Completing the Square

To **complete the square** for the expression $x^2 + bx$, add $\left(\frac{b}{2}\right)^2$, which is the square of half the coefficient of x. Consequently,

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$$

When solving quadratic equations by completing the square, you must add $\left(\frac{b}{2}\right)^2$ to *both sides* to maintain equality.

Completing the Square: Leading Coefficient is 1

Let's solve the equation $x^2 - 6x + 2 = 0$ by completing the square.

$x^2 + 6x + 2 = 0$	*Original Equation*
$x^2 + 6x = -2$	*Subtract 2 from both sides*
$x^2 + 6x + (3)^2 = -2 + (3)^2$ $\downarrow \nearrow$ $(b/2)^2$	*Divide the 6 by 2, square it, and then add to both sides*
$x^2 + 6x + 9 = 7$	*Simplify*
$(x+3)^2 = 7$	*Perfect square trinomial*
$x + 3 = \pm\sqrt{7}$	*Extract square roots*
$x = -3 \pm \sqrt{7}$	*Solutions*

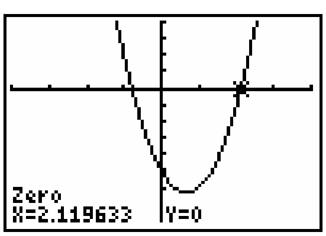
Completing the Square: Leading Coefficient is Not 1

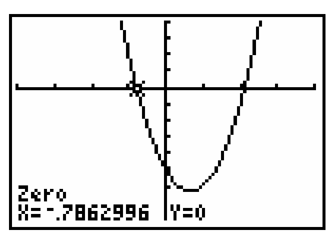
Let's solve the equation $3x^2 - 4x - 5 = 0$ by completing the square.

If the leading coefficient of a quadratic equation is not 1, you should divide both sides of the equation by this coefficient *before* completing the square.

$3x^2 - 4x - 5 = 0$	*Original equation*
$3x^2 - 4x = 5$	*Add 5 to both sides*
$x^2 - \frac{4}{3}x = \frac{5}{3}$	*Divide both sides by 3*
$x^2 - \frac{4}{3}x + \left(-\frac{2}{3}\right)^2 = \frac{5}{3} + \left(-\frac{2}{3}\right)^2$	*Divide* $-\frac{4}{3}$ *by 2, square it, and then add to both sides*
$\downarrow \quad \nearrow$ $(b/2)^2$	
$\left(x - \frac{2}{3}\right)^2 = \frac{19}{9}$	*Perfect square trinomial*
$x - \frac{2}{3} = \pm\frac{\sqrt{19}}{3}$	*Extract square roots*
$x = \frac{2}{3} \pm \frac{\sqrt{19}}{3}$	*Solutions*

Using a graphing calculator, you can see that the two solutions are approximately 2.11963 and –0.78630, which agree with the two graphical solutions shown below.





Completing the Square: One Term is Not Present

Let's solve the equation $4x^2 - 7x = 0$ by completing the square.

As you can see, we have no constant but we will treat the problem the same as if there was a constant present. We skip the step of moving the constant over to the other side of the equation and continue on from there.

$4x^2 - 7x = 0$	*Original equation*
$x^2 - \frac{7}{4}x = 0$	*Divide both sides by 4*
$x^2 - \frac{7}{4}x + \left(-\frac{7}{8}\right)^2 = 0 + \left(-\frac{7}{8}\right)^2$ $\downarrow \quad \nearrow$ $(b/2)^2$	*Divide* $-\frac{7}{4}$ *by 2, square it, and then add to both sides*
$\left(x - \frac{7}{8}\right)^2 = \frac{49}{64}$	*Perfect square trinomial*
$x - \frac{7}{8} = \pm\frac{7}{8}$	*Extract square roots*
$x = \frac{7}{8} \pm \frac{7}{8}$ or $x = 0, \frac{7}{4}$	*Solutions*