4.4 Build Quadratic Models from Verbal Descriptions and from Data

Learning Objectives

1. Build Quadratic Models from Verbal Descriptions
2. Build Quadratic Models from Data

use the calculator (quad reg)

do these manually (algebraic)

Example 1 - Revenue

- You run the college bookstore. Your supplier for calculators will give you x calculators at price p based on the following: $x = 21000 - 150p$
- The revenue you make on selling calculators is the quantity sold multiplied by the price: $R = xp$ (# of calculators sold × price)
- Write a revenue function in terms of price only.

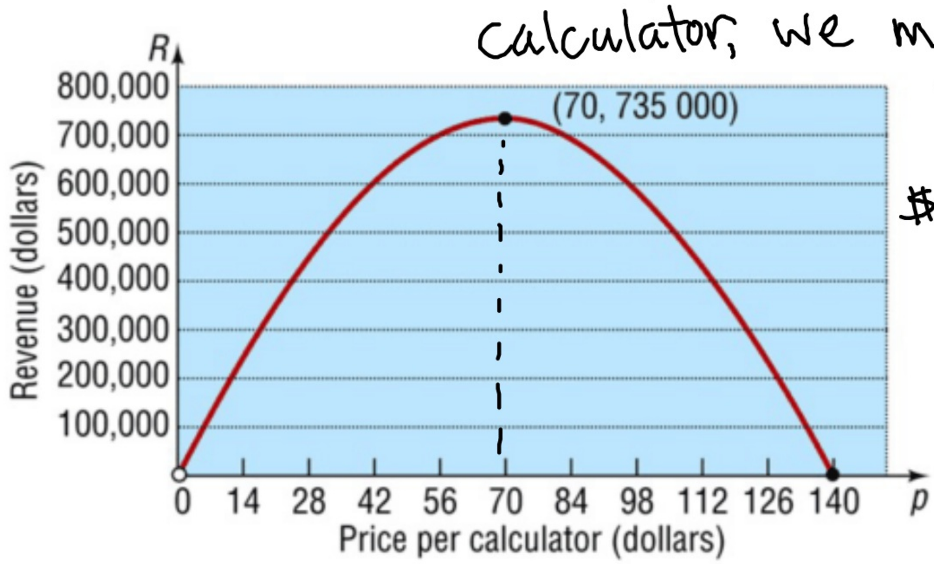
$$\begin{cases} R = x \cdot p \\ x = 21000 - 150p \end{cases}$$

$\rightarrow R = (21000 - 150p)\,p$

$$\boxed{R = 21000p - 150p^2}$$

Figure: $R(p) = -150p^2 + 21{,}000p$

At a price of \$70 for a calculator, we maximize revenue at \$735,000



Example 2

Maximizing the Area Enclosed by a Fence

A farmer has 1600 yards of fence to enclose a rectangular field. What are the dimensions of the rectangle that encloses the most area?

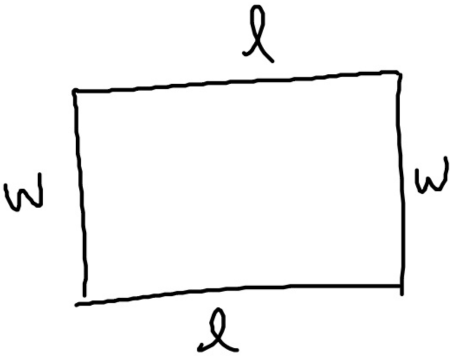
$A = lw$

$P = 2w + 2l$

$1600 = 2w + 2l$

$1600 - 2w = 2l$

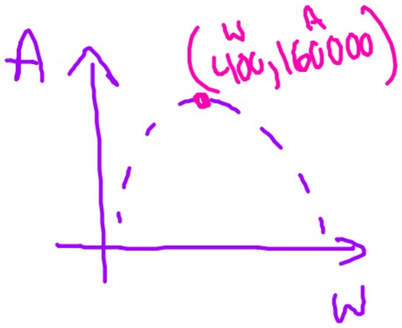
$800 - w = l$



$A = (800 - w)w$

$\boxed{A = 800w - w^2}$

$A = 800w - w^2$



$-\frac{b}{2a} = \frac{-800}{2(-1)} = 400$ (width)

$800(400) - (400)^2 = 160{,}000$ (area)

The dimensions that give us max. area are 400 ft x 400 ft.
(w) (l)

Example 2b

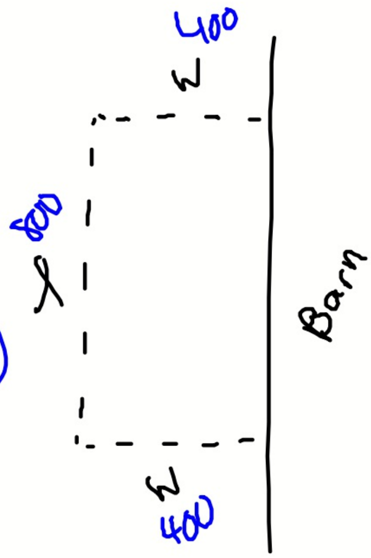
This time, the farmer has 1600 yards of fence, but he is going to enclose a rectangular area off the side of his barn, using the wall of the barn as one side of the enclosure. What are the dimensions that will enclose the maximum area?

$$A = lw \qquad 2w + l = 1600$$

$$l = 1600 - 2w$$

$$A = (1600 - 2w)w$$

$$\boxed{A = 1600w - 2w^2}$$



$$\frac{-b}{2a} = \frac{-1600}{2(-2)} = 400 \text{ (width)}$$

400 x 800

$a = \frac{-32}{(400)^2} = -2 E-4$
$= -2. \times 10^{-4}$
$= -.0002$

$b = 1$

$c = 500$

Example 3

Analyzing the Motion of a Projectile

A projectile is fired from a cliff 500 feet above the water at an inclination of 45° to the horizontal, with a muzzle velocity of 400 feet per second. From physics, the height *h* of the projectile above the water can be modeled by

$$\boxed{h(x) = \frac{-32x^2}{(400)^2} + x + 500} = -.0002x^2 + x + 500$$

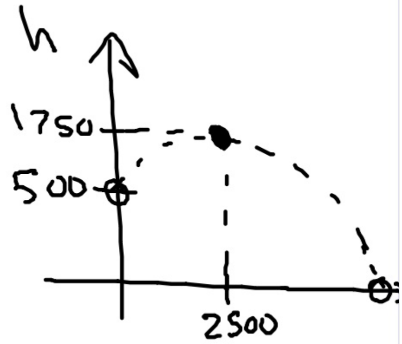
where *x* is the horizontal distance of the projectile from the base of the cliff. See Figure 27 on next slide.

(a) Find the maximum height of the projectile.

(b) How far from the base of the cliff will the projectile strike the water?

Pearson

$h(x) = -.0002x^2 + x + 500$



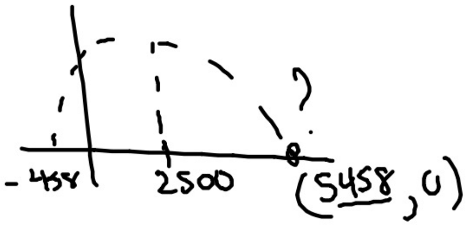
a) max. height? find the vertex!

$$\frac{-b}{2a} = \frac{-1}{2(-.0002)} = 2500$$

$$-.0002(2500)^2 + 2500 + 500 = 1750$$

height = 1750 ft

b) how far?



$$-.0002x^2 + x + 500 = 0$$

$$x = \frac{-1 \pm \sqrt{1 - 4(-.0002)(500)}}{2(-.0002)} = \frac{-1 \pm \sqrt{1.4}}{-.0004}$$

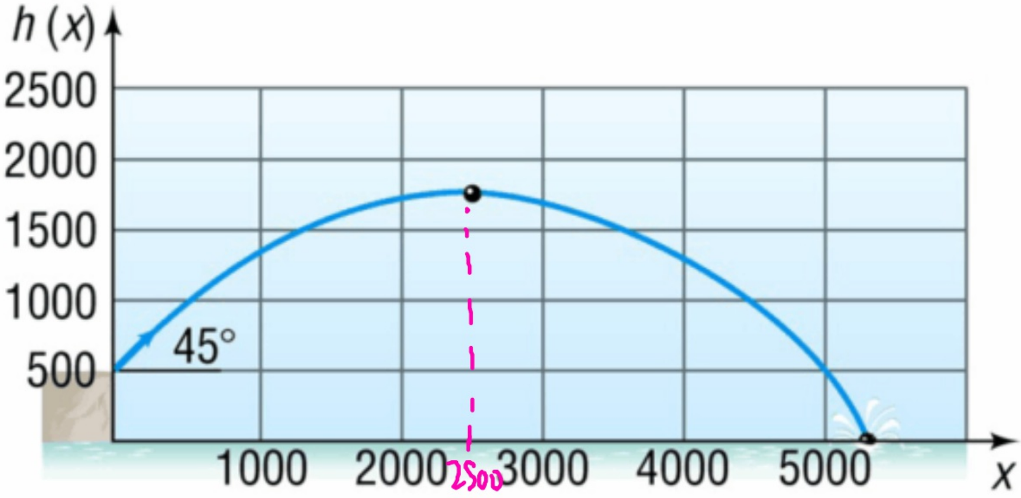
$$\approx \frac{-1 \pm 1.183216}{-.0004} \rightarrow \frac{-1 + 1.183216}{-.0004} \rightarrow -\cancel{458}$$

OR

$$\frac{-1 - 1.183216}{-.0004} \rightarrow 5458 \text{ ft.}$$

Example 3

Figure 27



Example 4 (1 of 2)

A shot-putter throws a ball at an inclination of 45° to the horizontal. The following data represent the height of the ball *h*, in feet, at the instant that it has traveled x feet horizontally.

Distance, x	Height, h
20	25
40	40
60	55
80	65
100	71
120	77
140	77
160	75
180	71
200	64

a) Use a graphing utility to draw a scatter plot of the data. Comment on the type of relation that may exist between the two variables.

Example 4 (2 of 2)

b) Use a graphing utility to find the quadratic function of best fit that models the relation between distance and height. Graph the quadratic function on the scatter plot.

$y = -.0037x^2 + 1.032x + 5.67$

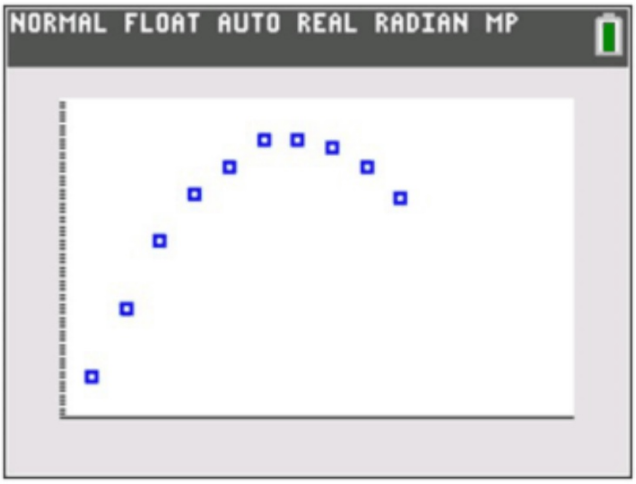
c) Use the function found in part (b) to find the horizontal distance the ball will travel before reaching the maximum height.

$\frac{-b}{2a} = \frac{-1.032}{2(-.0037)} = 139.46$

d) Find the maximum height. $-.0037(139.46)^2 + 1.032(139.46) + 5.67 = 77.63$

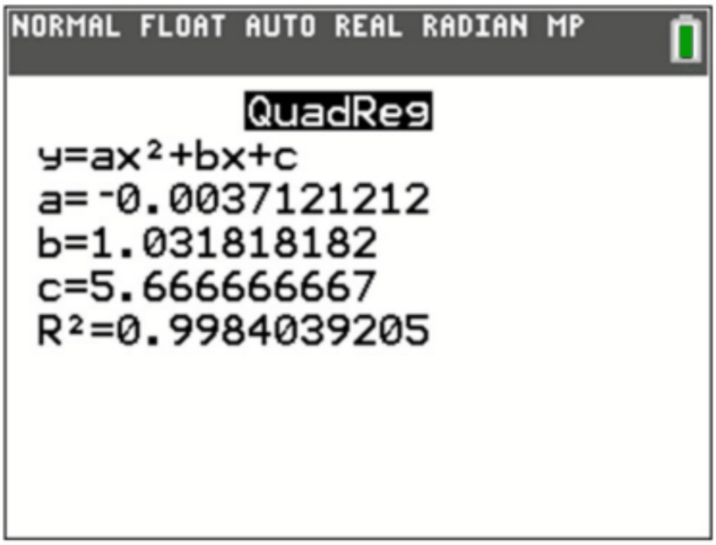
Solution 4 (3 of 5)

Part a:



Solution 4 (4 of 5)

Part b:



Solution 4 (5 of 5)

Part b: