2.2 Graphs of equations in 2 variables

Intercepts ; Symmetry.

I. Graphs of equations in 2 variables.

Equations in 2 variables

$3x + 3y = 3$ $\qquad$ $x^2 + r^2 = 32$

$5b + 4s = 3$

Note:

① Any values of x and y that make the equation true is said to satisfy the equation.

↳ solution

② If any point satisfies an equation then that point lies on the graph represented by the equation.

Ex: Determine which of the given points are on the graph of the given equation.

a) $y = x^4 - \sqrt{x}$

① $(\overset{x}{0}, \overset{y}{0})$ $\boxed{\text{yes}}$

$0 = (0)^4 - \sqrt{0}$

$0 = 0$

② $(1, 1)$ NO

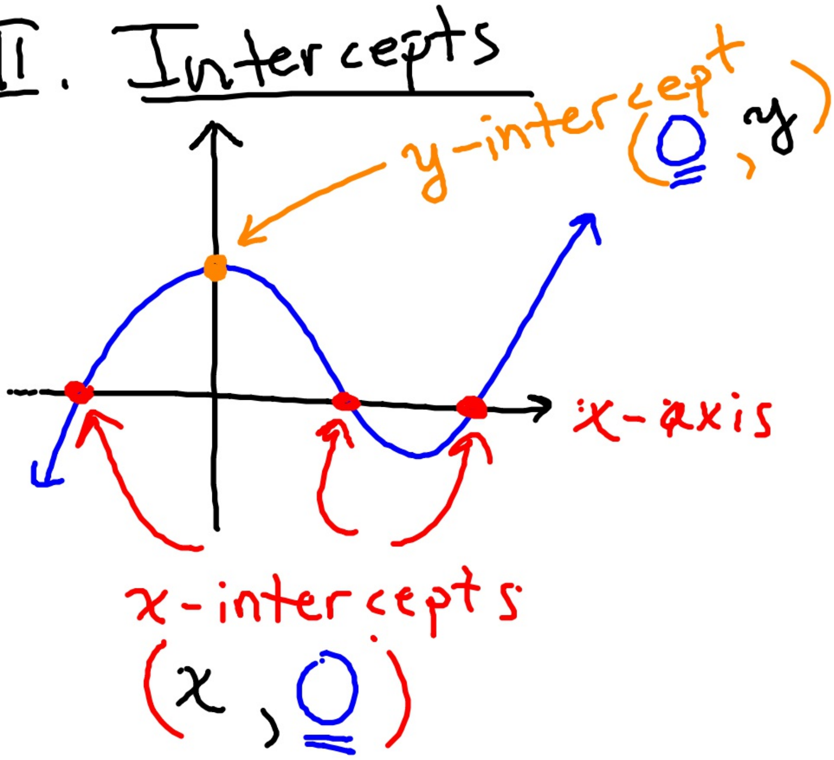
$1 = 0$

③ $(-1, 0)$ NO

$0 = (-1)^4 - \sqrt{-1}$

$0 = $ N.R.N

II. Intercepts



Finding intercepts

① To find x-int set $y = 0$ and solve for x.

② To find y-int set $x = 0$ and solve for y.

Ex: Find the intercepts and graph.

$$4x^2 + y^2 = 36$$

x-int: $y = 0$

$$4x^2 + (0)^2 = 36$$

$$\frac{4x^2}{4} = \frac{36}{4}$$

$$x^2 = 9$$

$$x = \pm 3$$

$(3,0)$ and $(-3,0)$

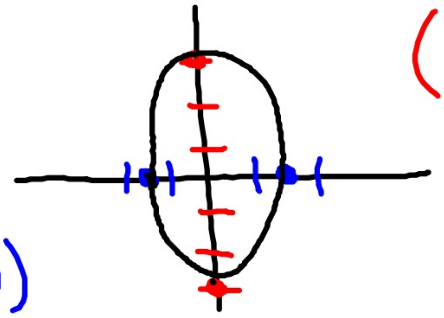
y-int: $\boxed{x = 0}$

$$4(0)^2 + y^2 = 36$$

$$y^2 = 36$$

$$y = \pm 6$$

$(0,-6)$ and $(0,6)$



How to graph:

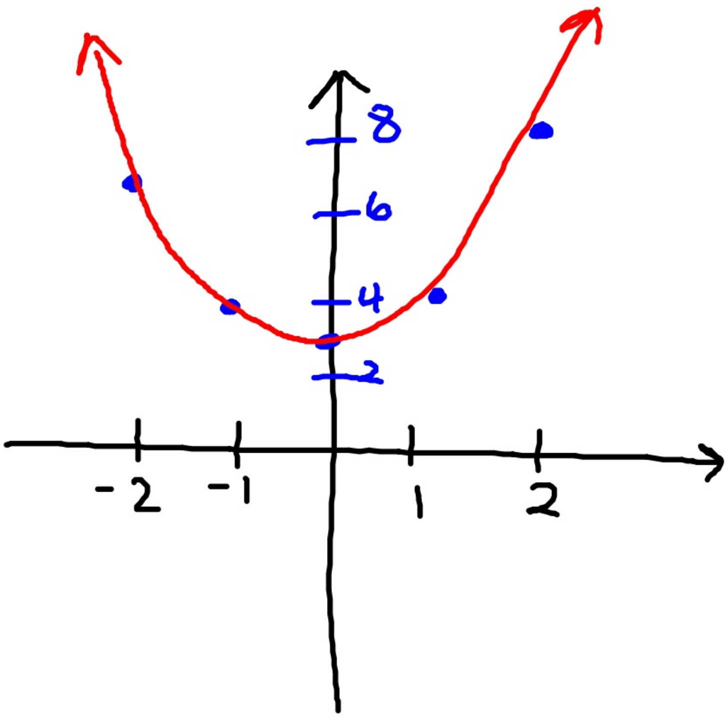
① By plotting:

$$y = x^2 + 3$$

x	y
-2	7
-1	4
0	3
1	4
2	7

$y = (-2)^2 + 3$

$y = 7$



(2) Using graphing calculator

$$4x^2 + y^2 = 36$$

$$\sqrt{y^2} = \sqrt{36 - 4x^2}$$

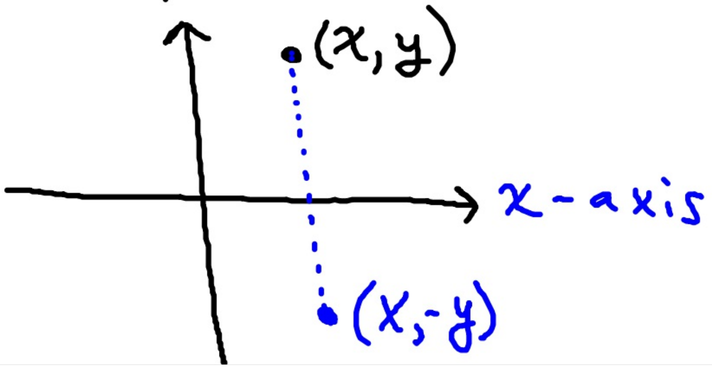
$$y = \pm\sqrt{36 - 4x^2}$$

$$y_1 = -\sqrt{36 - 4x^2}$$

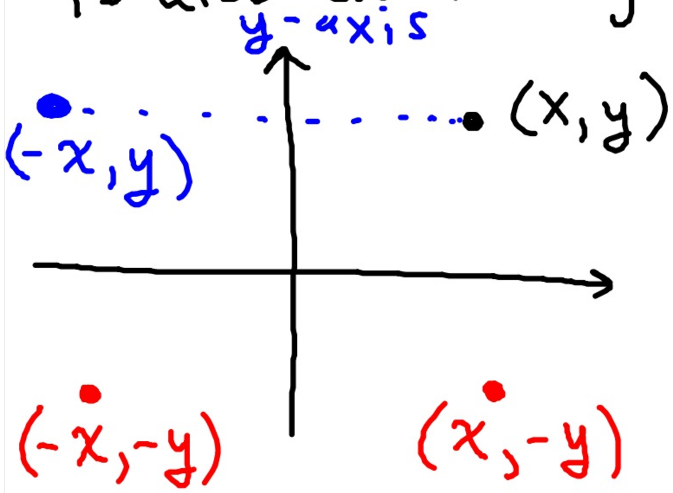
$$y_2 = \sqrt{36 - 4x^2}$$

III. Symmetry

① A graph is symmetric with respect to (w.r.t) the x-axis if, for every point (x, y) on the graph, the point $\underline{(x, -y)}$ is also on the graph



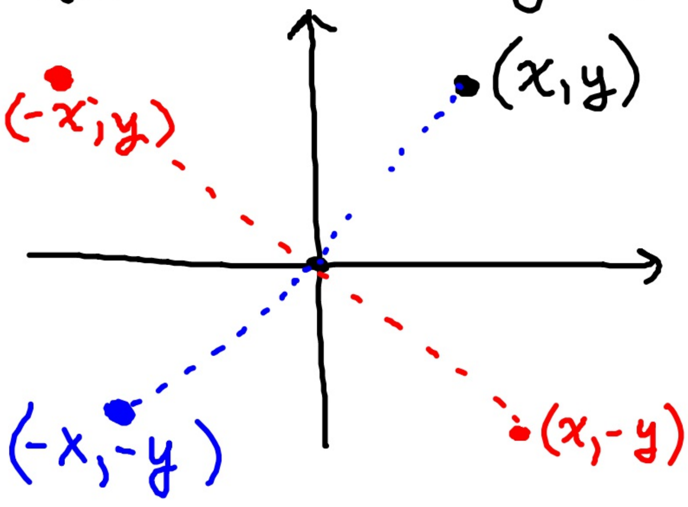
② A graph is symmetric w.r.t the y-axis if for every point (x, y) on the graph, the point $\underline{(-x, y)}$ is also on the graph.



y's are same

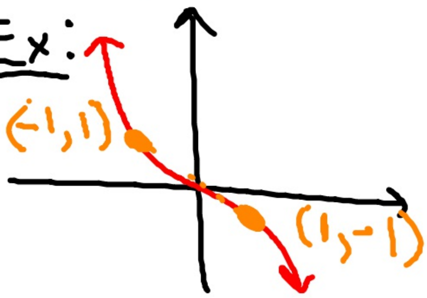
x's are opposites.

③ A graph is symmetric w.r.t the origin if for every point (x,y) on the graph, the point $\underline{(-x,-y)}$ is also on the graph.



both x and y are opposites.

Ex:



Testing for symmetry

(x, y) & $(x, -y)$

① x-axis symmetry:

Replace y's with $-y$'s and simplify the equation. If the simplified equation is $\boxed{\text{equivalent}}$ to original equation then it is sym. w.r.t the x-axis.

Ex: $-3x+4y^2=10x^2$

$-3x+4(-y)^2=10x^2$

$-3x+4y^2=10x^2$

equivalent

$\therefore$ sym. w.r.t x-axis

② y-axis symmetry. (x,y) & $(-x,y)$

Replace x's with $-x$'s ...... y-axis

③ origin: (x,y) and $(-x,-y)$

Replace x's with $-x$'s

and y's with $-y$'s ...... origin

Ex: Test for x-axis, y-axis, and origin symmetry.

$$y = \frac{-x}{x^2+5}$$

$\underline{x\text{-axis}}$: $-y \to y$'s

$$-1(-y) = \frac{-1}{1} \cdot \frac{-x}{x^2+5}$$

$\boxed{\text{Not sym.}}$

$$y = \frac{x}{x^2+5}$$

$\underline{y\text{-axis}}$: $-x \to x$

$$y = \frac{-(-x)}{(-x)^2+5}$$

$$y = \frac{x}{x^2+5} \to \text{not equivalent}$$

$\therefore$ not sym w.r.t y-axis

③ origin: $-x \to x$, $-y \to y$

$$-y = \frac{-(-x)}{(-x)^2+5}$$

$$-1(-y) = \frac{-1}{1} \cdot \frac{x}{x^2+5}$$

$$y = \frac{-x}{x^2+5}$$ ← equivalent

∴ sym w.r.t origin.