

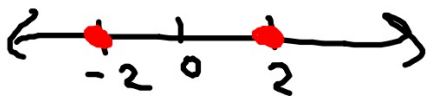
## 1.6 Equations and inequalities involving absolute values.

### I. Absolute value equations.

Ex: Solve

$$|x| = 2$$

$$x = -2 \text{ or } x = 2$$



Rule: If  $|u| = k$ ,  
then  $u = -k$  or  $u = k$ .

Ex: Solve.

$$\textcircled{1} \quad |2x - 3| = 7$$

$$2x - 3 = -7 \quad \text{or} \quad 2x - 3 = 7$$

$$2x = -4$$

$$2x = 10$$

$$x = -2$$

or

$$x = 5$$

$$\{-2, 5\}$$

$$\cancel{(-2, 5)}$$

$$\textcircled{2} \quad |2x-3| + 2 = 7$$

$\quad \quad \quad -2 \quad \quad -2$

$$|2x-3| = 5$$

$$2x-3=5$$

$+3 \quad +3$

$$2x=8$$

$$x=4$$

$$2x-3=-5$$

$+3 \quad +3$

$$2x=-2$$

$$x=-1$$

or

## II. Absolute value inequalities

i) less than  $<$  or  $\leq$

Ex: Solve

$$|x| \leq 2$$



$$[-2, 2]$$

$$-2 \leq x \leq 2$$

$$x \geq -2 \text{ and } x \leq 2$$

Rule: If  $|u| \leq K$ , then

$$-K \leq u \leq K$$

Ex: Solve. Write solution using interval notation and graph.

$$\textcircled{1} \quad |2x+5| < 7$$

$$-7 < 2x+5 < 7$$

$$-12 < 2x < 2$$

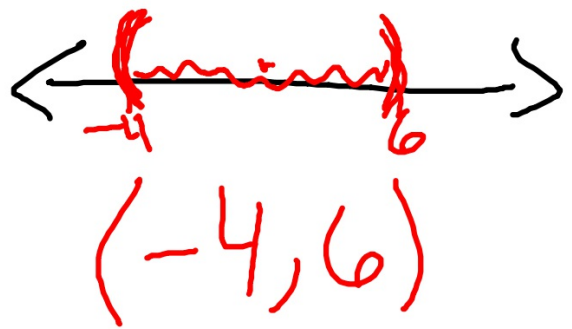
$$-6 < x < 1$$



$$(-6, 1)$$

$$\textcircled{2} \quad 3 + |1 - x| \leq 8$$

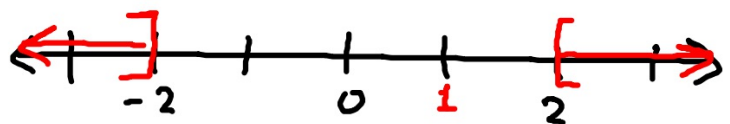
$$\begin{aligned} |1 - x| &\leq 5 \\ 5 \wedge 1 - x &\leq 5 \\ 6 \wedge 1 - x &\leq 4 \\ 6 > x &> -4 \end{aligned}$$



ii) greater than  $\geq$  or  $>$

Ex: Solve

$$|x| \geq 2$$



$$x \leq -2 \text{ or } x \geq 2$$

$$(-\infty, -2] \cup [2, \infty)$$

Rule: If  $|u| \geq K$ , then

$$u \leq -K \quad \text{or} \quad u \geq K$$

Ex: Solve. Write solution using interval notation and graph.

①  $\frac{1}{2} + |2x - 1| \geq \frac{5}{2}$

$-\frac{1}{2}$   $\frac{1}{2}$

$|2x - 1| \geq 2$

$2x - 1 \leq -2$  or  $2x - 1 \geq 2$

$2x \leq -1$   $2x \geq 3$

$x \leq -\frac{1}{2}$  or  $x \geq \frac{3}{2}$

$(-\infty, -\frac{1}{2}] \cup [\frac{3}{2}, \infty)$

