MAC1105 Test 5
(Deb Howard 3-16)

Name Key

$A = Pe^{rt}$

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$pH = -log[H^+]$

$L = 10log\left(\frac{I}{I_0}\right)$

$R = log\left(\frac{I}{I_0}\right)$

1. The concentration of hydrogen ions is 0.00316 moles per liter in a can of soda. Calculate the pH of the soda. (7) 5.2

$pH = -\log[H^+]$

$H^+ = 0.00316$

$pH = -\log(0.00316) \approx 2.5$

2. You invest $50,000 in a fund compounded monthly at 4% interest. How much will the amount become after 20 years? (7) 5.5

$P = 50{,}000$
$n = 12$
$r = 0.04$
$t = 20$

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$A = 50{,}000\left(1 + \frac{0.04}{12}\right)^{(12 \times 20)}$

$A = \$111{,}129.10$

3. Solve **algebraically.** How many years will it take $50,000 to grow to be $111,277 in a fund compounded continuously at 8%? (10) 5.5

$P = 50{,}000$
$A = 111{,}277$
$r = 0.08$

$A = Pe^{rt}$

$\frac{111{,}277}{50{,}000} = \frac{50{,}000\, e^{0.08t}}{50{,}000}$

$\frac{111{,}277}{50{,}000} = e^{0.08t}$

$\ln\left(\frac{111{,}277}{50{,}000}\right) = 0.08t$

$t = \frac{\ln\left(\frac{111{,}277}{50{,}000}\right)}{0.08}$

$t \approx 10$ years

4.4

4. Solve **algebraically.** $\left(\sqrt{x+1}\right)^2 = (x-5)^2$

10

$x+1 = (x-5)(x-5)$

$x+1 = x^2 - 10x + 25$

$0 = x^2 - 11x + 24$

$0 = (x-8)(x-3)$

$x-8=0$ $\quad x-3=0$

$x=8$ $\quad x=3$

check!

$\sqrt{8+1} = 8-5$ ☺

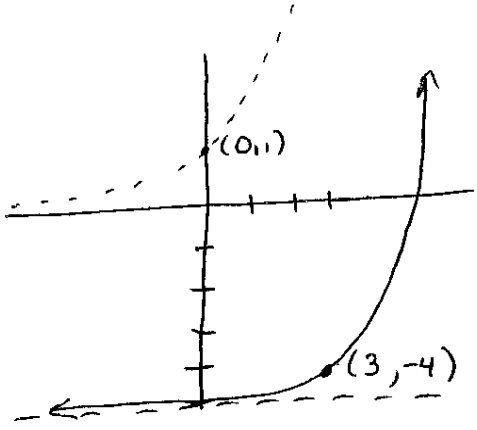
$\sqrt{3+1} = 3-5$ ☹

5. Show a complete graph of the function $f(x) = 4^{x-3} - 5$.

5.1

1

right 3 down 5



$y = -5$

1 6. Rewrite as a single logarithm. $2\ln(x) + \ln(4) - \ln(5)$

5.2

use: $\log_a(x^n) = n\log_a x$

$\log_a(xy) = \log_a(x) + \log_a(y)$

$\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$

$\ln(x^2) + \ln(4) - \ln(5)$

$\ln\left(\frac{4x^2}{5}\right)$

5.2

7. Simplify the following:

a.) $log_2\left(\frac{1}{32}\right) =$ (-5)

(1) $2^{?} = \frac{1}{32} = \frac{1}{2^5} = 2^{-5}$

b.) $log_{25}(5) =$ (1/2)

(1) $25^{?} = 5 \qquad \sqrt{25} = 5 \qquad 25^{1/2} = 5$

c.) $ln(e^4) =$ (4)

(1) $e^{?} = e^4$

d.) $log_3(1) =$ (0)

(1) $3^{?} = 1 \qquad 3^0 = 1$

8. The amount of a radioactive isotope present is given by $f(x) = 700e^{-0.00025t}$ grams where *t* is the time in years that the isotope decays.

5.1

a.) How many grams is the initial amount of the radioactive isotope?

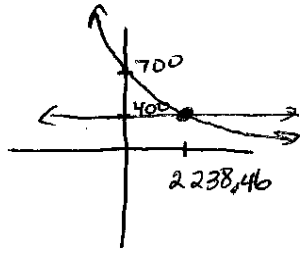
(1) $t = 0 \qquad y = 700e^{-.00025*0} = 700e^0 = 700(1)$

$Y = 700$

5.1 - 5.3

b.) How many years will it take for 400 grams of the radioactive isotope to be remaining? Solve by a method of your choice.

(1)



$\frac{400}{700} = \frac{700e^{-.00025t}}{700}$

$\frac{4}{7} = e^{-.00025t}$

$\frac{ln(4/7)}{-.00025} = \frac{-.00025t}{-.00025}$

$\frac{ln(4/7)}{-.00025} = t \approx$ 2238.46 years

5.3

9. Solve **algebraically.** $6 + \log(x) = 8 - 2\log(x)$

10

$\log(x) + 2\log(x) = 2$

$3\log(x) = 2$ OR $\log(x) + \log(x^2) = 2$

$\log(x) = \frac{2}{3}$ $\log(x^3) = 2$

$10^{2/3} = x$ $10^2 = x^3$

$4.64 = x$ $\sqrt[3]{100} = \sqrt[3]{x^3}$

$4.64 = x$