points

$\bar{x} = 78.76$

36, 68, 82, 96, 100

MAC1140 Test 2 Name Key

Please show all work for credit. You may use your notes and graphing calculator for all questions on the exam. Show all work for credit. Your cell phone should be turned off and not on your desk at any time of the exam.

1. Find the determinant, showing all steps by hand. $\begin{bmatrix} 3 & 5 \\ -2 & 8 \end{bmatrix}$

(2)

$3(8) - (-2)(5)$

$24 + 10$

34

2. Find the inverse, showing all steps by hand. $\begin{bmatrix} 3 & 5 \\ -2 & 8 \end{bmatrix}$

(4)

$\begin{bmatrix} 3 & 5 & 1 & 0 \\ -2 & 8 & 0 & 1 \end{bmatrix} \div 3$

$\begin{bmatrix} 1 & \frac{5}{3} & \frac{1}{3} & 0 \\ -2 & 8 & 0 & 1 \end{bmatrix}$

$2\,\text{Row}\,1 + \text{Row}\,2$

$\begin{array}{cccc} 2 & \frac{10}{3} & \frac{2}{3} & 0 \\ +\,-2 & 8\cdot\frac{3}{3} & 0 & 1 \\ \hline 0 & \frac{34}{3} & \frac{2}{3} & 1 \end{array}$

$\begin{bmatrix} 1 & \frac{5}{3} & \frac{1}{3} & 0 \\ 0 & \frac{34}{3} & \frac{2}{3} & 1 \end{bmatrix} \cdot \frac{3}{34}$

$\begin{bmatrix} 1 & \frac{5}{3} & \frac{1}{3} & 0 \\ 0 & 1 & \frac{1}{17} & \frac{3}{34} \end{bmatrix}$

$\text{Row}\,1 + -\frac{5}{3}\,\text{Row}\,2$

$\begin{array}{cccc} 1 & \frac{5}{3} & \frac{1}{3}\cdot\frac{17}{17} & 0 \\ +\,0 & -\frac{5}{3} & -\frac{5}{51} & -\frac{5}{34} \\ \hline 1 & 0 & \frac{12}{51} & -\frac{5}{34} \end{array}$

Reduce

$\begin{bmatrix} 1 & 0 & \frac{12}{51} & -\frac{5}{34} \\ 0 & 1 & \frac{1}{17} & \frac{3}{34} \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & \frac{4}{17} & -\frac{5}{34} \\ 0 & 1 & \frac{1}{17} & \frac{3}{34} \end{bmatrix}$

$\text{Inverse} = \begin{bmatrix} \frac{4}{17} & -\frac{5}{34} \\ \frac{1}{17} & \frac{3}{34} \end{bmatrix}$

$(2\times 2)\ (2\times 3) = 2\times 3$

3. Multiply, showing all steps by hand. $\begin{bmatrix} 3 & 5 \\ -2 & 8 \end{bmatrix}\begin{bmatrix} 1 & 0 & -5 \\ 3 & 2 & -1 \end{bmatrix}$

(3)

$$\begin{bmatrix} 3(1)+5(3) & 3(0)+5(2) & 3(-5)+5(-1) \\ -2(1)+8(3) & -2(0)+8(2) & -2(-5)+8(-1) \end{bmatrix}$$

$$\begin{bmatrix} 18 & 10 & -20 \\ 22 & 16 & 2 \end{bmatrix}$$

4. Solve by any method. Use fractions and not decimals. If there is an infinite solution, indicate the solution in terms of z. Show work for credit.

(3)

$$\begin{cases} 2x+5y+z=4 \\ -x+y-z=1 \\ 3x+2y-3z=0 \end{cases}$$

$$\text{rref}\begin{bmatrix} 2 & 5 & 1 & 4 \\ -1 & 1 & -1 & 1 \\ 3 & 2 & -3 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -13/37 \\ 0 & 1 & 0 & 33/37 \\ 0 & 0 & 1 & 9/37 \end{bmatrix}$$

$x = \frac{-13}{37} \quad y = \frac{33}{37} \quad z = \frac{9}{37}$

5. Solve by any method. Use fractions and not decimals. If there is an infinite solution, indicate the solution in terms of z. Show work for credit.

(3)

$$\begin{cases} x-z-y=10 \\ 2x-3y+z=-11 \\ y-x+z=-10 \end{cases}$$ rearrange order!

$$\text{rref}\begin{bmatrix} 1 & -1 & -1 & 10 \\ 2 & -3 & 1 & -11 \\ -1 & 1 & 1 & -10 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -4 & 41 \\ 0 & 1 & -3 & 31 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x-4z=41$
$y-3z=31$
∞ solution

$x = 41+4z$
$y = 31+3z$
$z = z$

6. Solve $e^{3x-1} + 10 = 17$

$-10 \quad -10$

$e^{3x-1} = 7$

(3)

$\log_e(7) = 3x - 1$

$\ln(7) = 3x - 1$

$+1 \qquad +1$

$\frac{\ln(7)+1}{3} = \frac{3x}{3}$

$x = \frac{\ln(7)+1}{3} \approx 0.9819700497$

7. Solve $\log_3(2x+1) - \log_3(3x-4) = 2$

(4)

$\log_3\left(\frac{2x+1}{3x-4}\right) = 2$

$3^2 = \frac{2x+1}{3x-4}$

$\frac{9}{1} = \frac{2x+1}{3x-4}$

$9(3x-4) = 1(2x+1)$

$27x - 36 = 2x + 1$

$-2x \quad +36 \qquad -2x \quad +36$

$\frac{25x}{25} = \frac{37}{25}$

$x = \frac{37}{25}$

Check domain

$2x+1 > 0$ Yes $x > -\frac{1}{2}$

$3x-4 > 0$ Yes $x > \frac{4}{3}$

domain $\left(\frac{4}{3}, \infty\right)$

$\frac{37}{25}$ is greater than $\frac{4}{3}$

8. You invest \$3000 in a fund compounded continuously at a rate of 5%. How long do you have to wait so that you have \$10,000?

(3)

$A = Pe^{rt}$

$P = 3{,}000$

$A = 10{,}000$

$r = 0.05$

$\frac{10{,}000}{3{,}000} = \frac{3{,}000\, e^{0.05t}}{3{,}000}$

$\frac{10}{3} = e^{0.05t}$

$\frac{\ln(\frac{10}{3})}{0.05} = \frac{0.05t}{0.05}$

$t = \frac{\ln(10/3)}{0.05} = 24.07945609$ years