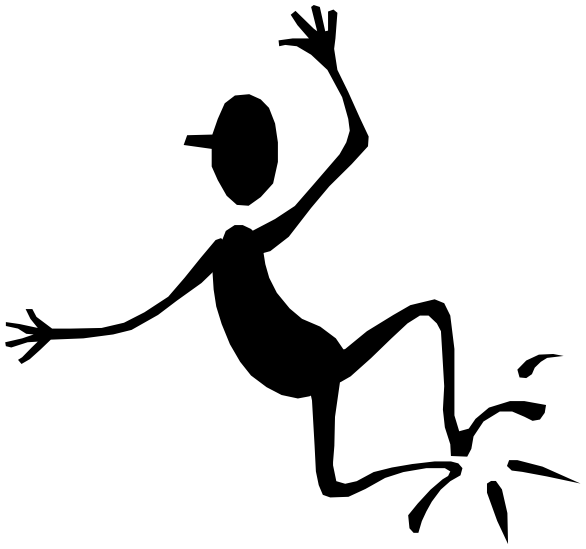
Beginning Algebra

Professor Sikora



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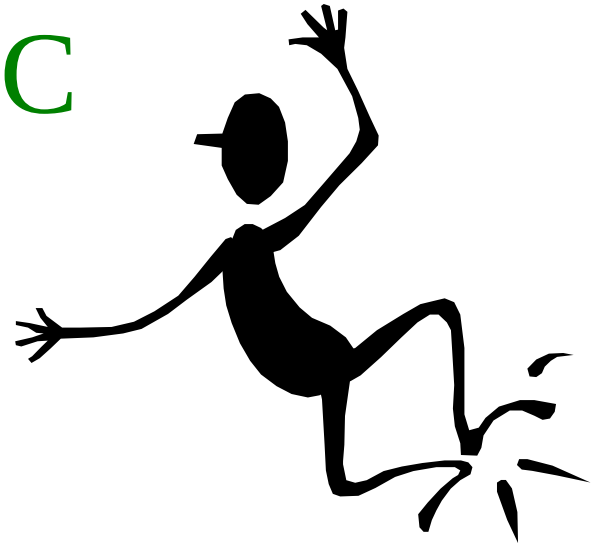
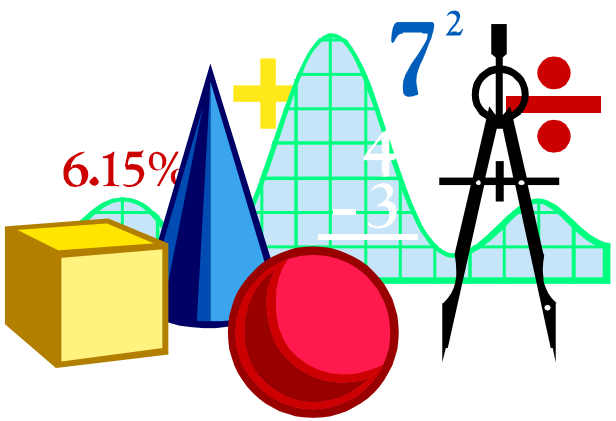


CHAPTER 2



SOLVING EQUATIONS & INEQUALITIES

Professor M. J. Sikora ~ Valencia Community College

2.1 **Solving Equations** [ie math sentences]

Solutions or roots = #s that when substituted for the variable(s), satisfies the equation ➔ makes it true

Ex: Is 25 a solution of $2(46 - x) = 41$? Y N

Ex: Is -2 a solution of $8x + 18 = 2$? Y N

To solve an eq. ➔ find all variable values that make eq. true

2.1 **Identity Eq.** = every real # is solution

To determine if an equation is an identity:

1) Simplify expressions on each side of = sign.

2) If, after simplifying, the expressions are identical, then the equation is an identity.

Ex: $2(3x - 4) - 10x = 15 - 4(x + 2)$ Identity? Y or N

Ex: $0.5(3y - 8) = y - 4 + 0.5y$ Identity? Y or N

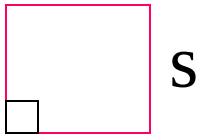
2.1 **Formulas from Geometry**

Perimeter = distance around

Area = surface enclosed

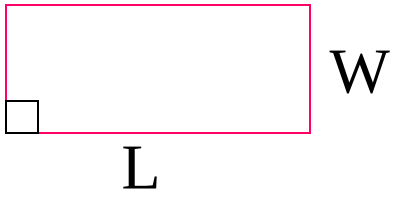
$P_{sq} = 4s$

$A_{sq} = s^2$



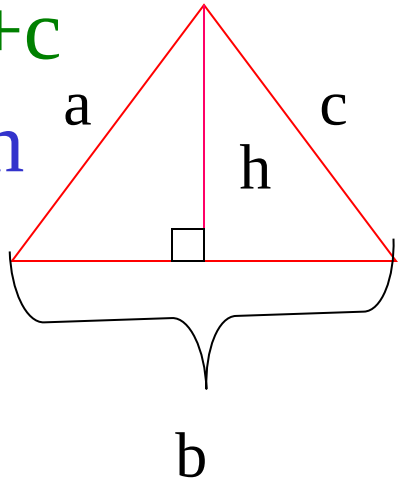
$P_{rect} = 2L + 2W$

$A_{rect} = LW$



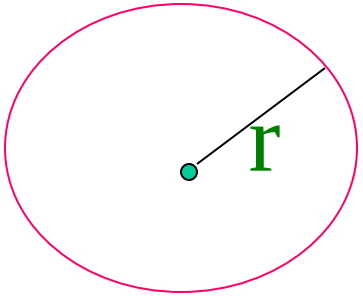
$P_{\Delta} = a+b+c$

$A_{\Delta} = \frac{1}{2}bh$



$C_{circle} = 2\pi r$

$A_{circle} = \pi r^2$



Memorize these! Other figures will have formulas given to you.

2.1 **Formulas=a relationship betw variables**

Sale price* = orig price – discount **s = p - d**

[x = orig. $. If discounted 25%, subtract .25x for sale $]

Interest* = principal • rate • time **I = Prt**

Distance* = rate • time **d = rt** [units same]

* Must memorize ‘cuz on competency test

2.1 **Problem Solving Technique** for word problems

Steps: **1)** Chose a variable for what you are to find [Write: Let x = _____] Write out facts [w/ your var.], pictures,....

2) Translate prob. to an equation [is ➔ =, etc.]

3) Solve equation [legally w/ properties]

4) Answer question(s) asked [may be more than what x equals]

5) Check [Sub. solution into original and work down 'til both sides of = sign the same. Then √]

2.1 Problems using **Formulas**

A 13’ x 20’ room needs crown molding where the walls & ceiling meet. a) Find the total molding length needed b) It comes in 12’ strips. How many strips needed? c) Strips cost $9 ea. Find total $.

a)

b)

c)

2.2 **Addition Principle of Equality**

Linear Eq. ➔ variables raised to power of 1 only

ADDITION Principle of equality: a,b,c∈REALS

Whatever you do to one side, you must do to the other!

If $a = b$

Then $a + c = b + c$

EQUIVALENT EQS. = same solution

Exs: Solve & check [by substituting ans. back into original and work down both sides until same]

$w - 15 = 45$ $\qquad\qquad$ $75 = b - 38$

2.2 **Addition Principle of Equality**

Used to SUBTRACT quantity from both sides: $a,b,c \in R$

Whatever you do to one side, you must do to the other!

If $a = b$

Then $a - c = b - c$

} EQUIVALENT EQS. = same solution

Exs: Solve & check [by substituting ans. back into original and work down both sides until same]

$x + 12 = 20$ $15 + y = 24$ $-20 + n = 29$

2.2 **Addition Principle of Equality**

Solve and Check:

Ex: $n - 4n + 8 = 6 n - 3 - 8n$

Ans: $10 = n$

2.2 **SOLVING EQS. with INFINITE # of Solutions**

Ex: $3y - y + 10 = 2y - 4 + 14$

Eqs. true for all values of variables = IDENTITY

2.2 **SOLVING EQS. with NO Solutions**

Ex: $3w + 8 = 6(w - 1) - 3w$

Final Eq.	# of Solutns.	Solutn Set	called
x=a # ex:x=2			solved
True ex: 0=0			identity
False ex:5=8			contradiction

2.2 **Addition Principle of Equality**

Ex: Raul wants to buy a membership to a gym which costs $395 for the year. He currently has saved $149. How much more does he need?

Let x = the amount Raul needs.

x = $246

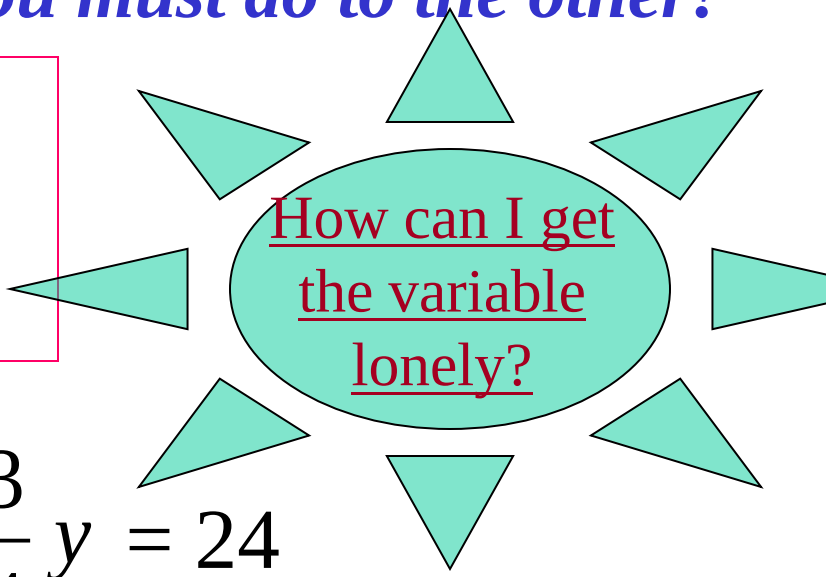
2.3 **Multiplication Principle of Equality**

MULT. Principle of equality: a,b,c∈REALS

Whatever you do to one side, you must do to the other!

If $a = b$

Then $ca = cb$ $(c \neq 0)$



Exs: Solve & check

$\frac{x}{4} = 5$ $\frac{s}{14} = 2$ $\frac{3}{4}y = 24$

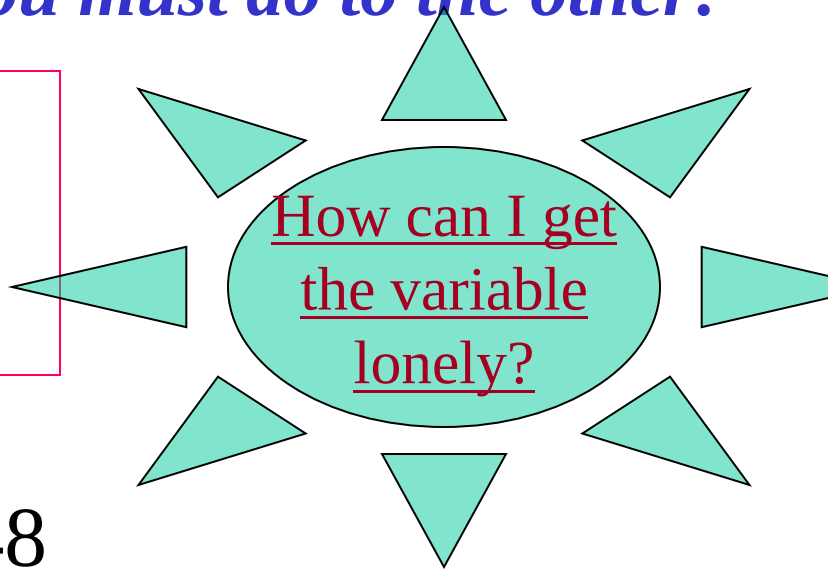
2.3 **Multiplication Principle of Equality**

Used to DIVIDE each side by same quantity : $a,b,c \in REALS$

Whatever you do to one side, you must do to the other!

If $a = b$

Then $\frac{a}{c} = \frac{b}{c}$ **$(c \neq 0)$**



Exs: Solve & check

$2w = 40$

$12k = 48$

2.3 Steps for **Solving Equations**

4 Steps to solve a Linear Equation: $ax + b = c$; $a,b,c \in R$, $a \neq 0$

1) Simplify each side separately [clear ( ) with Distrib, combine Like terms]
2) Isolate variable term on 1 side [use addition or subtraction prop. so var. term on 1 side & # term on other] **Tip:** Clear the variable term that has the lesser coefficient to avoid negative coefficients.
3) Isolate variable [use mult. or division prop. so variable term = #] Box ans.
4) Check [substitute solution into original to see if a true statement results √]

2.3 **Multiplication Principle of Equality**

Ex: Solve and check.

$$-\frac{5}{4}x = \frac{3}{8}$$

$$x = -\frac{3}{10}$$

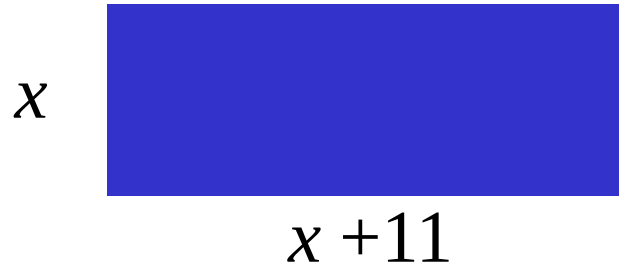
Ex: Solve and check.

$$2-5\ \ y+5\ \ =3\ \ y-2\ \ -1$$

$$y = -2$$

2.3 **Multiplication Principle of Equality**

Ex: The perimeter of the figure shown is 72 inches. Find the width and the length.



x = 12.5

2.3 **Multiplication Principle of Equality**

Ex: The Smith family is planning a 510-mile trip. If their average speed is 68 MPH, how long will it take them to complete the trip?

x = 7.5 hrs

Mini-Quiz 2.1 ➔2.3 ~ SHOW ALL WORK on bottom or back of strip

1) Solve: $2(x - 4) + x = 7$ 2) Ck #1)

3) Solve: $\dfrac{40 - 8w}{5} = -2w$ 4) Ck #3)

5) Solve: $4(y - 3) - y = 3(y - 4)$

6) Solve: $-3(v + 2) = -2v - 8 - v$

7) Check if $\dfrac{2}{3}$ is a solution of: $6 - (3x + 2) = 6x - 4$

8) In Solving: $\frac{2}{3}x = 8$ mult. each side by _____

9) Solve & ck: $\frac{2}{3}x = 8$

10) If an equation has fractions, multiply thru by their ___

2.4 **Solving Formulas – for a variable**

Solve $A = \frac{1}{2}bh$ for h

Solve $p = r - c$ for r

Solve $P = 2L + 2W$ for W

Solve $jm + c = n$ for m

2.5 **Translating Basic Phrases**

Fill in chart

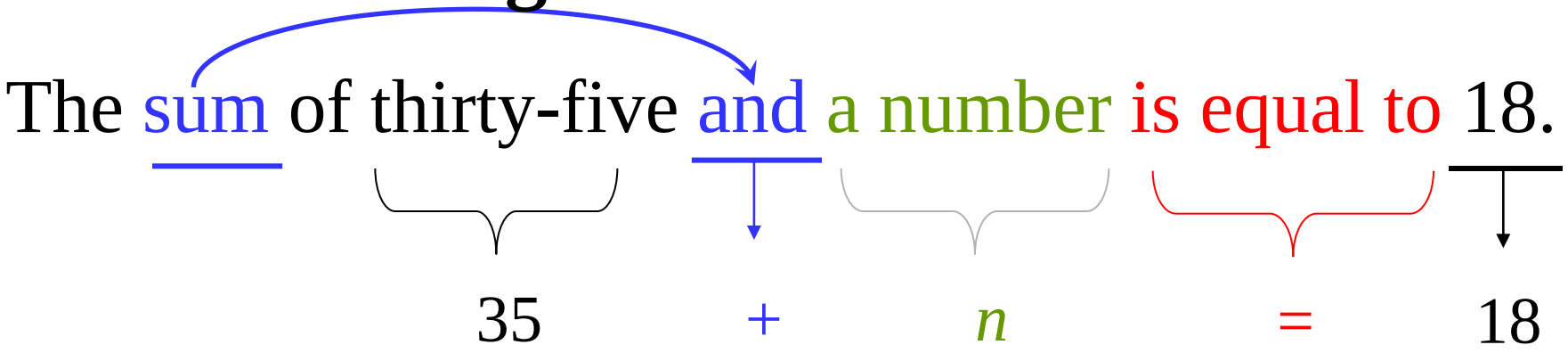
Addition	**Translatn**	**Subtraction**	**Translation**
The sum of x and 3		The difference of x and 3	
h plus k		h minus k	
7 added to t		7 subtracted from t	
3 more than a number		3 less than a number	
y increased by 2		y decreased by 2	

2.5 **Translating Basic Phrases**

Fill in chart

Multiplicatn	**Translation**	**Division**	**Translation**
The product of x and 3		The quotient of x and 3	
h times k		h divided by k	
Twice a number n		h divided into k	
Triple the number n		The ratio of a to b	
Two-thirds of a number n			

2.5 **Translating Basic Phrases**



Answer: n = -17

2.5 **Translating Basic Phrases**

Eight less than five times a number is equal to thirty-seven.

Answer: $n = 9$

2.5 **Translating Basic Phrases**

Ex: Nine times the sum of a number and seven subtracted from three times the number results in negative twenty-seven.

Answer $x = -6$

Mini-Quiz 2.4 ➔2.5 ~ SHOW ALL WORK on bottom or back of strip

1) Solve $A = P + Prt$, for t 2) Solve for x: $y = mx + b$

3) Solve for h: $V = \pi r^2 h$

4) Translate to an eq.: Four less than three times a # is 5 5) Solve eq. in 4)

6) Translate to an eq.: Three times the difference of a # and 5 equals twice the # and 3 7) Solve eq. in 6)

8) Solve: $2 - (3x + 4) - (10 - 3x) = 5x - 2$ 9) Ck 8)

10) Solve: $\frac{w}{3} - 5 = \frac{w}{5} - 3$

2.6 **Linear Inequality** ~ **Set-builder notation**

Linear inequality: An inequality containing expressions in which each variable term contains a single variable with an exponent of 1.

Examples of linear inequalities:

$x > 5$ $\quad$ $n + 2 < 6$ $\quad$ $2(y - 3) \leq 5y - 9$

Set builder notation

$$\{ x \mid x \geq 5 \}$$

The set of all x such that x is greater than or equal to 5.

2.6 **Linear Inequality** ~ **Interval Notation**

[#,___ or ___ ,#**]**: Graph **Includes** the associated point

(#,___ or ___ ,#**)**: **Do not Include** the associated point

($-\infty$, ___ Begin with lowest unbounded neg. #

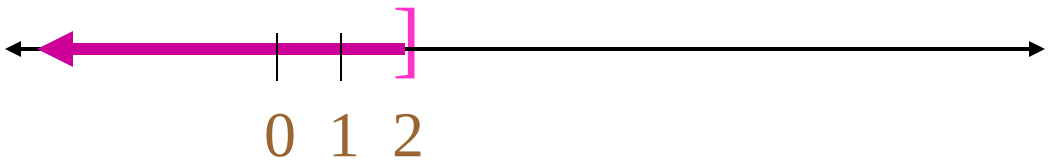
___ , ∞ **)** End with highest unbounded pos. #

Graph these solutions & use Interval Notation to describe:

$x > 4$ $x \leq -2$ $-1 \leq x < 3$

2.6 **Addition Property of Inequality**

$x \leq 2$ Graph:



$x > -2$ Graph:

$-1 < x \leq 2$:

= Interval

ADDITION PROP. OF INEQUALITY: A,B,C ∈ Reals:

A < B

A + C < B + C

EQUIVALENT EQS. [also > , ≤, and ≥]

Also for subtraction

Solve & Graph: $-1 + 8r < 7r + 2$

$4 < x + 2 < 10$

2.6 **Mult. Property of Inequality**

MULTIP. PROP. OF INEQUALITY: A,B,C $\in$ Reals, C $\neq$ 0:

If C is positive **A < B**

AC < B C

If C is negative **A < B**

AC > B C ***[reverse inequality]***

Solve & Graph: $-2r > -12$

Solve & Graph: $36 < -9y$

2.6 Solving Linear Inequalities

1. Simplify both sides of the inequality as needed.
   a. Distribute to clear parentheses.
   b. Clear fractions or decimals by multiplying through by the LCD just as we did for equations.
   c. Combine like terms.
2. Legally get all variable terms are on one side of the inequality and all constants are on the other
3. Use the multiplication principle to clear any remaining coefficient. **If you multiply (or divide) both sides by a negative number, reverse the direction of the inequality symbol!**

2.6 **Solving Linear Inequalities**

Solve $8x + 13 > 3x - 12$.

Set builder notation: $\{x|x > -5\}$

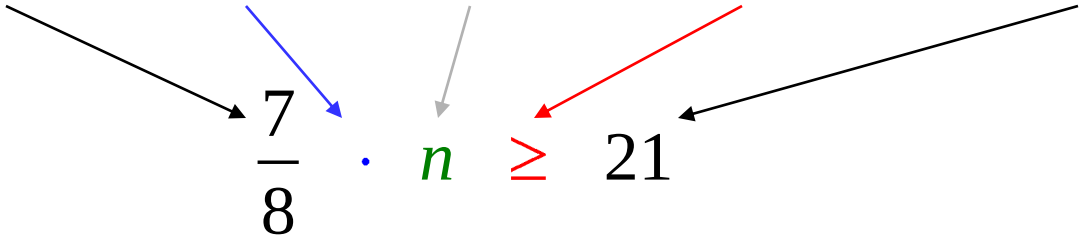
Interval notation: $(-5, \infty)$

2.6 Translations for Linear Inequalities

Less Than:		**Greater Than:**	
A number is less than 7.	$n < 7$	A number is greater than 2.	$n > 2$
A number must be smaller than 5.	$n < 5$	A number must be greater than 3.	$n > 3$
		A number must be more than –6.	$n > -6$
Less Than or Equal to:		**Greater Than or Equal to:**	
A number is at most 9.	$n \leq 9$	A number is at least 2.	$n \geq 2$
The maximum is 14.	$n \leq 14$	The minimum is 18.	$n \geq 18$

2.6 Translations for Linear Inequalities

Ex: Seven-eighths of a number is at least twenty-one.



$$\frac{7}{8} \cdot n \geq 21$$

$$n \geq 24$$

Mini-Quiz 2.6 & Rev ~ SHOW ALL WORK on bottom or back of strip

1) Solve: $6x + 12 > -12$

2) Write solution for 1) in set-builder notation

3) Graph solution in 1) & write solution in Interval Notation

4) Solve: $6(y + 1) \leq 4 - 8y + 3(5y - 1)$

5) Write solution for 4) in set-builder notation

6) Graph solution in 4) & write solution in Interval Notation

7) Solve for C: $F = \frac{9}{5}C + 32$

8) Solve: $-\frac{1}{2}(6c - 3) = \frac{1}{3}c - 1$

9) Solve: $5(b - 2) \geq -(b - 3) + 2b$

10) Will your Study Sheet for the Ch 2 Test include all of the important facts is RED, explanations in BLUE & examples in PENCIL?