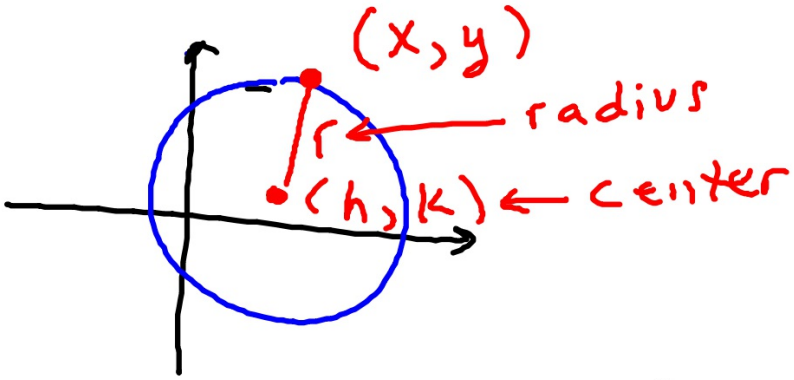
Section 2.4: Circles

A circle is a set of points in the xy-plane that are a fixed distance r from a fixed point (h, k).

The fixed distance r is called the radius, and the fixed point (h, k) is called teh center of the circle.



distance formula from (h,k) to (x,y)

$$(r)^2 = \left(\sqrt{(x-h)^2 + (y-k)^2}\right)^2$$

$$(x-h)^2 + (y-k)^2 = r^2$$

standard form of eqn. of a circle

When center is on the origin $(\overset{h}{0},\overset{k}{0})$

$$x^2 + y^2 = r^2$$

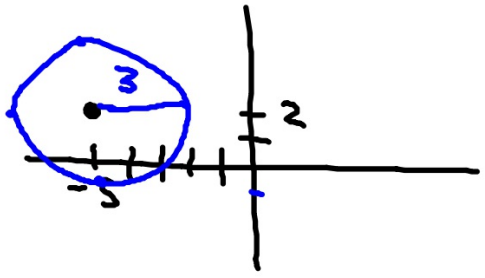
when center is on origin & radius = 1

$$x^2 + y^2 = 1$$ ← unit circle

Ex: Write the standard equation of a circle with radius $\overset{r}{3}$ and whose center is $(\overset{h}{-5},\overset{k}{2})$

$$(x-(-5))^2 + (y-2)^2 = 3^2$$

$$(x+5)^2 + (y-2)^2 = 9$$



Ex: Find the center and radius of the circle with the given equation.

i) $(x-2)^2 + (y+3)^2 = 81$

$(x-\underset{h}{2})^2 + (y-\underset{k}{-3})^2 = \underset{r}{9^2}$

center $(2,-3)$ radius: 9

ii) Find the intercepts.

$(0-2)^2 + (y+3)^2 = 81$

$4 + (y+3)^2 = 81$

$(y+3)^2 = 77$

$$(y+3)^2 = 77$$

$$y+3 = \pm\sqrt{77}$$

$$\boxed{y = -3 \pm \sqrt{77}}$$

$$(x-2)^2+(y+3)^2=81$$

$$(x-2)^2+(3)^2=81$$

$$(x-2)^2+9=81$$

$$\sqrt{(x-2)^2}=\sqrt{72}$$

$$x-2=\pm\sqrt{72}$$

$$x=2\pm\sqrt{72}$$

$$\boxed{x=2\pm 6\sqrt{2}}$$

$\sqrt{2}\sqrt{4}\sqrt{9}$ (2·9)

$\sqrt{2}\cdot 2\cdot 3$

General Form of an equation of a circle.

$$x^2 + y^2 + ax + by + c = 0$$

it is possible to have a circle, a point, or no circle at all.

a point $x^2 + y^2 = 0$ $(0,0)$

no circle at all $x^2 + y^2 = -5$

Ex: Graph the equation by finding the center and radius of the circle.

$$x^2 + y^2 + 4x - 6y + 12 = 0$$

$$x^2+y^2+4x-6y+12=0$$

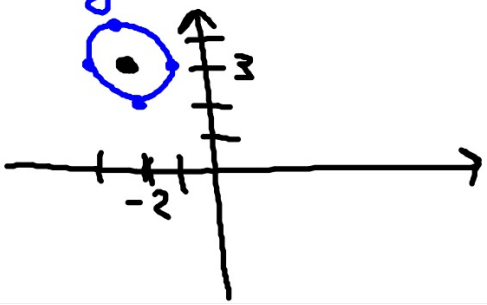
$$x^2+4x+\underbrace{4}_{\left(\frac{4}{2}\right)^2}+y^2-6y+\underbrace{9}_{\left(\frac{-6}{2}\right)^2}=-12+4+9$$

$$\underbrace{(x^2+4x+4)}+\underbrace{(y^2-6y+9)}=1$$

$$(x+2)^2+(y-3)^2=1$$

center: $(-2,3)$

radius: 1



Ex: Graph

$$\frac{2x^2}{2} + \frac{2y^2}{2} - \frac{12x}{2} + \frac{8y}{2} - \frac{24}{2} = \frac{0}{2}$$

$$x^2 + y^2 - 6x + 4y - 12 = 0$$

$$x^2 - 6x + 9 + y^2 + 4y + 4 = 12 + 4 + 9$$

$$(x-3)^2 + (y+2)^2 = 25$$

center $(3, -2)$

$r = 5$