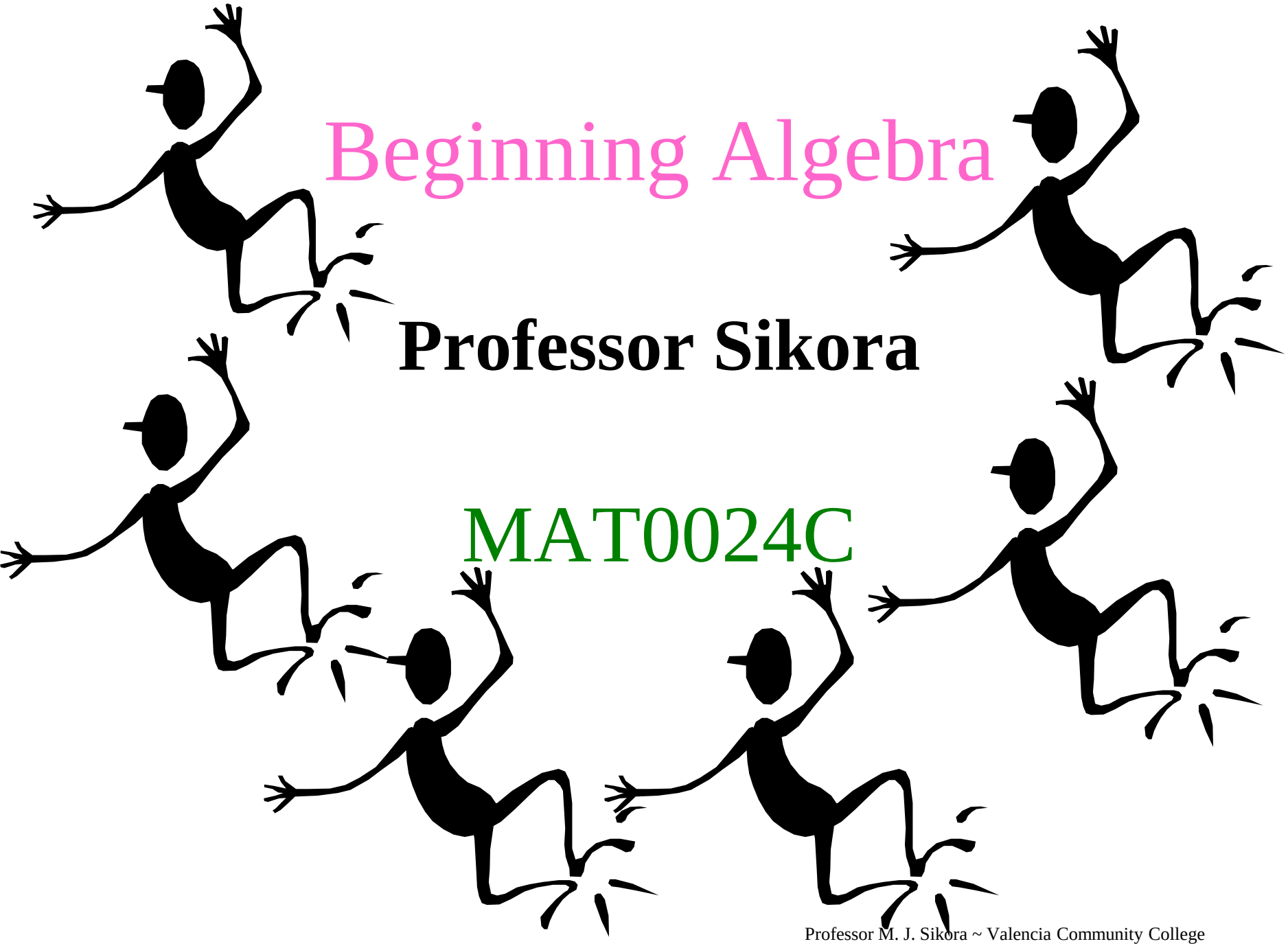
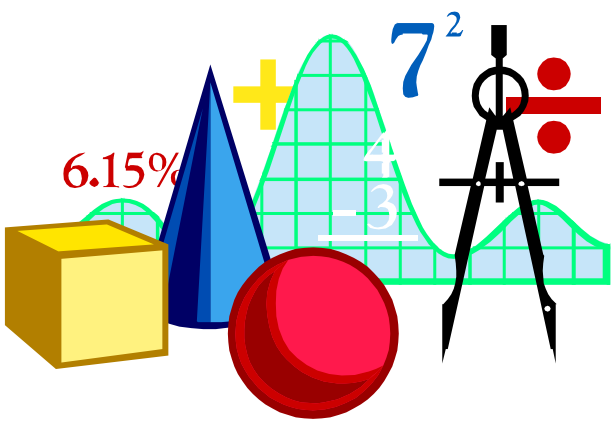
Beginning Algebra



Professor Sikora

MAT0024C

CHAPTER 6



POLYNOMIALS

6.1 Positive Integer Exponents

$x^n = x \cdot x \cdot x \cdot x \cdot x$ [n of these x factors]

base, exponent

Numerical:

Ex: $-3^4 =$ where as Ex: $(-3)^4 =$

Ex: $-3^3 =$ and Ex: $(-3)^3 =$

Rule: Neg # to even exponent = ___#, Neg # to odd exponent = ___#,

Ex: $\left(-\frac{2}{3}\right)^3 =$

6.1 Power Rules for Exponents

POWER OF A QUOTIENT $b \neq 0$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

Ex: Simplify $\left(\frac{3}{x}\right)^3$ $x \neq 0$

Ex: $\left(-\frac{2x^3}{3y^2}\right)^4$

6.1 Zero Exponents

ZERO EXPONENT $[x \neq 0]$

same

$x^0 = 1$ ‘cuz $\frac{4^3}{4^3} = 4^{3-3} = 4^0$ & $\frac{4^3}{4^3} = \frac{4}{4} \bullet \frac{4}{4} \bullet \frac{4}{4} = 1$

Ex: $(-7)^0 =$ _____

Ex: $-7^0 =$ _____

Ex: $-(-7)^0 =$ _____

6.1 Neg. Integer Exponents

NEGATIVE EXPONENT [$a \neq 0$, n integer]

$$a^{-n} = \frac{1}{a^n}$$

'cuz $\frac{4^2}{4^5} = 4^{2-5} = 4^{-3}$ & $\frac{4 \bullet 4}{4 \bullet 4 \bullet 4 \bullet 4 \bullet 4} = \frac{1}{4^3}$

same

Ex: Simplify 4^{-2}

Ex: Simplify -4^{-2}

Ex: Simplify $(-4)^{-2}$

$$\frac{1}{a^{-n}} = a^n$$

Ex: Simplify $\frac{1}{4^{-3}}$

6.1 Neg. Integer Exponents

NEGATIVE to POSITIVE EXPONENTS

[a, b ≠ 0; m,n integers]

$$\frac{x^{-m}}{y^{-n}} = \frac{y^{n}}{x^{m}}$$

Ex: Simplify $\dfrac{4h^{-5}}{m^{-2}k}$

Ex: $\dfrac{5^{-2}}{4^{-3}}$

Ex: $\dfrac{x^{-6}}{8y^{-7}}$

6.1 Scientific Notation: application of exponents

Scientific Notation form: $\mathbf{a \times 10^n}$

$\mathbf{1 \leq |a| < 10 \quad n \in \text{Integers}}$

Move Decimal pt. to right of 1st nonzero digit

Count # of places moved [no move ➔ 10^0]

Large #s have **positive** power of 10

Small #s have **negative** power of 10

Ex: Write in Sci. notation: .0571 75,000

Ex: Write w/o exponents: 8.7×10^5 4.28×10^{-4}

6.2 Defining Polynomials

Polynomial Ex: $3x^3 + 5x^2 + 2x^2 + 8x + 1$

Terms ➔ separated by + or - signs

Coefficient ➔number (w/ sign) in front of var.

Like Terms ➔ same variable to same power

Unlike Terms ➔ diff. variable or diff. power

Polynomial = a term or sum of terms where all variables have whole # exponents

{0,1,2,3,…}

6.2 Classifying Polynomials

Polynomial = Monomial or sum of Monomials

Monomial: a number **or Product**
a variable **of these**

Exponents must be **positive**

Names for Special Polynomials:

Monomial(1 term) Ex: $3y^2$ or $2abc^3$ or -5

Binomial(2 terms) Ex: $3y^2 + 2abc^3$ or $-5+x$

Trinomial (3 terms) Ex: $3y^2 + 2abc^3 - 5$

6.2 Classifying Polynomials

State if each of the following is a polynomial.

If it is, state if it is a **Monomial**, **Binomial**, or **Trinomial**

$3a - 7bc$ _____

$3x^2 + 7x - 4$ _____

$7y^3 - 4y^2 + 2$ _____

$10\,x^3y^2z$ _____

$\dfrac{8}{r^3} + 11r$ _____

6.2 Classifying Polynomials

Degree of a Monomial = *sum* of variables of exponents

Degree of a Polynomial = *greatest* degree of any monomial term

Monomial	Degree	Polynomial	Degree
$3y^2$	2	$3y^2 + 2abc^3$	5
$2abc^3$	___	$y^7 + y^6 + 3x^4m^4$	___
-14	___	$p^5 + p^3m^3 + 4m$	___
$9xyz$	___	$x^2 + xy^2 + 4abc$	___

6.2 Evaluating Polynomial Functions

Evaluate: $-x^3 + x - 2x + 3$

for $x = 0$ ➔

for $x = 1$ ➔

for $x = -3$ ➔

Evaluate: $c^2 + 4c + 7$

for $c = -6$ ➔

6.2 Arranging Polynomials

Polynomials are arranged in powers of one variable: ascending order or descending order

ascending order | descending order

$4 + 5a - 6a^2 + 2a^3$ | $2a^3 - 6a^2 + 5a + 4$

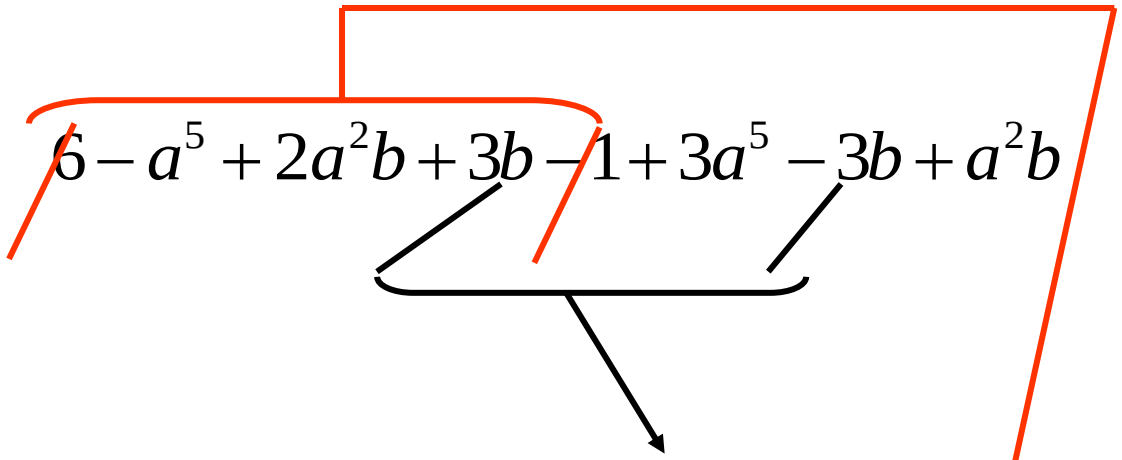
$-5 - 2x + 4x^2$ | **$4x^2 - 2x - 5$**

When several variables are in the terms, write in order of only one variable.

6.2 Combine Like Terms

$$6-a^5+2a^2b+3b-1+3a^5-3b+a^2b$$

Strike through like terms in the given polynomial as they are combined.



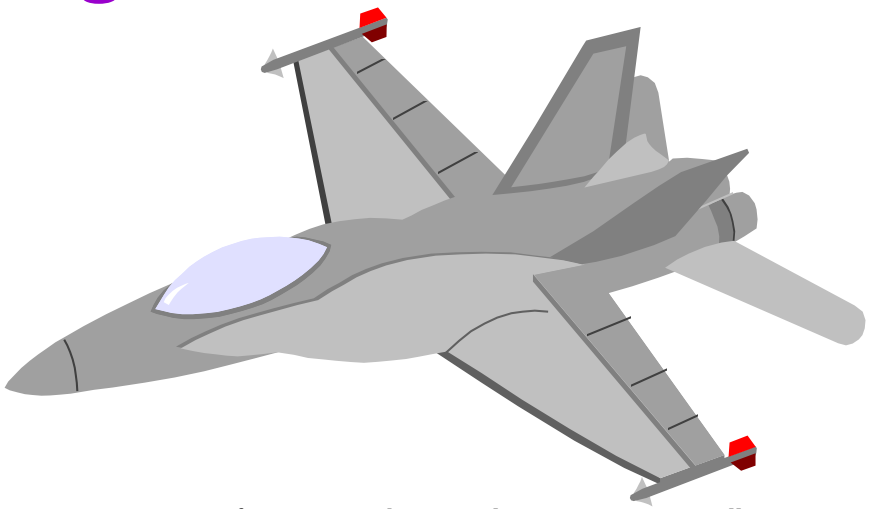
$$= 2a^5 \qquad +3a^2b \qquad +0 \qquad +5$$

$$= \boxed{2a^5+3a^2b+5}$$

6.3 Adding Polynomials

To ADD Polynomials:

- Group LIKE terms together **[LIKE terms have same variable to same power ==> can be combined!]** OR
- Place in COLUMN form **[In DESCENDING order with LIKE terms aligned!]**



6.3 Adding Polynomials

ADD these Polynomials:

Ex: $(9y - 7x + 15a) + (-3y + 8x - 8a)$

Ex: $(3a^2 + 3ab - b^2) + (4ab + 6b^2)$

6.3 Subtracting Polynomials

To SUBTRACT Polynomials:

- Find the Additive Inverse (opposite) of polynomial after (-) sign[◯ original signs & place opposite sign above]
- Group LIKE terms together & Add

OR

- Place in COLUMN form [◯ original signs & place opposite sign above]& Add

6.3 Subtracting Polynomials

SUBTRACT these Polynomials:

Ex: $(7a - 10b) - (3a + 4b)$

Ex: $(3y^2 + 7y + 8) - (2y^2 - 4y + 3)$

Ex: $(-8t^3 + 3t^2 - 7t - 9) - (-10t^3 - 5t^2 + 3t - 11)$

Mini-Quiz 6.1➔6.3

1) Simplify: $\left(-\frac{2y^3z}{3z^2}\right)^4$

2)–3) Classify the Polynomial & state degree

2) $3xy^3$ 3) $2x^3 + x^2 + 1$ 4) Eval: $-2x^2 + 3x - 1$ for $x = -2$

5-7) Simplify: 5) $4(3y^2 - 2y) - y(2y + 4)$

6) $(3x^2 + 2x) + (5x^2 - 8x)$

7) $3(9x^2 + 3x + 7) - 2(11x^2 - 5x + 9)$

8) Subtract: $20x^3 + 12x$ — $12x^3 + 7x^2 - 7x$

9) Copy & Complete the table:

x	2	1	0	-1	-2
4^x					

10a) Write in scientific notation: 728 & 0.00942

10b) Write in standard notation: 7.53×10^4

6.4 Product Rule for Same Base Exponents

PROD. OF POWERS

$$a^m \cdot a^n = a^{m+n}$$

Bases must be the same!

Simplify each [Multiply coefficents 1st]:

$(21c^6)(c^7)$ $(8x^4)(3x)$ $(2a^4)(2a^3b^2)(-3ab^3)$

6.4 Product Rule for Exponents

We multiply numbers in scientific notation using the same procedure we used to multiply monomials.

Monomials:

$$4a^3 \quad 2a^6 = 4 \bullet 2a^{3+6}$$

$$= 8a^9$$

Scientific notation:

$$4\times10^3 \quad 2\times10^6 = 4\times2\times10^{3+6}$$

$$= 8\times10^9$$

Ex: $(4.2 \text{ x } 10^5)(2.8 \text{ x } 10^8)$

Ans: $1.176 \text{ x } 10^{14}$

6.4 **Power of a power** Rule

$\mathbf{(a^m)^n = a^{mn}}$ Ex: $(x^3)^4 = x^3\,x^3\,x^3\,x^3 = x^{12}$

Simplify: Ex: $(2^2)^3 =$ ____ Ex: $(2a^2b^4)^5 =$ _____

Ex: $(-5a^2b^6)^3 =$ _____

Ex: Simplify $(-5mn^4)(-2mp^2)^3(1.5m^2n) =$

Ans: $60m^6n^5p^6$

6.4 Power Rules for Exponents

POWER OF A PRODUCT

$$(ab)^m = a^m b^m$$

Simplify means: No powers of powers, each base only once, & fractions reduced

Ex: $(3x^2y)^3 =$

Ex: $(a^3)^3 (a^4)^2 =$

6.4 Using more than 1 rule

Simplify: $(6b^4y)^2[(-y)^2]^4$

Simplify: $(x^4y^3z^6)^2[(2x^2y^2)^2]^4$

6.5 Multiply Polynomial by Monomial

Remove parentheses & Simplify:

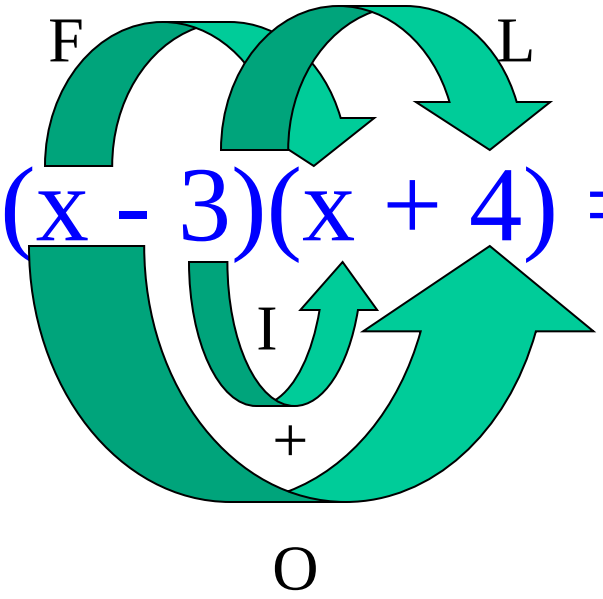
Use distributive Prop w/ ARROWS

Ex: $2(a^2 - 3a) + 5(a^2 + 2a)$

Ex: $5x(x^2 + 2x + 1) - x(x - 3)$

Ex: $-2a^2b \;\; 3a^3b + ab^3 - 5a^4 + 4bc$

6.5 Multiplying Binomial by Binomial



$(x - 3)(x + 4) =$

If 2 Binomials use FOIL - add outer & inner terms under = sign

$(3a + 11)(5a - 2) =$

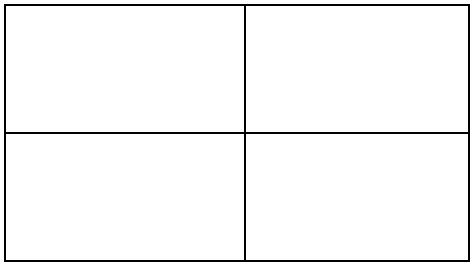
$(5y - 2z)(2y + 3z) =$

$(x + 3)(2x - 1) + 2x(x - 1) =$

6.5 Multiplying Polynomial by Binomial

Use 1 of these 3 Methods:

1. **BOX**



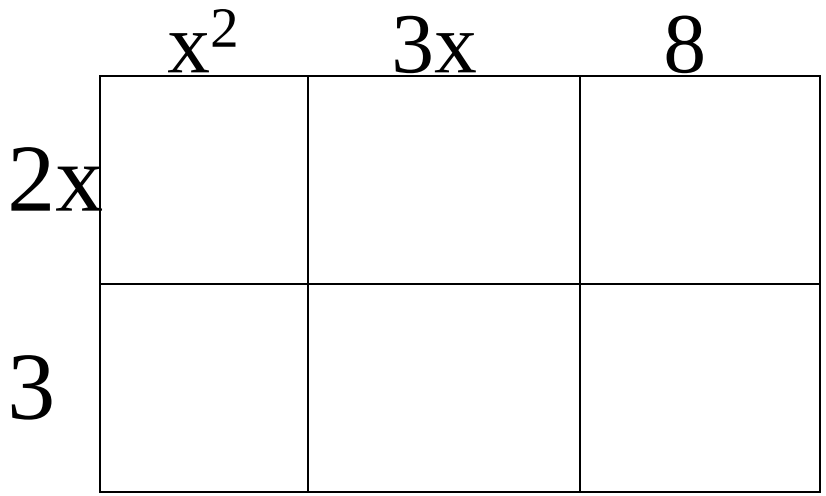
2. **LONG MULTIPLICATION**

3. **MULTIPLE DISTRIBUTIVE**

6.5 Multiplying Polynomial by Binomial

Use 1st of these 3 Methods:

$(2x + 3)(x^2 + 3x + 8) =$



6.5 Multiplying Polynomial by Binomial

Use 2nd of these 3 Methods:

2. LONG MULTIPLICATION

$(2x + 3)(x^2 + 3x + 8) =$

6.5 Multiplying Polynomial by Binomial

Use 3rd of these 3 Methods:

3. MULTIPLE DISTRIBUTIVE

$(2x + 3)(x^2 + 3x + 8) =$

6.5 Multiplying Polynomial by Binomial

Solve using your favorite Method:

1. BOX **2. LONG MULT.** **3. MULTIPLE DISTRIB.**

$(2x - 1)(x^2 - 4x + 3) =$

6.5 Multiplying Conjugates

FOIL

EASY!

$$(a + b)(a - b) = a^2 - b^2$$

Same binomials with different middle signs

$$(x - 3)(x + 3) =$$

$$(x - 2y)(x + 2y) =$$

$$(6x - 20y)(6x + 20y) =$$

6.5 Special Binomial Products

Exs. using **SQUARE OF SUM:**

$$(a + b)^2 = a^2 + 2ab + b^2$$

Sq. 1st term; twice prod. of 2 terms; sq. last term

$$(x + 5)^2 =$$

$$(3a + 2)^2 =$$

$$(5b + 7c)^2 =$$

6.5 Special Binomial Products

SQUARE OF DIFFERENCE: FOIL

$$(a - b)^2 = (a - b)(a - b) = a^2 - 2ab + b^2$$

Sq. 1st term; twice prod. of 2 terms; sq. last term

$(x - 3)^2 =$

$(4a - 2)^2 =$

$(6x^2 - 10y)^2 =$

6.6 Quotient Rule for Exponents

Develop a pattern: $\frac{2^4}{2^1} =$

$\frac{2^5}{2^3} =$

QUOTIENT OF POWERS [a ≠ 0]

$$\frac{a^m}{a^n} = a^{m-n}$$

6.6 Dividing By Monomials

Exs. of **QUOTIENT OF POWERS**:

$$\frac{18x^3y^6z^2}{24x^4y^3} = = \frac{3y^3z^2}{4x}$$

$$\frac{2.34\times10^7}{3.6\times10^3} = = 6.5\times10^3$$

$$\left(\frac{x^{-7}}{x^3}\right)^3 = \frac{1}{x^{30}}$$

Exponents Summary

Assume that no denominators are 0, that a and b are real numbers, and that m and n are integers.

Zero as an exponent: $a^0 = 1$, where $a \neq 0$.

0^0 is indeterminate.

Negative exponents: $a^{-n} = \frac{1}{a^n}$, $\frac{1}{a^{-n}} = a^n$, $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

Product rule for exponents: $a^m \cdot a^n = a^{m+n}$

Quotient rule for exponents: $a^m \div a^n = a^{m-n}$

Raising a power to a power: $\left(a^m\right)^n = a^{mn}$

Raising a product to a power: $(ab)^n = a^n b^n$

Raising a quotient to a power: $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

6.6 Division of Polynomials

÷Polynomial by Monomial: ÷ each term of poly. by mono.

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

$c \neq 0$

÷ Ex: Divide $12m^6 + 18m^5 + 30m^4$ by $6m^2$

Note: Omit Objective 5 pages 454 – 457=>Dividing by a Binomial

Ans: $2m^4 + 3m^3 + 5m^2$

6.6 Division of Polynomials

Ex: $\dfrac{36x^6y^2 - 9x^3y - 6x}{3x^2y}$

$= 12x^4y - 3x - \dfrac{2}{xy}$

Simplify first!

Ex: Divide: $(x + y)^2 - (x - y)^2$ by xy

Mini-Quiz 6.4➔6.6 + Review 1-5 Multiply

1) $(3x^3y^4)(-4x^5y)$

2) $-5x^3(4x^2 + 3x - 6)$

3) $(3a - 4)(5a + 2)$

4) $(5y - 3)^2$

5) $(2x - 1)(5x^2 + 4x - 3)$

6-9 Simplify: 6) $p^{12} \quad p^4$

7) $(2w)^4 - (3w^2)^2$

8) $\dfrac{(q^5)^{-2}}{a^6 \bullet a^{-3}}$

9) $\dfrac{(3z)^{-3}}{(y^5 z^2)^{-1}}$

10) $\dfrac{24w^5 - 12w^2}{4w^2}$