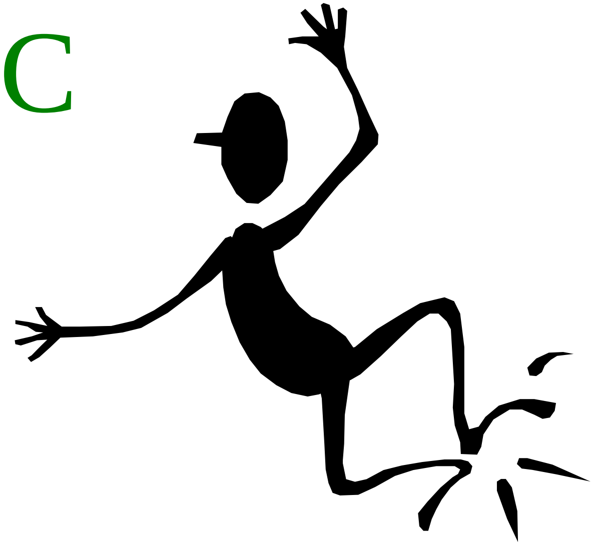
Beginning Algebra

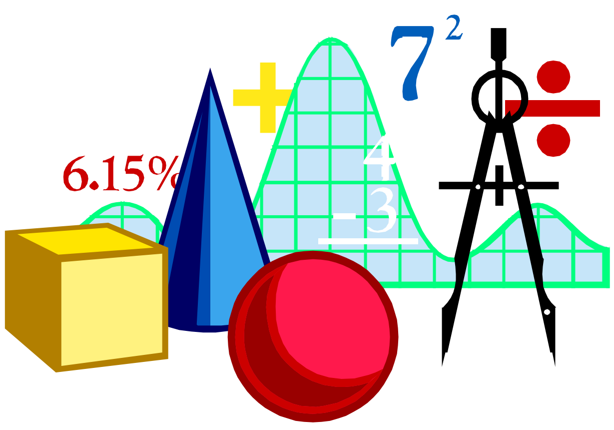
Professor Sikora

MAT0024C



CHAPTER 1

FOUNDATIONS OF ALGEBRA



1.1 **Variables**

VARIABLES = letters

= symbols

= represent possible number

What word do the **VARIABLES** spell?

m^2 4at -17h

CONSTANT = Symbol that does not vary in value. ex: 5, -2, 10, 200,…

1.1 **Expressions**

ALGEBRAIC EXPRESSIONS =

1 OR MORE: numbers

variables

arithmetic operators

$[+, - , \cdot, \div]$

Exs:

$3x - 1$, $\dfrac{w+z}{5}$, $4a + 7$, bc (factors of a product)

You <u>evaluate</u> an expression [when know values of the variables]

1.1 **Equations** have an = sign [the verb]

To **Solve** an eq. ➔ Find values of the variable [solutions] to make it true

Is 3 a solution of $2x^2 + 1 = 19$? Y N

From n = {1, 2, 3, 4}, find the solution for $n + 9 = 12$

1.1 **Inequalities** [the verb]

Math Relationship with the verb symbol ➔ **($\neq$, <, >, $\leq$, or $\geq$).**

Symbolic form	**Translation**
	Eight is not equal to three.
	Five is less than seven.
	Seven is greater than five.
	x is less than or equal to three.
	y is greater than or equal to two.

1.1 **<u>Set</u> = Collection of objects**

Objects are called elements or members.

Write the set containing the first four days of the week.

Answer:

{Sunday, Monday, Tuesday, Wednesday}

Write the set containing multiples of 3 to 15, inclusive.

Answer:

{3, 6, 9, 12, 15}

1.1 **Rational #**

Rational number: Any real number that can be expressed in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

Ex: $\frac{3}{4}$ -3

$.67$ $.153$

$.\overline{3}$ $.5$

Rules for Rationals:

Fractions

Terminating Decimals

Non-terminating, repeating decimals

1.1 **Irrational #** = not rational

Any real number that cannot be expressed in the form $\dfrac{a}{b}$, where a and b are integers and $b \neq 0$.

Ex: $\sqrt{3}$

$\sqrt{5}$

π

Rules for Irrationals:

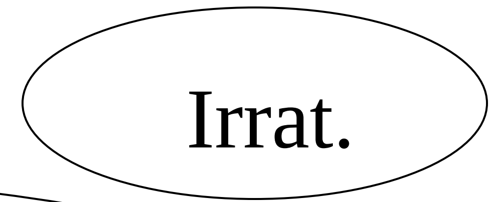
Non-terminating, non-repeating decimals

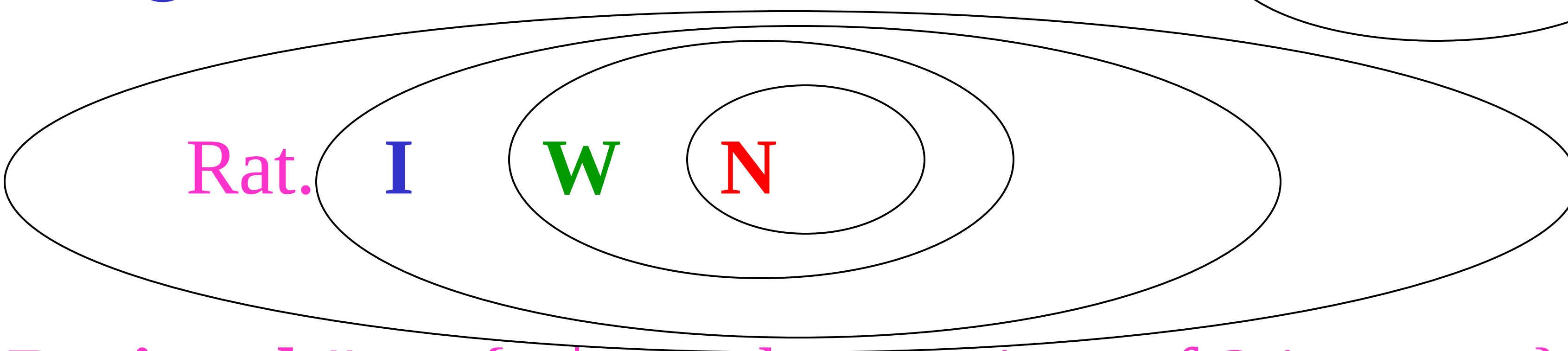
1.1 **Real #'s** categorized

Natural ==>{1, 2, 3, 4, ...} ie. Counting #"s

Whole #s ==> {0, 1, 2, 3, ...}

Integers = {...-2, -1, 0, 1, 2, 3, ...}





Rational #s = {x | x is the quotient of 2 integers}

Irrational #s = {x | x is <u>not</u> the quotient of 2 integers} Ex. $\sqrt{2}$, $\prod$

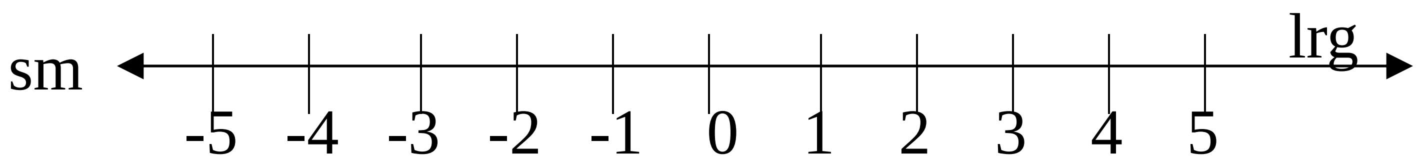
REAL #S = {x | x is rational or irrational #}

1.1 **<u>Real #’s</u>** have positions on # line

Graph on a number line:

Natural #s

Whole #s



Integers

Rational #s – fractions [proper, improper, mixed, terminating decimals, repeating decimals]

Irrational #s – non-terminating, non-repeating decimals

{see Venn Diagram on inside cover of book & page 5}

Be able to categorize any given real #

1.1 **Absolute Value**

Abs. Value of a # is it's distance [pos.] from 0 |5| = 5 and |-5| = 5

Distances [and thus ABSOLUTE VALUES] are always **positive!**

$$|x| = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$$

Simplify: |-23| =

|23| =

- |-23| =

$|-\sqrt{3}| =$

|-4.8| =

1.1 Compare #s on number line

a > b if a to right of b on number line

b < a if b to left of a on number line

Use =, < , or > in each box below:

-15	$\square$	-20
-4.1	$\square$	0
$2\frac{5}{6}$	$\square$	$2\frac{1}{4}$
$\lvert -.95 \rvert$	$\square$	9.5

1.2 **Fractions**

$$\frac{numerator}{denominator} \leftarrow fraction\_bar$$

denominator $\neq 0$ and if so, fraction **undefined**

Mult. fractions: $\dfrac{a}{b} \bullet \dfrac{c}{d} = \dfrac{ac}{bd}$ b & d $\neq 0$ Exs:

• reciprocal of that # = 1 [KEEP – CHANGE – FLIP]

Divide fractions: $\dfrac{a}{b} \div \dfrac{c}{d} = \dfrac{a}{b} \bullet \dfrac{d}{c}$ Exs:

reciprocals

1.2 Build/Simplify Fractions

2 fractions **Equivalent** if they represent the **same #**

Build a fraction ➔ Mult. by a form of 1

Mult. Prop of 1: $1 \cdot a = a$ & $a \cdot 1 = a$ $\quad a \in$ Reals

Exs. Write $\frac{5}{8}$ as an equivalent fraction $\frac{\quad}{24}$

Simplest form of a fraction [lowest terms]

Remove a factor of 1 Ex: Simplify $\frac{63}{42}$

1.2 **+ or – Fractions** often need an ***LCD***

- ***Least common multiple (LCM):*** The smallest number that is a multiple of each number in a given set of numbers.

  Ex: LCM of 2 and 3 is ____

  Ex: LCM of 3 and 4 is ____

- ***Least common denominator (LCD):*** The least common multiple of the denominators of a given set of fractions.

1.2 **+ or - Fractions**

A) **Same** Denominator

$$\frac{a}{d}+\frac{b}{d}=\frac{a+b}{d}$$ and $$\frac{a}{d}-\frac{b}{d}=\frac{a-b}{d}$$ Exs:

B) **Unlike** Denominators

Find LCD = smallest # each denom ÷s into evenly

1)Factor each denom

2)Take each factor that appears to its **highest** pwr

3) Mult these factors for LCD

Ex: $$\frac{3}{4}-\frac{2}{5}=$$

1.2 Fractions use factors & factorization

In multiplying: given factors → find the product [ex: 2 • 5 • 7 = 70]

In factoring: given product → find the factors [ex: 70 = 2 • 5 • 7]

A # w/ exactly 2 factors [the # itself & 1] = **PRIME number** (whole #s > 1)

Composite number (whole #s > 1) = not prime #s

Prime Factorization = when whole # expressed as product of prime factors

1.2 Fractions use factors & factorization

Find the PRIME FACTORIZATION of:

200

2 **100**

50 **2**

10 **5**

2 **5**

$\boxed{2^3 \cdot 5^2}$

140

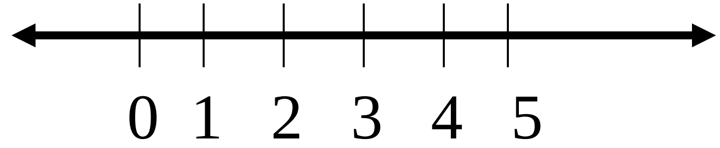
-420

1.3 **Properties of Addition** $[a,b,c \in R]$

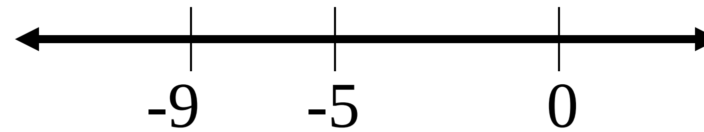
Property	***Addition***
Commutative	$\mathbf{a + b = b + a}$
Associative	$\mathbf{(a + b) + c = a + (b + c)}$
Additive Identity	$\mathbf{a + 0 = a}$ **[0=Identity Elem for +]**
Additive Inverse	$\mathbf{a + (-a) = 0}$

Be able to identify these properties when used!

1.3 **Add Real #s**



- Methods:
  - Using arrows on the Number Line
  - Applying RULES:



Add Integers [SAME SIGN]

* Add their Absolute Values

* Ans. has SAME sign

Ex: $2 + 3 =$

Ex: $-5 + (-4) =$

1.3 **Add Real #s**

Add Real #s [**SAME** SIGN]   *Review*

* Add their Absolute Values

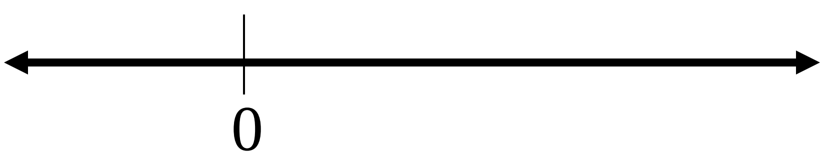
* Ans. has SAME sign Ex: -2 + (-5) = -7

Add Real #s [**DIFFERENT** SIGNS]

* Subtract their Absolute Values [big - sm.]

* Ans. has sign of larger Absolute Value

Ex: 4 + (-5) = -1   Ex: -14 + 5 = -9



1.3 **Add Several Real #s**

$(+9) + (+3) + (-7) =$

$-16 + (-20) + 5 + 11 =$

$-7 + 13 + (-5) + 10 =$

1.3 **Subtract Real #s**

To SUBTRACT a number, add its Additive Inverse:

$$\boxed{\mathbf{a - b = a + (-b)}}$$

Use all those addition rules!

Ex: $-8 - (-12) =$

Ex: $6 + [(-1 - 4) - 2\ ] =$

Ex: $-(-14) - |-6| =$

MQ **1.1 ➔1.3**

1. What does an algebraic equation have that an algebra expression doesn’t? Which one can be solved?
2. Translate: w is less than or equal to 7
3. Give the Prime Factorization of 270 using exponents
4. What do we call a number that is the quotient of 2 integers?
5. Write the set of natural #s less than 6
6. -|-52| = ____
7. Build an equivalent fraction to $\frac{9}{16} =$ w/ denom. of 64

8, $\frac{23}{25} \div \frac{46}{5} =$

9. $8\frac{2}{9} - 7\frac{2}{3} =$

10. Insert symbol: $\left|-2\frac{2}{3}\right| \_\_\_ \frac{7}{3}$

1.4 **Properties of Multiplication** [a,b,c $\in$ R]

Property	*Multiplication*
Commutative	**$ab = ba$**
Associative	**$(ab)c = a(bc)$**
Mult. Prop of zero	**$a \cdot 0 = 0$**
Mult. Identity	**$a \cdot 1 = a$** **[1=Identity Elem for $\cdot$]**
Distributive	**$a(b + c) = ab + ac$**

Be able to identify these properties when used!

1.4 **Multiplying Real #s**

Rules for products of signed #s:

Signs same

$+ \cdot + = +$ **and** $- \cdot - = +$

Signs different

& $+ \cdot - = -$ **and** $- \cdot + = -$

Ex: $(-3)(-5) =$

Ex: $(-0.4)(2) =$

Ex: $-\frac{5}{8} \bullet \frac{16}{25} =$

1.4 **Multiplying & Dividing Real #s**

* Multiplication & Division are Inverse Operations. Thus:

Signs same

If	$+ \cdot + = +$ **and** $- \cdot - = +$
&	$+ \cdot - = -$ **and** $- \cdot + = -$
Then	$+ \div + = +$ **and** $- \div - = +$
	$+ \div - = -$ **and** $- \div + = -$

Signs different

Ex: $\frac{-15}{-3} = 5$ ‘cuz 5(-3) = -15

1.4 **Divide Real #s**

The **Difference** of 2 numbers → add its Additive Inverse:

$$a - b = a + (-b)$$

The **Quotient** of 2 numbers → mult. by its Reciprocal or Multiplicative Inverse:

$$\frac{x}{y} = x \bullet \underbrace{\frac{1}{y}}_{\text{Product} = 1} \qquad \boxed{y \neq 0}$$

Dividing by 0 is <u>undefined</u>

Ex. $\frac{81}{-9}$ Ex. $\frac{0}{9}$

1.5 **Exponents**

x^n --> x to the n^{th} power

exponent

base

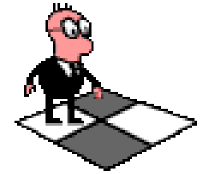
POWER ⟶ Repeated Multiples

$x \cdot x \cdot x \cdot x \ldots\ x$ --> n of these

Ex: $4^3 = 4 \cdot 4 \cdot 4 =$ __ $(-4)^3 = -4 \cdot -4 \cdot -4 =$ __

$-\left(\frac{1}{2}\right)^4 =$ ___ $\left(-\frac{1}{2}\right)^4 =$ ___

1.5 Square Roots



<u>Squares</u>: the square of 5 is 25 'cuz $5^2 = 25$

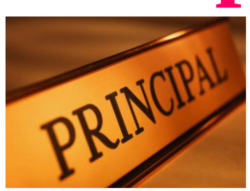
and the square of -5 is 25 'cuz $(-5)^2 = 25$

<u>Square Roots</u>: 5 is the square root of 25 'cuz $5^2 = 25$

and -5 is the square root of 25 'cuz $(-5)^2 = 25$

$\boxed{\text{b is square root of a if } b^2 = a}$

All positive #s have 2 sq. roots. It's <u>pos.</u> sq. root = **principal square root**



1.5 Square Roots

Square Root of a => $\pm\sqrt{a}$ **[a = positive real #]**

Note: $\sqrt{a}$ = represents the POSITIVE sq. root of a

Ex:

$$\sqrt{\frac{9}{16}} =$$

1.5 **Order of Operations**

PLEASE ( ), [ ], { }, | |, $\sqrt{}$

EXCUSE exponents or roots

MY multiplication DEAR division

AUNT add SALLY subtraction

LEFT → RIGHT

1.5 **Order of Operations**

Ex $28 - 36 \quad 9(-5) =$

Ex $6|-5-4| + 2(-3)^3 =$

P

E

M D

A S

1.5 **Order of Operations**

Ex: $88 - 2[7^2 - (12 + 8) \quad 4] =$

Ex: $-\frac{3}{5} \div \frac{1}{10} \bullet 4 + \sqrt{64 + 36}$

1.5 **Grouping Symbols**

- Nested Parentheses: $2[5+3(4-1)]$
  simplify inside → outside*
- Fraction Bar: simplify numerator * & denominator * then ÷ $\dfrac{2(7+8)+2}{3 \bullet 5+1}$
- Absolute Value Bars: work inside 1st
  $10^3 + 3\,|24 - 25\,|$

* Using Order of Operations

1.5 **Arithmetic Mean** [Average]

Mean of a set of values ➔ ÷ their sum by the # of values

average

Ex: Bruce has the following test scores in his biology class: 92, 96, 81, 89, 95, 93. Find the average of his test scores.

Ans:

$$\frac{92+96+81+89+95+93}{6} = \frac{546}{6}$$

$$= 91$$

1.6 **Translating Basic Phrases**

Fill in chart

Addition	**Translatn**	**Subtraction**	**Translation**
The sum of x and 3		The difference of x and 3	
h plus k		h minus k	
7 added to t		7 subtracted from t	
3 more than a number		3 less than a number	
y increased by 2		y decreased by 2	

1.6 **Translating Basic Phrases**

Fill in chart

Multiplicatn	**Translation**	**Division**	**Translation**
The product of *x* and 3		The quotient of *x* and 3	
h times *k*		*h* divided by *k*	
Twice a number n		*h* divided into *k*	
Triple the number n		The ratio of *a* to *b*	
Two-thirds of a number n			

1.6 **Translating Basic Phrases**

Fill in chart

Exponents	**Translatn**	**Roots**	**Translation**
c squared		The square root of *x*	
The square of *b*			
k cubed			
The cube of *b*			
n to the fourth power			
y raised to the fifth power			

MQ **1.4 ➔ 1.6**

1-4 State the property: **1)** -20 + (4 + 5) = -20 + (5 + 4)

2) -20•(4•5)=(-20•4)•5 **3)** a•1 = a **4)** 4(x+2)=4x+8

5-7) Add or Subtract **5)** $-\frac{3}{8}+\left(-\frac{1}{3}\right)$ **6)** –3 + 6 + (-9) + (-6)

7) –4 + 5 – (-3) – 13

8-10) Mult. or Divide **8)** $-2\frac{6}{25}\div\frac{4}{5}=$ **9)** $\left(-\frac{5}{6}\right)\left(-\frac{2}{15}\right)$

10) Simplify

$$(-3)^2+5\left[6-(2+1)\right]-\sqrt{49}$$

1.7 **Evaluating Algebraic Expressions**

Find the value of $2p^3$ if $p = 3$

Find the value of $\dfrac{4x - 2y}{x + 1}$

if $x = 6$ & $y = 9$

Find the value of 1) $3m^2$ 2) $(3m)^2$ if $m = 2$

Evaluate $m^3 – 6n^2$ when $m = -2$ and $d = -5$

1.7 Values causing undefined expressions

Dividing by 0 is undefined

Ex: $$\frac{8}{x+6}$$

Ex: $$\frac{2m}{(m-3)(m+4)}$$

1.7 **Distributive Prop.** for exp. rewrite

Distributive $a(b + c) = ab + ac$ **[also subtraction]**

Ex: $3(x + 12) =$

Ex: $-3(x - 10) =$

Ex: $\frac{4}{5}\left(\frac{1}{2}y - 10\right) =$

1.7 **Expressions**

Terms separated by + or - sign

Product or quotient of #s and/or variables

The **coefficient** is the numerical factor of a term ~ Identify it in these exs:

$8y^3 - 12y^2 + 3y - 4$

$\frac{3}{5}x$

$-\frac{y}{5}$

1.7 **Combine Like Terms** for exp. rewrite

Use properties Ex: **-** (7 – 6k) + 9 =

Be sure to arrow –1 thru parentheses

Terms of an Alg. Expression: separated by + or - sign

Combine **like terms** [same variables to same pwr]

Ex: 5(2a – 6) –3(4a – 9) =

Ex: $\frac{3}{8}y - x + 2 - \frac{3}{4}y + 5x$ =

MQ 1.7 & Review

Evaluate: 1) $50 - 2(5) - 7$ 2) $(3 \bullet 4)^2 - 4$

3) $9 \bullet 5 - 6 \quad 3$ 4) $(2^3 - 14 \quad 7 \bullet 2) - 2 \bullet 2 + 1$

5) $$\frac{4|9-7|+|-7|}{3^2-2^2}$$

6) Translate to algebraic expression: The difference between 2 times a number (x) and 4

7) Evaluate: $3x^2 - \dfrac{x}{2}$ for $x = -2$ 8) When undefined? $$\frac{3}{(x+3)(x-1)}$$

9) Translate to algebraic expression: The absolute value of the quotient of a and two

10) Combine Like Terms: $$\frac{1}{12}a + 4b + 3 + \frac{1}{6}a - b.$$