Exponent Properties

1. **Product of like bases:** $a^m a^n = a^{m+n}$

   To multiply powers with the same base, add the exponents and keep the common base.

   Example: $x^5 x^3 = x^{5+3} = x^8$

2. **Quotient of like bases:** $\dfrac{a^m}{a^n} = a^{m-n}$

   To divide powers with the same base, subtract the exponents and keep the common base.

   Example: $\dfrac{x^5}{x^3} = x^{5-3} = x^2$

3. **Power to a power:** $(a^m)^n = a^{mn}$

   To raise a power to a power, keep the base and multiply the exponents.

   Example: $\left(x^5\right)^3 = x^{5*3} = x^{15}$

4. **Product to a power:** $(ab)^m = a^m b^m$

   To raise a product to a power, raise each factor to the power.

   Example: $\left(x^4 y^5\right)^3 = x^{12} y^{15}$

5. **Quotient to a power** $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

   To raise a quotient to a power, raise the numerator and the denominator to the power.

   Example: $\left(\dfrac{x^3}{y^2}\right)^4 = \dfrac{x^{12}}{y^8}$

6. **Zero Exponent:** $a^0 = 1$

   Any number raised to the zero power is equal to “1”.

   Example: $(8x^4)^0 = 1$

7. **Negative exponent:** $a^{-n} = \dfrac{1}{a^n}$ or $\dfrac{1}{a^{-n}} = a^n$

   Negative exponents indicate reciprocation, with the exponent of the reciprocal becoming positive. You may want to think of it this way: unhappy (negative) exponents will become happy (positive) by having the base/exponent pair “switch floors”!

   Example: $8^{-2} = \dfrac{1}{8^2} = \dfrac{1}{64}$ *or* $\dfrac{4}{x^{-3}} = 4x^3$

Common Algebraic Errors

Error	Reason/Correct/Justification/Example
$\frac{2}{0} \ne 0$ and $\frac{2}{0} \ne 2$	Division by zero is undefined!
$-3^2 \ne 9$	$-3^2 = -9$, $(-3)^2 = 9$ Watch parenthesis!
$(x^2)^3 \ne x^5$	$(x^2)^3 = x^2 x^2 x^2 = x^6$
$\frac{a}{b+c} \ne \frac{a}{b} + \frac{a}{c}$	$\frac{1}{2} = \frac{1}{1+1} \ne \frac{1}{1} + \frac{1}{1} = 2$
$\frac{1}{x^2+x^3} \ne x^{-2} + x^{-3}$	A more complex version of the previous error.
$\frac{\cancel{a} + bx}{\cancel{a}} \ne 1 + bx$	$\frac{a+bx}{a} = \frac{a}{a} + \frac{bx}{a} = 1 + \frac{bx}{a}$ Beware of incorrect canceling!
$-a(x-1) \ne -ax - a$	$-a(x-1) = -ax + a$ Make sure you distribute the "-"!
$(x+a)^2 \ne x^2 + a^2$	$(x+a)^2 = (x+a)(x+a) = x^2 + 2ax + a^2$
$\sqrt{x^2+a^2} \ne x + a$	$5 = \sqrt{25} = \sqrt{3^2+4^2} \ne \sqrt{3^2} + \sqrt{4^2} = 3 + 4 = 7$
$\sqrt{x+a} \ne \sqrt{x} + \sqrt{a}$	See previous error.
$(x+a)^n \ne x^n + a^n$ and $\sqrt[n]{x+a} \ne \sqrt[n]{x} + \sqrt[n]{a}$	More general versions of previous three errors.
$2(x+1)^2 \ne (2x+2)^2$	$2(x+1)^2 = 2(x^2+2x+1) = 2x^2 + 4x + 2$ $(2x+2)^2 = 4x^2 + 8x + 4$ Square first then distribute!
$(2x+2)^2 \ne 2(x+1)^2$	See the previous example. You can not factor out a constant if there is a power on the parethesis!
$\sqrt{-x^2+a^2} \ne -\sqrt{x^2+a^2}$	$\sqrt{-x^2+a^2} = (-x^2+a^2)^{\frac{1}{2}}$ Now see the previous error.
$\frac{a}{\left(\frac{b}{c}\right)} \ne \frac{ab}{c}$	$\frac{a}{\left(\frac{b}{c}\right)} = \frac{\left(\frac{a}{1}\right)}{\left(\frac{b}{c}\right)} = \left(\frac{a}{1}\right)\left(\frac{c}{b}\right) = \frac{ac}{b}$
$\frac{\left(\frac{a}{b}\right)}{c} \ne \frac{ac}{b}$	$\frac{\left(\frac{a}{b}\right)}{c} = \frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{1}\right)} = \left(\frac{a}{b}\right)\left(\frac{1}{c}\right) = \frac{a}{bc}$