}$$

$\underline{R.T.}$ positive ☺

$$\lim_{n\to\infty} \frac{\frac{2^{n+1}(n+1)!}{(n+3)!}}{\frac{2^n\, n!}{(n+2)!}} = \lim_{n\to\infty} \frac{\cancel{2^n}\cdot 2^1 (n+1)\cancel{n!}\ \cancel{(n+2)!}}{(n+3)\cancel{(n+2)!}\ \cancel{2^n}\ \cancel{n!}} = \lim_{n\to\infty} \frac{2(n+1)}{n+3} = 2 > 1$$

Diverges.

$\therefore$ Diverges

$$\sum_{n=1}^{\infty} n^2 e^{-n^3}$$

$\therefore$ since all positive AC

$\underline{I.T}$: positive, decreasing, continuous ☺

$$\int_1^{\infty} n^2 e^{-n^3}\, dn \qquad u = -n^3$$

$$du = -3n^2\, dn$$

$$\frac{du}{-3} = n^2\, dn$$

$$\int e^u \frac{du}{-3} = \frac{-1}{3} e^u$$

$$= -\frac{1}{3} e^{-n^3} = \frac{-1}{3e^{n^3}}\Big|_1^{\infty} = 0 + \frac{1}{3e}$$ converges.