

Formula List for College Algebra – Sullivan 10th ed.
DO NOT WRITE ON THIS COPY.

Intercepts: Learn how to find the x and y intercepts.

Symmetry: Learn how “test for symmetry” with respect to the x -axis, y -axis and origin.

Linear Equation Formulas:

Standard or General Form: $Ax + By = C$

Slope formula: $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$ also $m = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(b) - f(a)}{b - a} = \frac{f(x+h) - f(x)}{h}$

Slope y -intercept form: $y = mx + b$ or Linear Function: $f(x) = mx + b$

Point Slope form: $y = y_1 + m(x - x_1)$ or $y = m(x - x_1) + y_1$

Systems of Linear Equations:

Inconsistent – the system has NO SOLUTIONS (*Contradiction*)

Dependent – the system has INFINITELY OR MANY SOLUTIONS (*Identity*)

Consistent – the system has at least ONE SOLUTION (*Conditional*)

Independent – the system has different lines (may have one solution or none).

Quadratic Equation:

A equation is an equation of the form :

$$ax^2 + bx + c = 0, a, b \text{ and } c \text{ are real numbers and } a \neq 0.$$

Square Root Method: Learn square root method.

Quadratic Formula:

The solutions of the equation $ax^2 + bx + c = 0$, where $a \neq 0$, are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

The Discriminant: For $ax^2 + bx + c = 0$, $a \neq 0$:

- a. If $b^2 - 4ac > 0$, there are two unequal real solutions.
- b. If $b^2 - 4ac = 0$, there are "two equal" or "one" real solutions.
- c. If $b^2 - 4ac < 0$, there is no real solution.

Zero-Factor principle:

$ab = 0$ if and only if $a = 0$ or $b = 0$.

Factorization Formulas:

The Difference of Two Squares $A^2 - B^2 = (A + B)(A - B)$

The Sum of Two Squares $A^2 + B^2 = \text{prime}$

The Difference of Two Cubes $A^3 - B^3 = (A - B)(A^2 + AB + B^2)$

The Sum of Two Cubes $A^3 + B^3 = (A + B)(A^2 - AB + B^2)$

Trinomial Squares – The Square of a Binomial

$$A^2 + 2AB + B^2 = (A + B)(A + B) = (A + B)^2$$

$$A^2 - 2AB + B^2 = (A - B)(A - B) = (A - B)^2$$

Cube of a Binomial:

$$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

$$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$$

Difference quotient of a function f at x is given by:

$$\frac{f(x+h) - f(x)}{h}, \text{ where } h \neq 0.$$

The Algebra of Functions:

Sum: $(f + g)(x) = f(x) + g(x)$

Difference: $(f - g)(x) = f(x) - g(x)$

Product: $(f \cdot g)(x) = f(x) \cdot g(x)$

Quotient: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ where $g(x) \neq 0$

...

Quadratic Function:

A quadratic function is one in the form: $f(x) = ax^2 + bx + c$

where a , b , and c are constants and a is not equal zero.

Quadratic Equation in Vertex Form:

The vertex form of the equation $ax^2 + bx + c = 0$, where $a \neq 0$, is :

$$y = a(x - x_v)^2 + y_v \text{ or } f(x) = a(x - h)^2 + k,$$

where $(x_v, y_v) = (h, k)$ is called the vertex.

Vertex of a parabola:

$$x_v = \frac{-b}{2a} \text{ is the AXIS of Symmetry and } y_v = f\left(\frac{-b}{2a}\right).$$

$$\text{In other words – the vertex is : } (h, k) = (x_v, y_v) = \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right) = \left(\frac{-b}{2a}, \frac{4ac - b^2}{4a}\right).$$

Distance Formula: The distance between points (x_1, y_1) and (x_2, y_2) in the coordinate plane is

$$\text{given by: } d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint Formula: The midpoint $M = (x_m, y_m)$ of a line segment with endpoints

(x_1, y_1) and (x_2, y_2) in the coordinate plane is given by:

$$M = (x_m, y_m) = \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$$

Modeling and Regression Analysis:

Scatterplot:

Go to $Y=$ and press enter on STATPLOT #1 to turn ON.
STAT, select 1. EDIT (enter x values in list 1 and y values in list 2)
WINDOW (set viewing window) or press Zoom #9
GRAPH

Find the Best Graphical Model or Regression Line / Curve:

STAT CALC #4 for Linear Modeling, #5 for Quadratic Modeling, etc.
and press enter once on the screen or press CALCULATE

To paste your answer onto $Y=$ and graph line on scatterplot:

Go to $Y1 =$ and make sure is blank
VARS select #5, arrow to EQ, select #1 (pastes eq. in $Y1$)
GRAPH (graphs plot and line)
CALC #1 (evaluates for an input)

The Equation of a Circle:

The standard form of the Equation of a Circle:

$$(x - h)^2 + (y - k)^2 = r^2.$$

The equation of a circle with center $(0,0)$ and radius r is given by: $x^2 + y^2 = r^2$

The general form of the equation of a Circle:

For $A, B, C, D,$ and E real numbers, $A \neq 0$, A and B not zero, the general form of the equation of the circle is given by: $Ax^2 + By^2 + Cx + Dy + E = 0$. Other textbooks may have the general form as: $x^2 + y^2 + ax + by + c = 0$.

Vertical Line Test: Know what is and how to do a "vertical line test."

...

Polynomial Function:

$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x^1 + a_0$ where n is degree of the polynomial.

Power Function:

$f(x) = ax^n$ where a is a real number, $a \neq 0$, and $n > 0$ is an integer.

Rational Function:

A rational function is one of the form $f(x) = \frac{P(x)}{Q(x)}$

where $P(x)$ and $Q(x)$ are polynomials and $Q(x) \neq 0$.

Vertical Asymptotes:

If $Q(a) = 0$, but $P(a) \neq 0$, then the graph of the rational function

$f(x) = \frac{P(x)}{Q(x)}$ has a vertical asymptote at $x = a$.

Horizontal Asymptotes:

Suppose $f(x) = \frac{P(x)}{Q(x)}$ is a rational function where the degree of

$P(x)$ is m and the degree of $Q(x)$ is n , ($\frac{m}{n}$).

a) If $m < n$, (the degree of the numerator is less than the degree of the denominator) then the graph of f has a horizontal asymptote at $y = 0$.

b) If $m = n$, then the graph of f has a horizontal asymptote at $y = \frac{a}{b}$,

where a is the lead coefficient of $P(x)$ and b is the lead coefficient of $Q(x)$.

c) If $m > n$, then the graph of f does not have a horizontal asymptote.

d) If $m = n + 1$ (the degree of the numerator is one more than the degree of the denominator), then the line $y = ax + b$ is an oblique asymptote, which is the quotient found using long division.

e) If $m \geq n + 2$ (the degree of the numerator is two or more than the degree of the denominator), then there are no horizontal or oblique asymptotes.

Note : A rational function will never have both a horizontal asymptote and an oblique asymptote.

Composition of Functions:

Let $f(x)$ and $g(x)$ represent two functions. The composition of f and g , written $(f \circ g)(x)$, is defined as $(f \circ g)(x) = f(g(x))$. Here, $g(x)$ must be in the domain of $f(x)$. If it is not, then $f(g(x))$ will be undefined.

One-to-one Functions:

The inverse of a function f is also a function if and only if f is one-to-one.

The graph of a one-to-one function f and the graph of its inverse function f^{-1} are symmetric with respect to the line $y = x$.

Inverse Functions:

Suppose the inverse of f is a function, denoted by f^{-1} . Then

$f^{-1}(y) = x$ if and only if $f(x) = y$.

Composition of a Function and its Inverse:

If a function, $f(x)$ has an inverse $f^{-1}(x)$, then :

$(f^{-1} \circ f)(x) = x$ for every x in the domain of f , and

$(f \circ f^{-1})(x) = x$ for every x in the domain of f^{-1} .

Exponents:

1. $a^m \cdot a^n = a^{m+n}$

2. $\frac{a^m}{a^n} = a^{m-n}, a \neq 0$

3. $(a^m)^n = a^{m \cdot n}$

4. $(ab)^m = a^m b^m$

5. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$

6. $\frac{1}{a^{-m}} = a^m, a \neq 0$

7. $\frac{a^{-m}}{b^{-n}} = \frac{b^n}{a^m}, a \neq 0, b \neq 0$

8. $\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$

9. $a^0 = 1, a \neq 0$

10. $a^{1/n} = \sqrt[n]{a}, n$ is an integer $n \geq 2$.

11. $a^{m/n} = \left(a^{1/n}\right)^m = \left(a^m\right)^{1/n} = \sqrt[n]{a^m}$

Exponential Function:

$f(x) = ab^x$, where b, a and x are real numbers, $b > 0, b \neq 1$ and $a \neq 0$

The base b the growth factor and because $f(0) = ab^0 = a$, a is called the initial value.

The number e is defined by the expression:

$$\left(1 + \frac{1}{n}\right)^n \text{ and as } n \rightarrow \infty \quad e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n.$$

Exponential Formulas:

Simple Interest Formula: $I = Prt$

Compound Interest: $A = P(1 + r)^n$

Compound Interest with n Compound n times per year: $A = P\left(1 + \frac{r}{n}\right)^{nt}$,

P = principal, r = annual rate, n = number of compoundings per year,

t = number of years, A = amount after t years.

Compound Interest Continuously: $A = Pe^{rt}$

Exponential Equality:

If $b^x = b^y$, then $x = y$ where $b > 0$ and $b \neq 1$.

Logarithms and Exponents: Conversion Equations

If $b > 0$ and $x > 0$, then

$y = \log_b x$ if and only if $x = b^y$.

$y = \ln x$ if and only if $e^y = x$.

Useful Logarithm Properties:

$\log_b b = 1$, because $b^1 = b$

$\ln e = 1$, because $e^1 = e$.

$\log_b 1 = 0$, because $b^0 = 1$

$\ln 1 = 0$, because $e^0 = 1$.

$\log_b b^x = x$, because $b^x = b^x$

$\ln e^x = x$, because $e^x = e^x$.

$b^{\log_b x} = x$, for $x > 0$

$e^{\ln x} = x$, for $x > 0$.

Other Properties of Logarithms:

If x, y and $b > 0$, then

a. $\log_b (xy) = \log_b x + \log_b y$

If x and $y > 0$, then

a. $\ln (xy) = \ln x + \ln y$

b. $\log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y$

b. $\ln \left(\frac{x}{y}\right) = \ln x - \ln y$

c. $\log_b (x)^k = k \log_b x$

c. $\ln (x)^k = k \ln x$

Properties of Natural Logarithms:

If x and $y > 0$, then

a. $\ln (xy) = \ln x + \ln y$ b. $\ln \left(\frac{x}{y}\right) = \ln x - \ln y$ c. $\ln (x)^k = k \ln x$

The Natural log and e^x :

$\ln e^x = x$, for all x and $e^{\ln x} = x$, for $x > 0$.

Change the base of a logarithm:

$$\log_b a = \frac{\log_{10} a}{\log_{10} b} = \frac{\ln a}{\ln b}$$

Absolute Value:

Definition of Absolute Value: $|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$

Absolute Value Equations and Inequalities:

a. $|ax+b| = c$ ($c > 0$) is equivalent to: $ax+b = c$ or $ax+b = -c$

b. $|ax+b| < c$ ($c > 0$) is equivalent to: $-c < ax+b < c$

c. $|ax+b| > c$ ($c > 0$) is equivalent to: $ax+b > c$ or $ax+b < -c$