Beginning Algebra

Professor Sikora
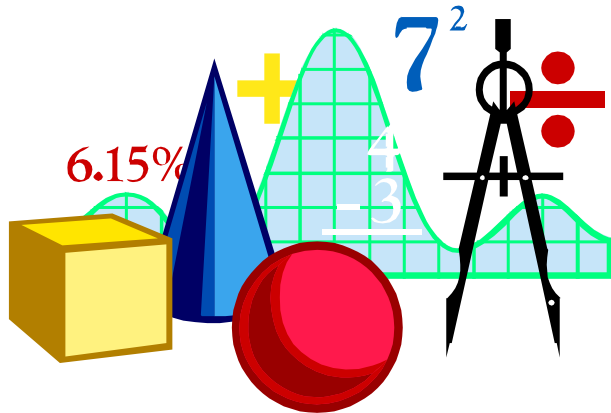
MAT0024C

CHAPTER 3

PROBLEM

SOLVING

6.15%

7^2

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3.1 **Ratios & Proportions**

Ratio = **Quotient of 2 #s or 2 quantities**

[a way to compare numerical quantities]

Ex: $\dfrac{7}{9}$ Ex: 21 to 27   Ex: 35:50

Are any of these ratios equal?

Ex: Express the ratio 3.2 to 16 in lowest terms

Ex: Express the ratio 1 foot to 2 yds in lowest terms
[make units same]

3.1 Ratios, Rates, & Proportions

<u>Unit Ratio</u> = **Ratio w/ denominator = 1**

[ex: cost per pound or cost per oz.]

Ex:　Which is a better buy? 12 oz Coke for 79¢ or a 16 oz Coke for 99¢

<u>Rate</u> = **Compare quantities of different units**

Ex: miles per hour [MPH]

Ex: What is the hourly rate for $640 earned for 40 hrs work?

3.1 Ratios, Rates, & Proportions

Ratio = Comparison by division a to b,

a : b or $\dfrac{a}{b}$ [units the same]

Proportion = 2 equal ratios: [b,d ≠ 0]

$$\dfrac{a}{b} = \dfrac{c}{d}$$ iff [if & only if] **ad = bc**

extremes   means

[Read *a* is to *b* as *c* is to *d*]

Ex: Is $\dfrac{21}{15} = \dfrac{62}{45}$ a proportion? Y or N

3.1 Solving Proportions

Solve equations involving Proportions
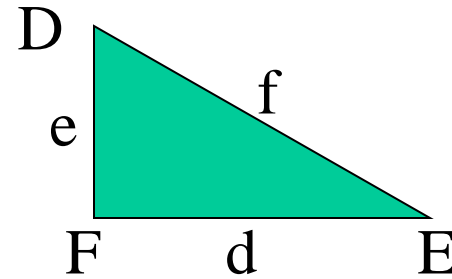
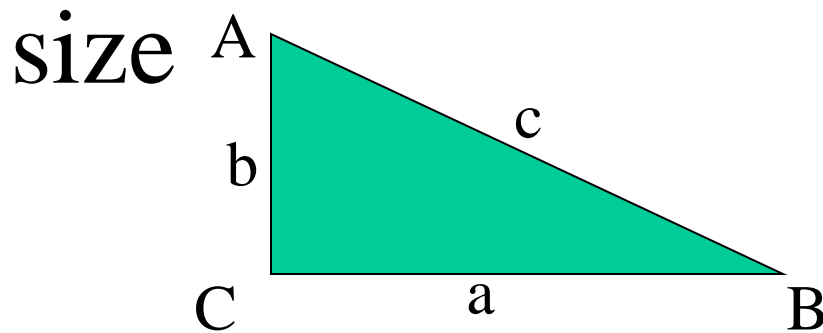Ex $\dfrac{12}{18} = \dfrac{3}{x}$ Ex $\dfrac{a+6}{2a} = \dfrac{1}{5}$

Set up $\left[\; \text{----} \;\right]$ for word problems:

Ex: If 7 shirts cost \$87.50, find the cost of 11 shirts.

3.1 Similar Δs have corresponding sides pptnl

Similar Δ s = same shape; different size

Δ ABC~ Δ DEF means $\dfrac{a}{d} = \dfrac{b}{e} = \dfrac{c}{f}$

Ex: A tree casts a shadow of 18' when a 5' person casts a shadow of 1.5' How tall is the tree?

Note: Similar figures [trapezoids, pentagons, …] also have pptnl sides.

3.2 ~ <u>**% Problems**</u> – Rewriting

Percent: Ratio representing some part of 100

Ex: Write as reduced fraction & as a decimal: $26\% =$

[by 100]

Ex: Write a fraction or decimal as a %: $\dfrac{3}{8} =$

[mult. By 100%] $0.486 =$

3.2 ~ <u>**% Problems**</u> – Use translation & algebra

Translation:

What => n

of => times

is => equals

_ _% => ._ _

What is 28% of 270?

14 is what % of 52?

80 is 20% of what number?

3.2 ~ <u>% of Increase/Decrease</u>

1) **Find amt. of incr. by subtracting initial from final amt.**

2) **Solve for P in proportion:** $\dfrac{P}{100} = \dfrac{amt\_inc.}{initial\_amt.}$

Ex: A pediatrician plots the growth of a child as growing from 27.5 inches to 31.25 inches. What is the percent of increase?

$$\dfrac{P}{100} = \dfrac{3.75}{27.5}$$ ← Part
← Whole

Amt of increase $= 31.25 - 27.5 = 3.75$

What percent of 27.5 is 3.75?

$$27.5P = 375$$

$$\dfrac{27.5P}{27.5} = \dfrac{375}{27.5}$$

$$P = 13.6$$

Answer The percent of increase in growth is 13.6%.

3.3 **Problems** Translate word situation ➜ equation

Steps: **1)** Chose a variable for what you are to find [Write: Let x = _____] Write out facts [w/ your var.], pictures,....

2) Translate prob. to an equation [is ➜ =, etc.]

3) Solve equation [legally w/ properties]

4) Answer question(s) asked [may be more than what x equals]

5) Check [Sub. solution into original and work down 'til both sides of = sign the same. Then √]

3.3 ~ <u>**Problems**</u> Translate word situation ➔ equation

Ex: One positive # is one-third another positive #. The larger

minus the smaller # is equal to 15. Find both #s.

Let x = _____ then _____ = _____

Eq:

Solve:

22.5 & 7.5

Ex: The perimeter of a rectangular frame is 36 cm. The

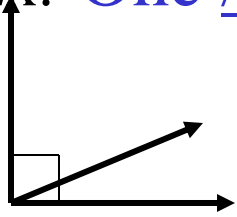length is 3 less than twice the width. Find length & width.
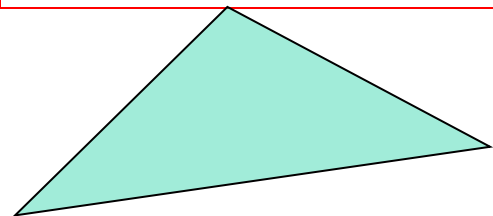
11cm & 7cm

3.3 ~ __Problems__ w/ 2 or more unknowns

Ex: One / is 6° less than 3 times it's complement. Find both /s

Ex: The sum of 3 consecutive integers is 96. Find them. 66° & 24°

31, 32, 33

Ex: Find the meas. of all 3 /s of a Δ if the 2nd / is twice the 1st / and the 3rd is 20 less than the 2nd /

Note: Sum of / measures of a Δ = **180°**

40°, 80°, 60°

3.3 ~ Given a Total in a problem

Let x = one item & Total # - x = other item

Ex: Jon has some $5 bills & $10 bills in his wallet, If he has a total of 16 bills worth a total of $110, how many of each bill is in his wallet.

Note: $ · amt + $ · amt = Total amt

ten $5 bills,
six $10 bills

Mini-Quiz 3.1 ➔ 3.3 ~ SHOW ALL WORK

1) Translate to eq.: What is 60% of 200?   2) Solve 1)

3) Translate to eq.: 75 is 20% of what #? 4) Solve 2)

5-8) Show set up: [---] & use proportions to solve: 5) If a car travels 91 mi. in 2 hrs., how far will it travel in 7 hrs?

6) 2 shirts cost $25, how much will 5 shirts cost?

7) A chef needs 4 large bottles of ketchup to make $2\frac{1}{3}$ gal. of sauce. How many bottles for 10 gal. sauce?

8) If a 5 ft. person casts a 6 ft. shadow, how tall is a building that casts a 30 ft. shadow in the same sun?

9) Translate to eq.: The sum of Sam & Jan's age is 51. Sam is 2 yrs older than 6 times Jan's age. Their ages? 10) Solve 9)

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3.4 ~ % Decrease or % Increase Probs

Decrease: $Orig - %Decr. of $Orig = $Sale
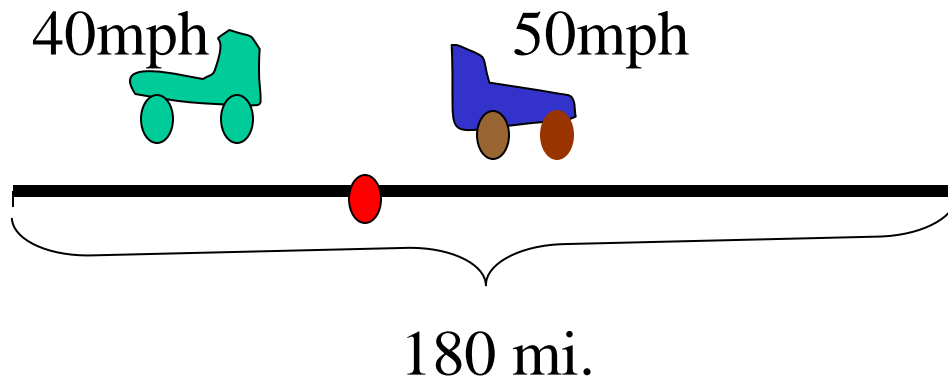
Ex: Find the Original price (x) of a shirt that is now 30% off resulting in a sale price of $28.00

Increase: $Orig + %Incr. of $Orig = $Final

3.4 ~ __Rate Problems__ ~ *DISTANCE*

Distance = Rate • Time **[d = rt]**

Diagram Motion Problems ~ Ex: 2 cars are traveling in opposite directions, 1 @ 40mph & the other @ 50mph. When will they be 180 miles apart?

40mph   50mph

Looking for t

180 mi.

Eq.

3.4 ~ <u>Rate Problems</u>

Note: **d = r·t**

Objects traveling in **<u>opposite</u>** directions: Ex: **2 trains leave the station at the same time in opposite directions. One @ 40 MPH & other @ 55 MPH. In how many hrs will they be 190 miles apart?**
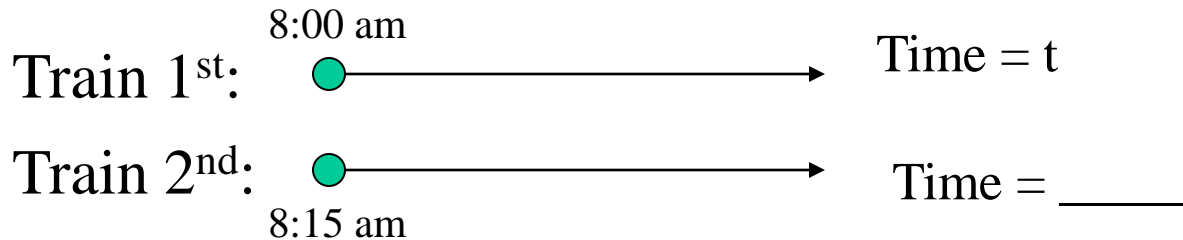
2 hrs

3.4 ~ <u>Rate Problems</u>

Note: $d = r \cdot t$

Objects traveling in **same** direction: Ex:

Two trains are delivering to the same site. One leaves at 8:00 a.m. and the other leaves at 8:15 a.m. If the first train is traveling at 55 mph and the second at 60 mph, at what time will the second catch up to the first?

8:00 am

Train 1st: ●———————————→ Time = t

Train 2nd: ●———————————→ Time = _____

8:15 am

11:00 am

3.5 ~ <u>Investment Problems</u> $I = Prt$

(I)nterest=**(P)**rincipal•**(r)**ate [% as decimal]•**(t)**ime[in yrs]

Ex [easy]: **Plug & Chug:** How much interest on $2000 invested at 5% for 8 years?

Problem below = Extra Credit

Ex: You invest a total of $4000 in two different accounts. The first account earns 6% while the second account earns 4%. If the total interest earned is $210 after one year, what principle was invested in each account?

Accounts	Principal	Rate	Interest
First	$4000 - P$	0.06	
Second	P	0.04	

$$0.06(4000 - p) + 0.04p = 210$$

What can you mult. each term by?

Answer The principal invested in the second account is $1500. In the first account is $4000 − $1500 = $2500.

3.5 ~ <u>Investment Problems</u> $\boxed{\textbf{I = Prt}}$

(I)nterest=**(P)**rincipal•**(r)**ate [% as decimal]•**(t)**ime[in yrs]

Ex: You invest some \$ @ 8% and \$3000 more than twice as much @ 10%. Total annual income from this is \$2540. How much invested @ 8%?

| Principal	Rate	Interest
P	.08	.08P
2P + 3000	.10	.10(2P+3000)

Eq: .08P + .10(2P + 3000) = 2540

What can you mult. each term by?

Ans: \$8000

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Mini-Quiz 3.4 ➔ 3.5 ~ SHOW ALL WORK

1-5) Distance Prob:

Two trains are traveling toward each other from a distance of 208 miles. One train is traveling at 18 miles per hour and the other at 46 miles per hour. How long will it take for them to pass each other?
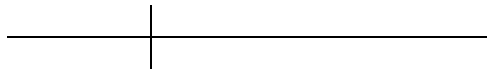
3.25 hours

Problem below = Extra Credit

6-10) Distance Prob: [see page 214 ~ Your Turn 2]

Juan & Angela are bicycling along the same trail. Juan passes a marker @ 9:00 am, and Angela passes the same marker @ 9:05 am. Juan is traveling 8mph while Angela is traveling 10 mph. What time will Angela catch up to Juan?

J: 9:00 am ⟶ Time = t

A: 9:05 am ⟶ Time =_____

9:25 am