MAC2312 Test 3 Summer 2007 Name Key

Prof Howard 3-16

1. **Set up but do not evaluate.**

Find the arc length of the curve $f(x) = x - \sinh x$ for $x \in [0,1]$.

$f'(x) = 1 - \cosh x$

$$L = \int_0^1 \sqrt{1 + (1-\cosh x)^2}\, dx$$

2. **Set up but do not evaluate.**

$y = \frac{1}{\cos x}$ $x = \frac{1}{\cos y}$

Find the area of the surface obtained by rotating the curve $f(x) = \sec x$ about the x axis for $x \in \left[0, \frac{\pi}{4}\right]$.

$f'(x) = \sec x \tan x$

$f(y) = \cos^{-1}\left(\frac{1}{y}\right)$

$f'(y) = \frac{-1}{\sqrt{1-(1/y)^2}} \cdot \frac{-1}{y^2}$

$$SA = \int_0^{\pi/4} 2\pi \sec x \sqrt{1 + (\sec x \tan x)^2}\, dx$$

OR

$$SA = \int_1^{2/\sqrt{2}} 2\pi y \sqrt{1 + \left(\frac{1}{y^2\sqrt{1-(1/y)^2}}\right)^2}\, dy$$

3. **Set up but do not evaluate.**

Find the arc length for the curve $\begin{matrix} x = e^t \\ y = \sin t \end{matrix}$ where $t \in [0, \pi]$.

$\frac{dy}{dx} = \frac{\cos t}{e^t}$

$\frac{dy}{dt} = \cos t$ $\frac{dx}{dt} = e^t$

$$L = \int_0^{\pi} \sqrt{(\cos t)^2 + (e^t)^2}\, dt$$

4. **Set up but do not evaluate.**

Find the area of the region enclosed by $r = \sin 2\theta$.

$$A = \int_0^{2\pi} \frac{1}{2} (\sin 2\theta)^2\, d\theta$$

5. Given $\begin{array}{l} x = 2\cos t \\ y = \sin^2 t \end{array}$ $\Rightarrow x^2 = 4\cos^2 t$

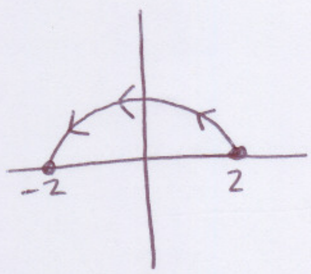
a. Eliminate the parameter to find the Cartesian equation.

$x^2 = 4\cos^2 t$

$4y = 4\sin^2 t$

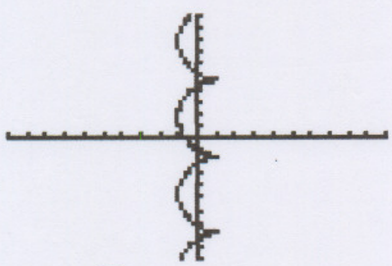
$x^2 + 4y = 4 \qquad x \in [-2, 2]$

b. Sketch the curve and indicate with an arrow the direction of the curve.

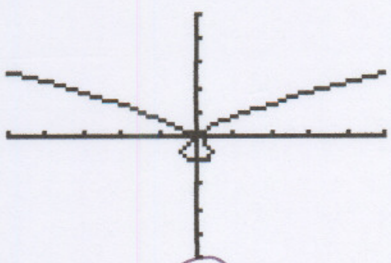


6. Match the following parametric equations with their graphs.

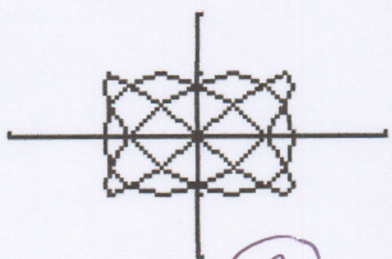
(A)	$x = -\sin t$ $y = t - \cos t$	(B)	$x = \sin 3t$ $y = \sin 4t$	(C)	$x = t^3 - t$ $y = t^2 - 1$



A



C



B

7. Find the equation of the tangent line to the curve $\begin{aligned} x &= e^{\sqrt{t}} \\ y &= t - \ln t \end{aligned}$ for $t = 1$.

$$m_{tan} = \frac{dy}{dx} = \frac{1 - \frac{1}{t}}{e^{\sqrt{t}} \cdot \frac{1}{2\sqrt{t}}} \Bigg|_{t=1} = 0$$

$$x = e^{\sqrt{1}} = e$$

$$y = 1 - \ln 1 = 1$$

$(e, 1)$ $m_{tan} = 0$

$$y - 1 = 0(x - e)$$

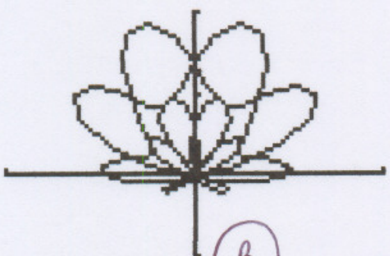
$$\boxed{y = 1}$$

8. Match the equation with the appropriate graph.

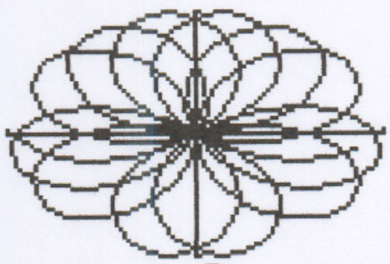
(A) $r = 1 + 4\cos(5\theta)$

(B) $r = \sin\theta + \sin^3(5\theta/2)$

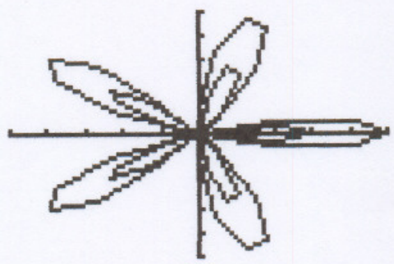
(C) $r = \sin(8\theta/5)$



B



C



A

9. Given $\dfrac{(x-3)^2}{4} + \dfrac{(y+1)^2}{16} = 1$

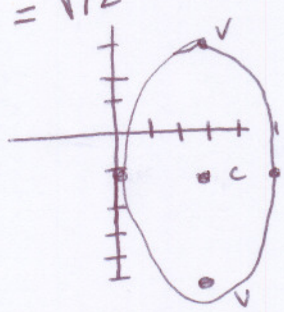
$a = 2$ $b = 4$

$c = \sqrt{|4 - 16|} = \sqrt{12} = 2\sqrt{3}$

a. Find the center. $C(3, -1)$

b. Find the verticies. $V(3, 3)$ $(3, -5)$

c. Find the foci. $F(3, -1 \pm 2\sqrt{3})$



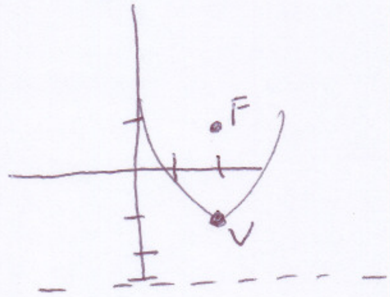
10. Given $(x-2)^2 = 8(y+1)$ $= 4p(y+1) = 4(2)(y+1)$

$p = 2$

a. Find the vertex. $V(2,-1)$

b. Find the focus. $F(2,1)$

c. Find the directrix. $y = -3$



11. Given $r = \dfrac{ed}{1 \pm e\cos\theta}$ and $r = \dfrac{ed}{1 \pm e\sin\theta}$ determine which is an ellipse, parabola, and hyperbola.

$r = \dfrac{10}{3-2\cos\theta} = \dfrac{10/3}{1-\frac{2}{3}\cos\theta}$; $e = 2/3 \Rightarrow 0 < e < 1$ ellipse

$r = \dfrac{6}{1+2\sin\theta} = \dfrac{6}{1+2\sin\theta}$; $e = 2 \Rightarrow e > 1$ hyperbola

$r = \dfrac{6}{1-\sin\theta} = \dfrac{6}{1-1\sin\theta}$; $e = 1 \Rightarrow e = 1$ parabola