Domain & Range

A. The Domain of a Function

"The Domain of a function is the set of all the allowable x or input values."

By using the word "allowable" one could assume there are some values which are simply not allowed into the function.

Think of a function as a mathematical machine that "does" something. For instance, f(x) = 2x + 1 is a machine that first multiplies whatever you put into it by two, and then adds one.

So, this machine sits in the corner with its door open waiting for you to pick up the shovel and shovel "stuff" into it. All the "stuff" you put into it is the DOMAIN.

B. There are TWO RESTRICTIONS on the domain.

1. WE DO NOT ALLOW ANY VALUES OF X WHICH WOULD CAUSE US TO DIVIDE BY ZERO.

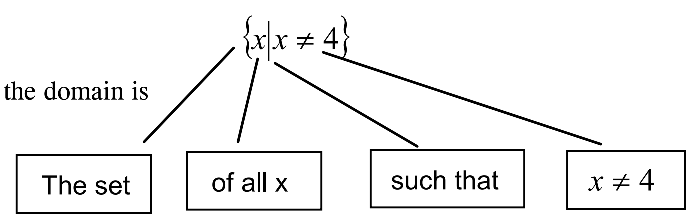
Sometimes we can determine the domain by inspection (just by looking at it).

Example: Find the domain of the following function:

$$f(x) = \frac{3x}{x-4}$$

Can you see that if we let x = 4 the denominator will be zero?

So the domain is



Sometimes it is too difficult to determine the domain by inspecting, so we use a kind of reverse logic. We set the denominator equal to zero and solve, in order to find the values of x which would make the bottom zero. We then DO NOT LET THESE VALUES OF X INTO THE DOMAIN of the function.

Example: Find the domain of the following function:

$$f(x) = \frac{3x}{x^2 - 4x - 12}$$

We can't let the bottom equal zero, so we will SET THE BOTTOM EQUAL TO ZERO TO FIND THE VALUES OF X WE WILL EXCLUDE FROM THE DOMAIN.

$$X^2 - 4X - 12 = 0$$

$$(X - 6)(X+2) = 0$$

$$X - 6 = 0 \text{ and } X + 2 = 0$$

$$X = 6 \qquad X = -2$$

These values of x would make us divide by zero, so we don't let them into the domain.

Domain = $\{x | x \neq 6, x \neq -2\}$

2. The second restriction involves taking the even root (like square root) of a negative number.

WE DO NOT ALLOW ANY VALUE OF X WHICH WOULD CAUSE US TO TAKE THE SQUARE ROOT OF A NEGATIVE NUMBER.

(Note: It is not that we cannot take the square root of a negative number --- we did that earlier and used i for the square root of a negative one. It is that the result, an imaginary number, is not something we can use for our present application of graphing, since graphing deals with distances.)

Follow my logic chain through the next example:

Example: Find the domain of the following:

$$f(x) = \sqrt{6 - 3x}$$

Since we are not allowed to take the square root of a negative number, 6-3x **CANNOT BE NEGATIVE.**

What is the implication of that? Since the 6 - 3X cannot be negative, then **IT CAN BE POSITIVE OR ZERO.**

The way mathematicians would say, "6 - 3X can be positive or zero." Is

$$6 - 3X \geq 0$$

Solve this to FIND THE VALUES YOU WOULD INCLUDE IN THE DOMAIN.

$$6 - 3X \geq 0$$

$$-3X \geq -6$$

$$X \leq 2$$

Therefore, the Domain = $\{x | x \leq 2\}$ or,
In interval notation that would be $(-\infty, 2]$.

VERY IMPORTANT INSIGHT: Notice that our two restrictions involve division and radicals. If your particular problem has no division and contains no radicals, the Domain is the set of all real numbers, you don't have to consider further.

C. The Range of a Function

"The Range of the function is the set of resulting y or output values."

I've emphasized the word "resulting" to call your attention to the fact that you first determine what can go into a function (the Domain) and then the function produces an output --- the Range.

Though thee are several ways in which to determine the range of a given function, we will only look at the one method, one which emphasizes the concept of the Range.

Example: Find the range of the following function:

$$f(x) = x^2 - 3$$

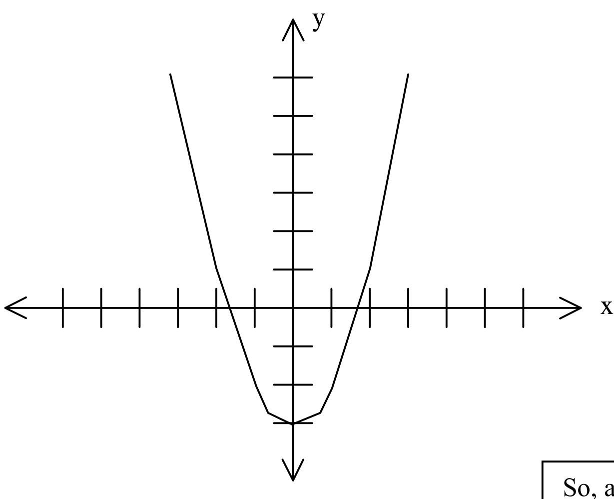
First, note that the domain is the set of all real numbers. We'll let any real numbers into this function. We don't need to worry about the denominator becoming zero, since there is no division. We also don't need to worry about taking the square root (or even root) of a negative number since this function does not involve radicals.

We'll make a table of values and then analyze that table according to the definition of the Range given above.

This is a sample of the domain of all real numbers.

x	f(x)
-3	6
-2	1
-1	-2
0	-3
1	-2
2	1
3	6

The Range is an attempt to describe this set of numbers --- a sample of the numbers we get back from the function.



We may make the following observations:

If we let x become more negative, f(x) will get larger and larger.

If we let x become more positive, f(x) will get larger and larger.

And, it looks as if the smallest value f(x) will ever be is -3.

So, a description of the f(x) values might be $[-3, \infty)$.

That is the Range: $[-3, \infty)$.