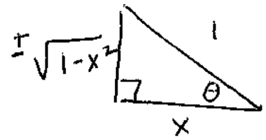
Take Home Quiz

1. $x = \cos(e^t) \implies e^t = \cos^{-1} x$
   $y = \sin(e^t)$

a) $y = \sin(\cos^{-1} x)$



$\boxed{y = \pm\sqrt{1-x^2}}$; $-1 \le x \le 1$

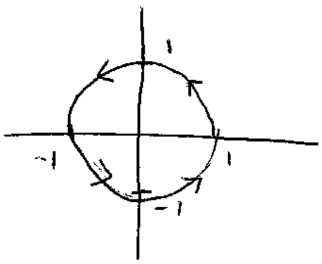
OR

$y^2 = \left(\pm\sqrt{1-x^2}\right)^2$

$y^2 = 1 - x^2$

$\boxed{x^2 + y^2 = 1}$

b)



$x' = -\sin(e^t) \cdot e^t$

$y' = \cos(e^t) \cdot e^t$

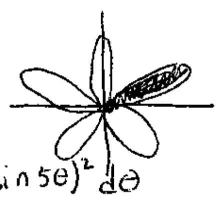
c) $L = \int_0^1 \sqrt{\left(-\sin(e^t) \cdot e^t\right)^2 + \left(\cos(e^t) \cdot e^t\right)^2}\, dt$

$L = \int_0^1 \sqrt{e^{2t}\left(\sin^2(e^t) + \cos^2(e^t)\right)}\, dt$

$L = \int_0^1 \sqrt{e^{2t}(1)}\, dt = \int_0^1 e^t\, dt = e^t \Big|_0^1$

$L = e^1 - e^0 = \boxed{e - 1}$

2. $r = \sin 5\theta$



$0 = \sin 5\theta$

$0, \pi, 2\pi, \ldots = 5\theta$

$0, \frac{\pi}{5}, \frac{2\pi}{5}, \ldots = \theta$

$A = \frac{5}{2} \int_0^{\pi/5} (\sin 5\theta)^2\, d\theta$ or $\int_0^{\pi} \frac{1}{2} (\sin 5\theta)^2\, d\theta$

$A = \frac{5}{2} \int_0^{\pi/5} \left(\frac{1}{2} - \frac{1}{2}\cos 10\theta\right) d\theta = \frac{5}{2}\left(\frac{1}{2}\theta - \frac{1}{2}\frac{\sin 10\theta}{10}\right)\Big|_0^{\pi/5}$

$= \frac{5}{2}\left[\frac{\pi}{10} - 0 - (0 - 0)\right] = \boxed{\frac{\pi}{4}}$

3.

$$y^2 - 5x^2 - 10y - 40x - 80 = 0$$

$$y^2 - 10y + \square - 5x^2 - 40x + \square = 80$$

$$y^2 - 10y + \square - 5(x^2 + 8x + \square) = 80$$

$$y^2 - 10y + 25 - 5(x^2 + 8x + 16) = 80 + 25 - 80$$

$$\frac{(y-5)^2}{25} - \frac{5(x+4)^2}{25} = \frac{25}{25}$$

$$\frac{(y-5)^2}{5^2} - \frac{(x+4)^2}{(\sqrt{5})^2} = 1$$

$a = \sqrt{5}$ left/right

$b = 5$ up/down

a.) $\boxed{C(-4, 5)}$

b.) $V(-4, 5 \pm 5) = \boxed{\begin{matrix} V(-4, 10) \\ V(-4, 0) \end{matrix}}$

c.) $c = \sqrt{25 + 5} = \sqrt{30}$ $\boxed{F(-4, 5 \pm \sqrt{30})}$

d.) $\boxed{y - 5 = \pm \frac{5}{\sqrt{5}}(x + 4)}$