4.1 Properties of Linear Functions and Linear Models

Learning Objectives

1. Graph Linear Functions
2. Use Average Rate of Change to Identify Linear Functions
3. Determine Whether a Linear Function Is Increasing, Decreasing, or Constant
4. Build Linear Models from Verbal Descriptions

Definition 1

A **linear function** is a function of the form

$$f(x) = mx + b$$

The graph of a linear function is a line with slope *m* and *y*-intercept *b*. Its domain is the set of all real numbers.

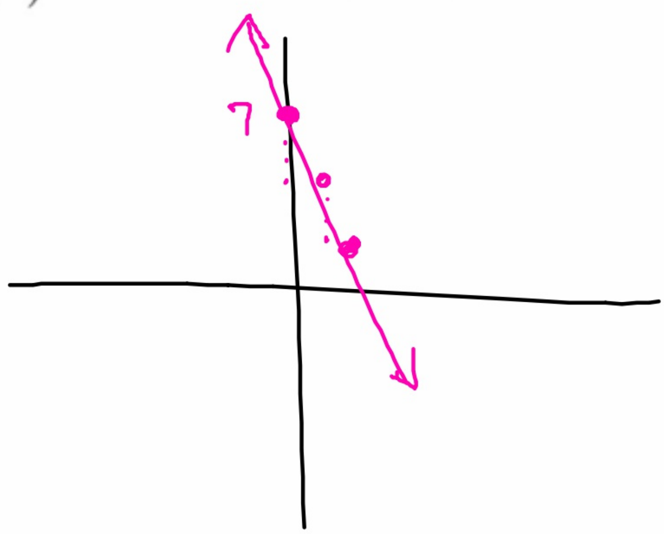
Example 1

Graphing a Linear Function

Graph the linear function $f(x) = -3x + 7$. What are the domain and the range of *f* ?

D: all reals

R: all reals



Average Rate of Change of a Linear Function

Linear functions have a constant average rate of change. That is, the average rate of change of a linear function $f(x) = mx + b$ is

$$\frac{\Delta y}{\Delta x} = m$$

Example 2 (a)

Using the Average Rate of Change to Identify Linear Functions

a) A strain of *E.* **coli** known as Beu 397-recA441 is placed into Petridish at 30° Celsius and allowed to grow. The data shown in Table 2 on the next page are collected. The population is measured in grams and the time in hours. Plot the ordered pairs (x, y) in the Cartesian plane, and use the average rate of change to determine whether the function is linear.

Time (hours), x	Population (grams), y	(x, y)
0	0.09	(0, 0.09)
1	0.12	(1, 0.12)
2	0.16	(2, 0.16)
3	0.22	(3, 0.22)
4	0.29	(4, 0.29)
5	0.39	(5, 0.39)

$m = \frac{.03}{1} = .03$

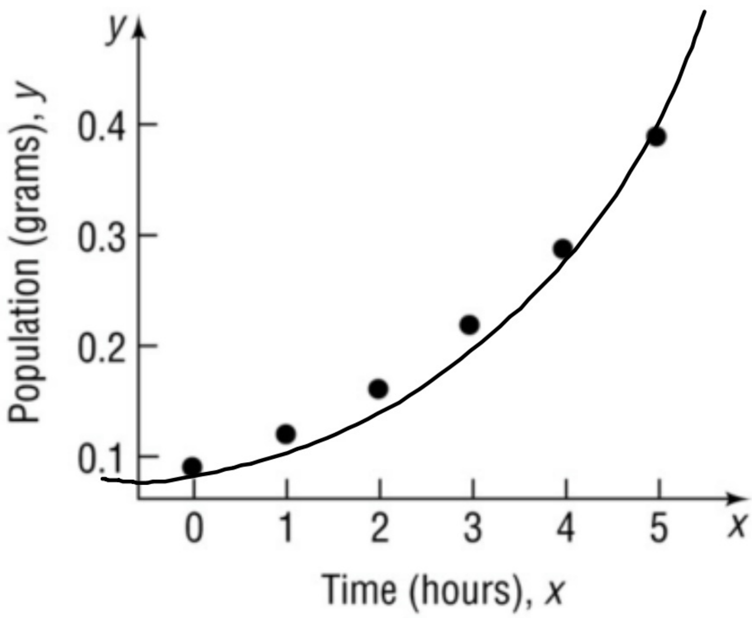
$m = .04$

$m = .06$

Solution 2 (a)

Part a: Figure 2

Non-linear



Example 2 (b)

b) The data represent the maximum number of heartbeats that a healthy individual of different ages should have during a 15-second interval of time while exercising. Plot the ordered pairs (x, y) in the Cartesian plane, and use the average rate of change to determine whether the function is linear.

Age, x	Maximum Number of Heartbeats, y	(x, y)
20	50	(20, 50)
30	47.5	(30, 47.5)
40	45	(40, 45)
50	42.5	(50, 42.5)
60	40	(60, 40)
70	37.5	(70, 37.5)

$m = \frac{-2.5}{10} = -.25$

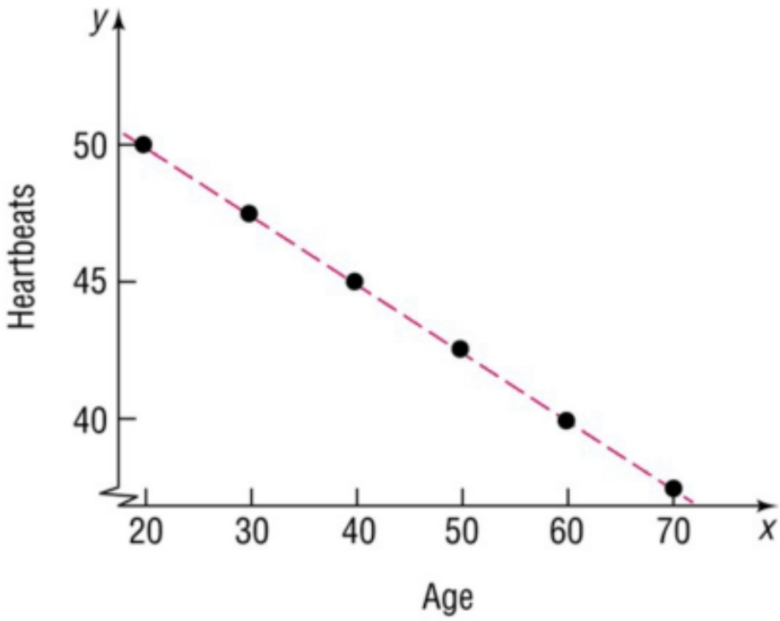
$m = -.25$

$m = -.25$

Solution 2 (b)

Part b: Figure 3

Linear



Example 3

Determining Whether a Linear Function Is Increasing, Decreasing, or Constant

Determine whether the following linear functions are increasing, decreasing, or constant.

(a) $f(x) = 5x - 2$ $(m > 0)$ increasing

(b) $g(x) = -2x + 8$ $(m < 0)$ decreasing

(c) $s(t) = \frac{3}{4}t - 4$ $(m > 0)$ increasing

(d) $h(z) = 7$ $(m = 0)$ constant

Definition 2

Modeling with a Linear Function

If the average rate of change of a function is a constant m, a linear function f can be used to model the relation between the two variables as follows:

$$f(x) = mx + b$$

where b is the value of f at 0; that is, $b = f(0)$.

Example 4

Straight-line Depreciation

Book value is the value of an asset that a company uses to create its balance sheet . Some companies depreciate assets using straight-line depreciation so that the value of the asset declines by a fixed amount each year. The amount of the decline depends on the useful life that the company assigns to the asset. Suppose a company just purchased a fleet of new cars for its sales force at a cost of $31,500 per car. The company chooses to depreciate each vehicle using the straight-line method over 7 years. This means that each car will depreciate by $\frac{\$31,500}{7} = \$4500$ per year. (slope)

Example 4 (cont.)

(a) Write a linear function that expresses the book value V of each car as a function of its age, x, in years.

$$V(x) = -4500x + 31500$$

(b) Graph the linear function. (next page)

(c) What is the book value of each car after 3 years?

$$V(3) = -4500(3) + 31500 = 18000$$

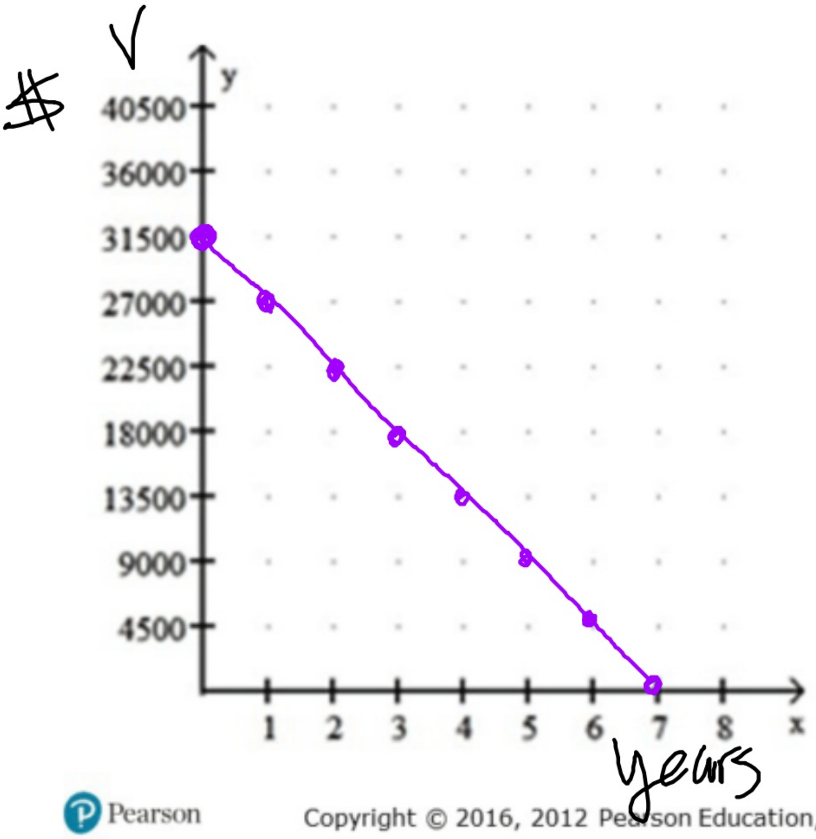
(d) Interpret the slope.

Every year the car depreciates \$4500.

(e) When will the book value of each car be \$9000?

$$9000 = -4500x + 31500$$
$$-22500 = -4500x \rightarrow x = 5 \text{ years}$$

Example 4 (cont.)



$V(x) = -4500x + 31500$

implied domain

D: $[0, 7]$

Example 5

Supply and Demand

The **quantity supplied** of a good is the amount of a product that a company is willing to make available for sale at a given price. The **quantity demanded** of a good is the amount of a product that consumers are willing to purchase at a given price. Suppose that the quantity supplied, *S*, and the quantity demanded, *D*, of cellular telephones each month are given by the following functions:

$$S(p) = 60p - 900$$
$$D(p) = -15p + 2850$$

where *p* is the price (in dollars) of the telephone.

Example 5

$$S(p) = 60p - 900$$
$$D(p) = -15p + 2850$$

The **equilibrium price** of a product is defined as the price at which quantity supplied equals quantity demanded. That is, the equilibrium price is the price at which $S(p) = D(p)$.

a) Find the equilibrium price of cellular telephones.

$$60p - 900 = -15p + 2850$$
$$75p = 3750$$
$$p = \$50$$

b) What is the **equilibrium quantity**, the amount demanded (or supplied) at the equilibrium price?

$D(50) = S(50)$ ... so $S(50) = 60(50) - 900$
$= 2100$ phones

Example 5 (cont.)

$$S(p) = 60p - 900$$
$$D(p) = -15p + 2850$$

c) Determine the prices for which quantity supplied is greater than quantity demanded. That is, solve the inequality ***S*(*p*) > *D*(*p*).**

$60p - 900 > -15p + 2850$

$p > 50$ (surplus)

d) Graph ***S* = *S*(*p*) and *D* = *D*(*p*),** and label the **equilibrium point**, the point of intersection of *S* and *D*. (next page)

Example 5 (cont.)

$$S(p) = 60p - 900$$
$$D(p) = -15p + 2850$$